United Kingdom Mathematics Trust

Junior Olympiad Past Paper

1999 – 2024 Collection

www.CasperYC.club

Last updated: September 26, 2025

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

Comments and suggestions to DrYuFromShanghai@QQ.com .

A1. What is the sum of the first nine primes?

A2. Forty-two cubes of side-length 1 cm are stuck together to form a solid cuboid. The perimeter of the base of the cuboid is 16 cm. What is its height, in cm?

A3. $PQRS$ is a square which has been divided into four regions: two identical rectangles, one square of area $9\,\text{cm}^2$ and a second square of area $25\,\text{cm}^2$. What is the area of square $PQRS$, in cm^2?

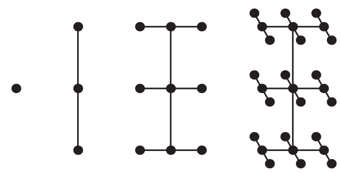
A4. Note that $49 = 4 \times 9 + 4 + 9$. How many two-digit numbers are equal to the product of their digits plus the sum of their digits?

A5. The difference between an interior angle of a regular polygon and an exterior angle of the same polygon is $150°$. How many sides does the polygon have?

A6. When a group of five friends met up, Alice shook hands with one person; Bill shook hands with two people; Cara shook hands with three people; Dhriti shook hands with four people. How many people did Erin shake hands with?

A7. The time is 20:24 (expressed in 24-hour time). What is the angle between the hour hand and the minute hand on an accurate analogue clock, in degrees?

A8. We make a sequence of diagrams. The first diagram consists of a single node. The second diagram is made from the first diagram by drawing two edges of length 1 cm from that node, and putting a node at the other end of each new edge. After that, we make each new diagram from the previous diagram by adding two new edges to each node, with these new edges each having half the length of the edges that were added in the previous diagram. We also attach a node to the end of each new edge. The first four diagrams are shown. Find the total length of all the edges in the fifth diagram, in cm.



A9. The numbers x and y satisfy the equations:

$$xy = \frac{7}{6} \qquad x(y+1) = \frac{5}{3} \qquad y(x+1) = \frac{7}{2}$$

What is the value of $(x+1)(y+1)$?

A10. What is the last digit of $2^{(2^{2024})}$?

B1. What is the smallest positive integer that only contains the digits 0 and 1, and is divisible by 36?

B2. Natasha and Rosie are running at constant speeds in opposite directions around a running track. Natasha takes 70 seconds to complete each lap of the track and meets Rosie every 42 seconds.

How long does it take Rosie to complete each lap?

B3. The positive integers from 1 to n ($n \geq 2$) inclusive are to be spaced equally around the circumference of a circle so that:

(a) no two even numbers are adjacent;

(b) no two odd numbers are adjacent;

(c) no two numbers differing by 1 are adjacent.

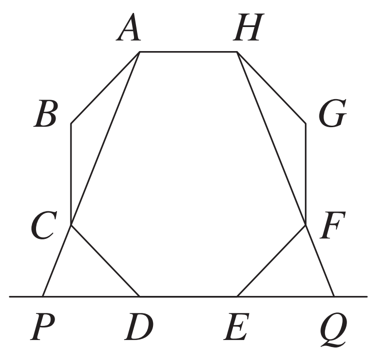
What is the smallest value of n for which the above is possible?

B4. My piggy bank contains x pound coins and y pennies and rattles nicely. If instead it contained y pound coins and x pennies, then I would only have half as much money.

What is the smallest amount of money my piggy bank could contain?

B5. A regular octagon $ABCDEFGH$ has sides of length 1. The lines AC and HF meet the line going through D and E at P and Q respectively.

What is the length of the line PQ?



B6. Let A be the set of $2n$ positive integers $1, 2, 3, \ldots, 2n$, where $n \geq 1$.

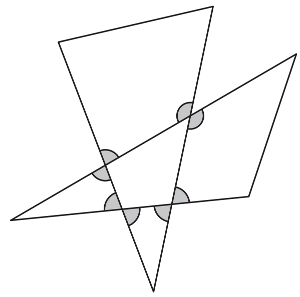
For which values of n can this be split into n pairs of integers in such a way that every pair has a sum which is a multiple of 3?

As always in Olympiad problems such as this, you also need to explain why no other values of n are possible.

A1. What is the integer nearest to $\frac{59}{13}$?

A2. What is the solution of the equation $24 \div (3 \div 2) = (24 \div 3) \div m$?

A3. Two triangles are drawn so that they overlap as shown. What is the sum of the marked angles?

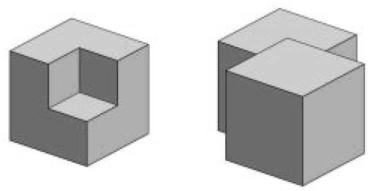


A4. What is the value of $\frac{(1^2+1)(2^2+1)(3^2+1)}{(2^2-1)(3^2-1)(4^2-1)}$? Give your answer in its simplest form.

A5. A number line starts at -55 and ends at 55. If we start at -55, what percentage of the way along is the number 5.5?

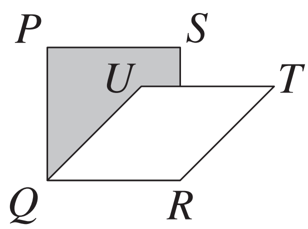
A6. Tea and a cake cost £4.50. Tea and an éclair cost £4. A cake and an éclair cost £6.50. What is the cost of tea, a cake and an éclair?

A7. A $2 \times 2 \times 2$ cm cube has a $1 \times 1 \times 1$ cm cube removed from it to form the shape shown in the left-hand diagram. One of these shapes is inverted and put together with a second of the shapes on a flat surface, as shown in the right-hand diagram.
What is the surface area of the new shape?



A8. Alex chooses three from the six primes 2003, 2011, 2017, 2027, 2029 and 2039.
The mean of his three primes is 2023.
What is the mean of the other three primes?

A9. The diagram shows the square $PQRS$, which has area $25\,\text{cm}^2$, and the rhombus $QRTU$, which has area $20\,\text{cm}^2$.
What is the area of the shaded region?



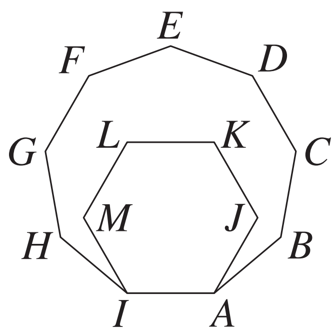
A10. What is the remainder when $\overbrace{23\cdots\cdots23}^{\text{twenty-three 23s}}$ is divided by 32?

B1. The sum of four fractions is less than 1. Three of these fractions are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{10}$. The fourth fraction is $\frac{1}{n}$, where n is a positive integer. What values could n take?

B2. Laura went for a training ride on her bike. She covered the first 10% of the total distance in 20% of the total time of the ride. What was the ratio of her average speed over the first 10% of the distance to her average speed over the remaining 90% of the distance?

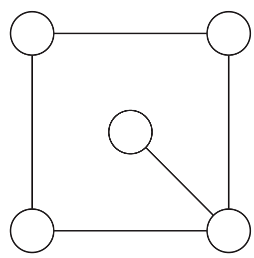
B3. As Rachel travelled to school, she noticed that, at each bus stop, one passenger got off and x passengers got on, where $x \geq 2$. After five stops, the number of passengers on the bus was x times the number of passengers before the first stop. How many passengers were on the bus before the first stop?

B4. The regular nonagon $ABCDEFGHI$ shares two of its vertices with the regular hexagon $AJKLMI$.
Show that the points H, M and D lie on the same straight line.



B5. The eleven-digit number '$A123456789B$' is divisible by exactly eight of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. Find the values of A and B, explaining why they must have these values.

B6. The diagram shows five circles connected by five line segments.
Three colours are available to colour these circles.
In how many different ways is it possible to colour all five circles so that circles which are connected by a line segment are coloured differently?

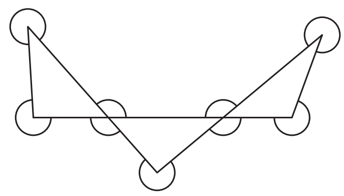


A1. What is the value of $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6}$?

A2. Seven consecutive odd numbers add up to 105. What is the largest of these numbers?

A3. In a class, 55% of students scored at least 55% on a test. 65% of students scored at most 65% on the same test. What percentage of students scored between 55% and 65% (inclusive) on the test?

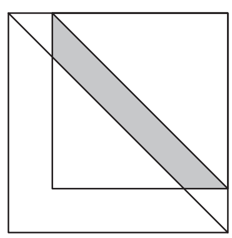
A4. What is the sum of the marked angles in this diagram?



A5. Consider the six-digit multiples of three with at least one of each of the digits 0, 1 and 2, and no other digits. What is the difference between the largest and the smallest of these numbers?

A6. Two positive numbers a and b, with $a > b$, are such that twice their sum is equal to three times their difference. What is the ratio $a : b$?

A7. The diagram on the right shows a 4 by 4 square placed on top of a 5 by 5 square, so that they have one vertex in common as shown. One diagonal of each square is also drawn. What is the area of the shaded region that is inside the 4 by 4 square and between the two diagonals?



A8. The sum of the numbers 1 to 123 is 7626. One number is omitted so that the sum is now a multiple of 4. How many different numbers could be omitted?

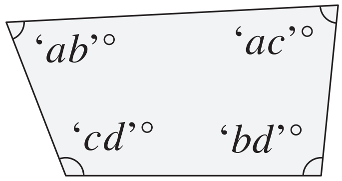
A9. Dividing 52 by 12 gives 4 remainder 4. What is the sum of all the numbers for which dividing by 12 gives a whole number answer which is the same as the remainder?

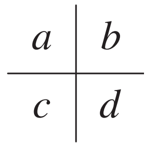
A10. Farmer Alice has an alpaca, a cat, a dog, a gnu and a pig. She also has five fields in a row. She wants to put one animal in each field, but doesn't want to put two animals in adjacent fields if their names contain the same letter. In how many different ways can she place her animals?

B1. The sum of two numbers is 90.
40% of the first number is 15 more than 30% of the second number.

Find the two numbers.

B2. In a certain quadrilateral, the four angles are each two-digit numbers. These four numbers can be placed in the 2 by 2 grid shown, with one digit in each cell.



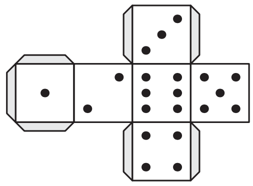


Find all the possibilities for the set of four angles.

B3. You start with a regular pentagon $ABCDE$. Then you draw two circles: one with centre A and radius AB, and the other with centre B and radius BA. Let the point inside the pentagon at which these two circles intersect be X.

What is the size of $\angle DEX$?

B4. Seth creates n standard dice by folding up n identical copies of the net shown. He then repeatedly puts one on top of another until there are none left, creating a vertical tower.



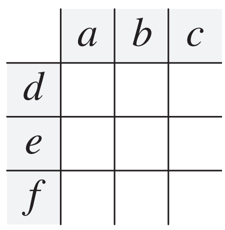
For each of the four vertical walls of the tower, he finds the total number of dots that are visible.

Given that the four totals calculated are all odd, what are the possible values for n?

B5. Charlie chooses one cell from a blank $n \times n$ square grid and shades it. The resulting grid has no lines of symmetry.

In terms of n, how many different cells could be shaded?

B6. The descriptors **'even'**, **'factors of 240'**, **'multiple of 3'**, **'odd'**, **'prime'** and **'square'** are to be placed in some order as row and column headings around the grid in positions a, b, c, d, e and f.



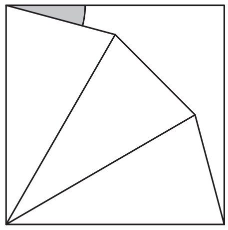
The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are to be placed in the empty cells inside the grid so that each digit satisfies both the relevant row and column headings.

(i) Show that it is possible to complete the grid.

(ii) In how many different ways can the grid be completed?

A1. What is the value of $\left(1+\frac{1}{1^2}\right)\left(2+\frac{1}{2^2}\right)\left(3+\frac{1}{3^2}\right)$?

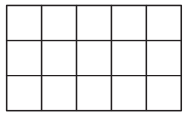
A2. Three identical isosceles triangles fit exactly (without overlap) into a square, with two of the triangles having edges in common with the square, as shown in the diagram.
What is the size, in degrees, of the shaded angle?



A3. What is the value of $\left(\frac{4}{5}\right)^3$ as a decimal?

A4. In the first week after Bill's birthday, his uncle gave him some money for his new piggy bank. Every week after that, Bill put £2 into his piggy bank.
By the end of the ninth week after his birthday, Bill had trebled the amount his uncle gave him. How much, in pounds, did Bill have in total at the end of the ninth week?

A5. Aston wants to colour exactly three of the cells in the grid shown so that the coloured grid has rotational symmetry of order two. Each of the cells in the grid is a square. In how many ways can Aston do this?

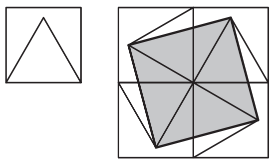


A6. To travel the 140 kilometres between Glasgow and Dundee, Sam travels half an hour by bus and two hours by train. The train travels 20 km/h faster than the bus. The bus and the train both travel at constant speeds. What is the speed, in km/h, of the bus?

A7. The product of five different integers is 12. What is the largest of the integers?

A8. In my desk, the number of pencils and pens was in the ratio $4:5$. I took out a pen and replaced it with a pencil and now the ratio is $7:8$. What is the total number of pencils and pens in my desk?

A9. A unit square has an equilateral triangle drawn inside it, with a common edge. Four of these squares and triangles are placed together to make a larger square. Four vertices of the triangles are joined up to form a square, which is shaded and shown in the diagram. What is the area of the shaded square?



A10. Amy, Bruce, Chris, Donna and Eve had a race. When asked in which order they finished, they all answered with a true and a false statement as follows:

Amy: Bruce came second and I finished in third place.

Bruce: I finished second and Eve was fourth.

Chris: I won and Donna came second.

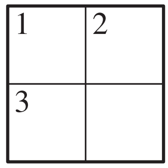
Donna: I was third and Chris came last.

Eve: I came fourth and Amy won.

In which order did the participants finish?

B1. The solution to each clue of this crossnumber is a two-digit number, that does not begin with a zero.

Across	Down
1. A prime	1. A square
3. A square	2. A square



Find all the different ways in which the crossnumber can be completed correctly.

B2. The Smith family went to a restaurant and bought two Pizzas, three Chillies and four Pastas. They paid £53 in total.

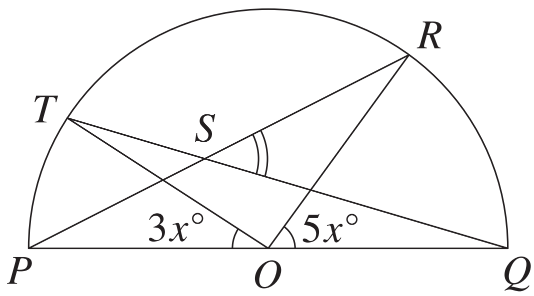
The Patel family went to the same restaurant and bought five of the same Pizzas, six of the same Chillies and seven of the same Pastas. They paid £107 in total.

How much more does a Pizza cost than a Pasta?

B3. Two overlapping triangles POR and QOT are such that points P, Q, R and T lie on the arc of a semicircle of centre O and diameter PQ, as shown in the diagram.

Lines QT and PR intersect at the point S. Angle TOP is $3x^\circ$ and angle ROQ is $5x^\circ$.

Show that angle RSQ is $4x^\circ$.



B4. The letters A, B and C stand for different, non-zero digits.

Find all the possible solutions to the word-sum shown.

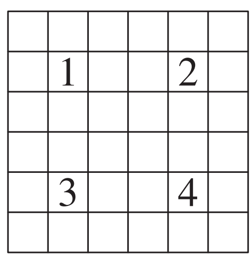
$$\begin{array}{cccc} & A & B & C \\ & B & C & A \\ + & C & A & B \\ \hline A & B & B & C \end{array}$$

B5. In Sally's sequence, every term after the second is equal to the sum of the previous two terms. Also, every term is a positive integer. Her eighth term is 400.

Find the minimum value of the third term in Sally's sequence.

B6. The integers 1 to 4 are positioned in a 6 by 6 square grid as shown and cannot be moved.

The integers 5 to 36 are now placed in the 32 empty squares. Prove that no matter how this is done, the integers in some pair of adjacent squares (i.e. squares sharing an edge) must differ by at least 16.



A1. What is the time 1500 seconds after 14:35?

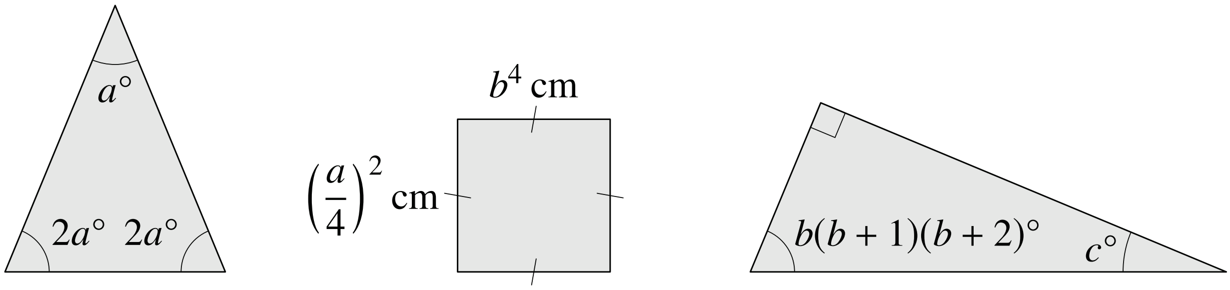
A2. Six standard, fair dice are rolled once. The total of the scores rolled is 32.

What is the smallest possible score that could have appeared on any of the dice?

A3. A satellite orbits the Earth once every 7 hours.

How many orbits of the Earth does the satellite make in one week?

A4.



What is the value of c?

A5. Dani wrote the integers from 1 to N. She used the digit 1 fifteen times. She used the digit 2 fourteen times.

What is N?

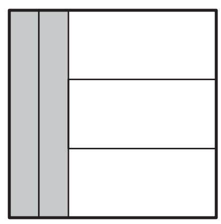
A6. How many fractions between $\frac{1}{6}$ and $\frac{1}{3}$ inclusive can be written with a denominator of 15?

A7. Two 2-digit multiples of 7 have a product of 7007.

What is their sum?

A8. The diagram shows a square made from five rectangles. Each of these rectangles has the same *perimeter*.

What is the ratio of the area of a shaded rectangle to the area of an unshaded rectangle?



A9. The number 3600 can be written as $2^a \times 3^b \times 4^c \times 5^d$, where a, b, c and d are all positive integers. It is given that $a + b + c + d = 7$.

What is the value of c?

A10. Three positive integers add to 93 and have a product of 3375. The integers are in the ratio $1 : k : k^2$.

What are the three integers?

B1. In this word-sum, each letter stands for one of the digits 0–9, and stands for the same digit each time it appears. Different letters stand for different digits. No number starts with 0.

$$\begin{array}{r} J\ M\ O \\ J\ M\ O \\ +\ J\ M\ O \\ \hline I\ M\ O \end{array}$$

Find all the possible solutions of the word-sum shown here.

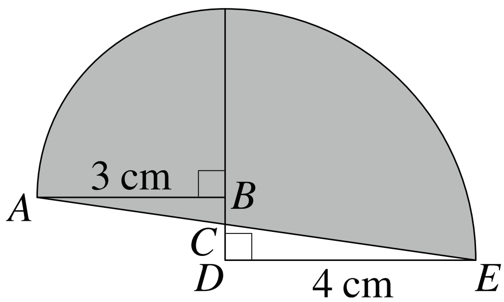
B2. The product $8000 \times K$ is a square, where K is a positive integer.

What is the smallest possible value of K?

B3. It takes one minute for a train travelling at constant speed to pass completely through a tunnel that is 120 metres long. The same train, travelling at the same constant speed, takes 20 seconds from the instant its front enters the tunnel to it being completely inside the tunnel.

How long is the train?

B4. The diagram alongside shows two quarter-circles and two triangles, ABC and CDE. One quarter-circle has radius AB, where $AB = 3\,\text{cm}$. The other quarter-circle has radius DE, where $DE = 4\,\text{cm}$.



The area enclosed by the line AE and the arcs of the two quarter-circles is shaded.

What is the total shaded area, in cm^2?

B5. My 24-hour digital clock displays hours and minutes only.

How many displayed times in a 24-hour period contain at least one occurrence of the digit 5?

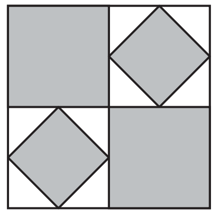
B6. An equilateral triangle is divided into smaller equilateral triangles.

The diagram on the left shows that it is possible to divide it into 4 equilateral triangles. The diagram on the right shows that it is possible to divide it into 13 equilateral triangles.

What are the integer values of n, where $n > 1$, for which it is possible to divide the triangle into n smaller equilateral triangles?

A1. What is the value of 0.8×0.12?

A2. A large square is split into four congruent squares, two of which are shaded. The other two squares have smaller shaded squares drawn in them whose vertices are the midpoints of the sides of the unshaded squares.
What fraction of the large square is shaded?



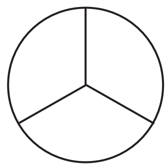
A3. What is the largest integer for which each pair of consecutive digits is a square?

A4. What is the value of $\dfrac{10^5}{5^5}$?

A5. The sizes in degrees of the interior angles of a pentagon are consecutive even numbers.
What is the size of the largest of these angles?

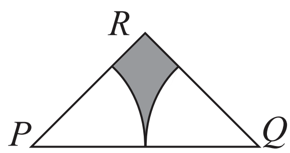
A6. A two-digit number '*ab*' is multiplied by its reverse '*ba*'. The ones (units) and tens digits of the four-digit answer are both 0.
What is the value of the smallest such two-digit number '*ab*'?

A7. The diagram shows a circle divided into three equal sectors.
What is the ratio of the length of the perimeter of one of these sectors to the length of the circumference of the circle?



A8. How many three-digit integers less than 1000 have exactly two different digits in their representation (for example, 232, or 466)?

A9. The triangle PQR is isosceles with $PR = QR$. Angle $PRQ = 90°$ and length $PQ = 2$ cm. Two arcs of radius 1 cm are drawn inside triangle PQR. One arc has its centre at P and intersects PR and PQ. The other arc has its centre at Q and intersects QR and PQ.
What is the area of the shaded region, in cm^2 ?



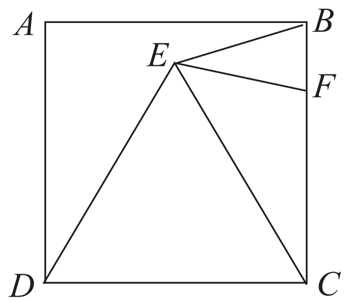
A10. A four-digit integer has its digits increasing from left to right. When we reverse the order of the digits, we obtain a four-digit integer whose digits decrease from left to right. A third four-digit integer uses exactly the same digits, but in a different order. The sum of the three integers is 26 352.
What is the value of the smallest of the three integers?

B1. Polly Garter had her first child on her 20th birthday, her second child exactly two years later, and her third child exactly two years after that.

How old was Polly when her age was equal to the sum of her three children's ages?

B2. In the diagram shown, $ABCD$ is a square and point F lies on BC. Triangle DEC is equilateral and $EB = EF$.

What is the size of $\angle CEF$?



B3. The letters a, b and c stand for non-zero digits. The integer 'abc' is a multiple of 3; the integer '$cbabc$' is a multiple of 15; and the integer '$abcba$' is a multiple of 8.

What is the integer 'abc'?

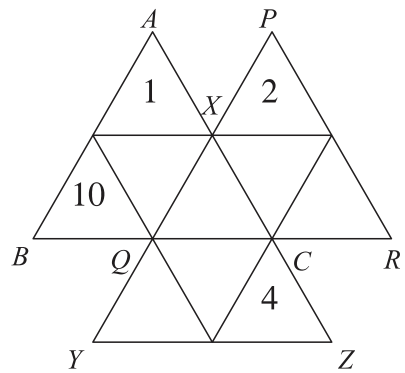
B4. A rectangular sheet of paper is labelled $ABCD$, with AB one of the longer sides. The sheet is folded so that vertex A is placed exactly on top of the opposite vertex C. The fold line is XY, where X lies on AB and Y lies on CD.

Prove that triangle CXY is isosceles.

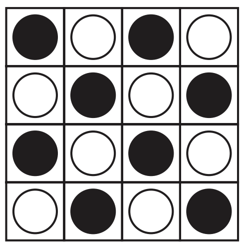
B5. The diagram shows three triangles, ABC, PQR and XYZ, each of which is divided up into four smaller triangles. The diagram is to be completed so that the positive integers from 1 to 10 inclusive are placed, one per small triangle, in the ten small triangles. The totals of the numbers in the three triangles ABC, PQR and XYZ are the same.

Numbers 1, 2, 4 and 10 have already been placed.

In how many different ways can the diagram be completed?



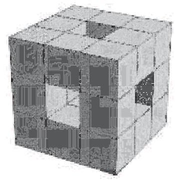
B6. Sixteen counters, which are black on one side and white on the other, are arranged in a 4 by 4 square. Initially all the counters are facing black side up. In one 'move', you must choose a 2 by 2 square within the square and turn all four counters over once. Describe a sequence of 'moves' of minimum length that finishes with the colours of the counters of the 4 by 4 square alternating (as shown in the diagram).



A1. How many centimetres are there in 1 km 2 m 3 cm 4 mm?

A2. The solid shown is formed by taking a 3 cm × 3 cm × 3 cm cube and drilling a 1 cm × 1 cm square hole from the centre of each face to the centre of the opposite face.

What is the volume in cm^3 of the solid?



A3. Howard is out running. He is now $\frac{3}{5}$ of the way through the second half of his run. What fraction of the whole run has he completed?

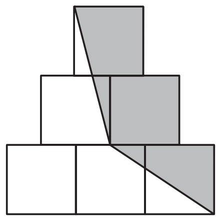
A4. A bookmark-maker sells bookmarks for £1 each or 7 for £6. What is the smallest amount you could pay for 2017 of her bookmarks?

A5. In 1866, the yacht *Henrietta* – with Gordon Bennett aboard – won the Great Ocean Yacht Race, travelling a distance of approximately 3000 nautical miles. The winning time was 13 days and 22 hours, to the nearest hour.

What was the yacht's average speed in nautical miles per hour, to the nearest integer?

A6. The diagram shows six identical squares arranged symmetrically.

What fraction of the diagram is shaded?



A7. A fully-grown long-tailed tit − *Aegithalos caudatus* − weighs only 9 g, whereas a £1 coin weighs 9.5 g.

To the nearest 1 %, what percentage of the weight of a £1 coin is the weight of a fully-grown long-tailed tit?

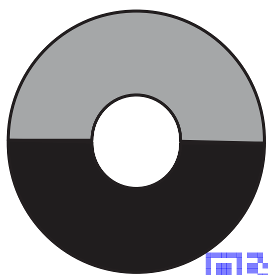
A8. A jar contains red and white marbles in the ratio 1 : 4. When Jenny replaces 2 of the white marbles with 7 red marbles, the ratio becomes 2 : 3.

What is the ratio of the total number of marbles in the jar now to the total number in the jar before?

A9. How many multiples of 3 that are less than 1000 are not divisible either by 9 or by 10?

A10. Two concentric circles are drawn, as shown in the diagram. Concentric circles share the same point as their centre. The radius of the smaller circle is a third of the radius of the larger circle. The top half of the larger circle which is outside the smaller circle, is shaded in grey.

The ratio of the grey shaded area to the area of the smaller circle in its simplest form is $a : b$. What are the values of a and b?



B1. An amount of money is to be divided equally between a group of children. If there was 20p more than this amount, then there would be enough for each child to receive 70p. However, if each child was to receive 60p, then £2.10 would be left over.

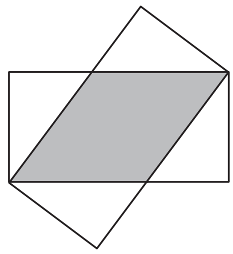
How many children are there in the group?

B2. A 3-digit integer is called a 'V-number' if the digits go 'high-low-high' – that is, if the tens digit is smaller than both the hundreds digit and the units (or 'ones') digit.

How many 3-digit 'V-numbers' are there?

B3. Two identical rectangles overlap in such a way that a rhombus is formed, as indicated in the diagram. The area of the rhombus is five-eighths of the area of each rectangle.

What is the ratio of the length of the longer side of the rectangle to the length of the shorter side?



B4. My uncle lives a long way away and his letters always contain puzzles. His three local teams are the Ants (A), the Bees (B), and the Cats (C), who play each other once a year.

My uncle claimed that the league table part way through the year looked like this:

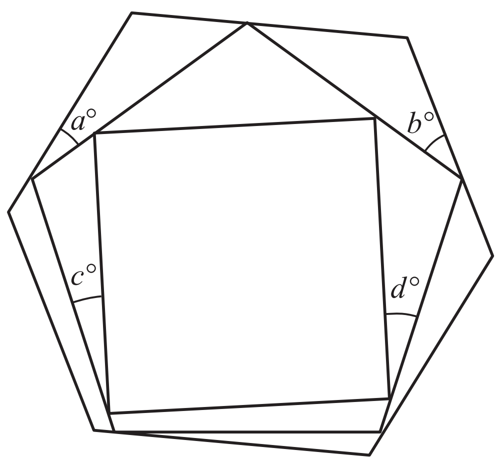
	Played	Won	Drawn	Lost	Goals for	Goals against
A	1	0	0	1	4	2
B	2	1	1	0	2	2
C	2	1	0	1	3	1

When we complained that this is impossible, he admitted that every single number was wrong but he excused himself because every number was exactly '1 out'.

Find the correct table, explaining clearly how you deduced the corrections.

B5. The diagram shows a square whose vertices touch the sides of a regular pentagon. Each vertex of the pentagon touches a side of a regular hexagon.

Find the value of $a + b + c + d$.



B6. The 9-digit positive integer N with digit pattern $ABCABCBBB$ is divisible by every integer from 1 to 17 inclusive.

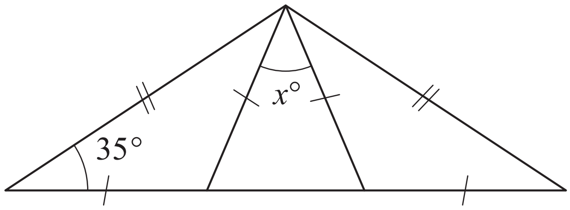
The digits A, B and C are distinct. What are the values of A, B and C?

A1. Roger picks two consecutive integers, one of which ends in a 5. He multiplies the integers together and then squares the result.

What are the last two digits of his answer?

A2. Three isosceles triangles are put together to create a larger isosceles triangle, as shown.

What is the value of x?

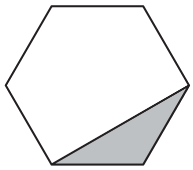


A3. The first term of a sequence is 0. Each term of the sequence after the first term is equal to $10p + 1$, where p is the previous term.

What is the sum of the first ten terms?

A4. The diagram shows a regular hexagon with area 48 m^2.

What is the area of the shaded triangle?



A5. Linda has a very thin sheet of paper measuring 20 cm by 30 cm. She repeatedly folds her paper in half by folding along the shorter line of symmetry. She finishes when she has a rectangle with area 75 cm^2.

What is the perimeter of her final rectangle?

A6. The points A, B, C, D and E lie in that order along a straight line so that $AB : BC = 1 : 2$, $BC : CD = 1 : 3$ and $CD : DE = 1 : 4$. What is $AB : BE$?

A7. A certain positive integer has exactly eight factors. Two of these factors are 15 and 21.

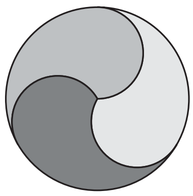
What is the sum of all eight factors?

A8. Julie and her daughters Megan and Zoey have the same birthday. Today, Julie is 32, Megan is 4 and Zoey is 1.

How old will Julie be when her age is the sum of the ages of Megan and Zoey?

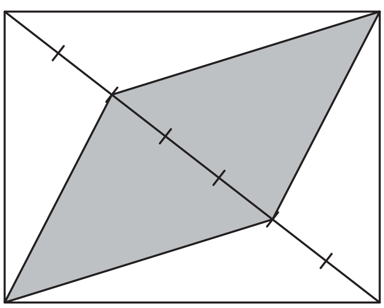
A9. A circle of radius 18 cm is divided into three identical regions by the three semicircles, as shown.

What is the length of the perimeter of one of these regions?



A10. The diagram shows a rectangle with length 9 cm and width 7 cm. One of the diagonals of the rectangle has been divided into seven equal parts.

What is the area of the shaded region?



B1. In a certain triangle, the size of each of the angles is a whole number of degrees. Also, one angle is 30° larger than the average of the other two angles.

What is the largest possible size of an angle in this triangle?

B2. The points A, B and C are the centres of three circles. Each circle touches the other two circles, and each centre lies outside the other two circles. The sides of the triangle ABC have lengths 13 cm, 16 cm and 20 cm.

What are the radii of the three circles?

B3. A large cube consists of a number of identical small cubes. The number of small cubes that touch four other small cubes face-to-face is 168.

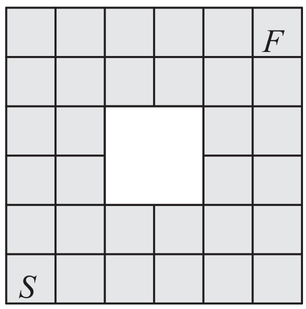
How many small cubes make up the large cube?

B4. In the trapezium $ABCD$, the lines AB and CD are parallel. Also $AB = 2DC$ and $DA = CB$. The line DC is extended (beyond C) to the point E so that $EC = CB = BE$. The line DA is extended (beyond A) to the point F so that $AF = BA$.

Prove that $\angle FBC = 90^\circ$.

B5. The board shown has 32 cells, one of which is labelled S and another F. The shortest path starting at S and finishing at F involves exactly nine other cells and ten moves, where each move goes from cell to cell 'horizontally' or 'vertically' across an edge.

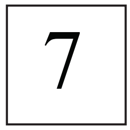
How many paths of this length are there from S to F?



B6. For which values of the positive integer n is it possible to divide the first $3n$ positive integers into three groups each of which has the same sum?

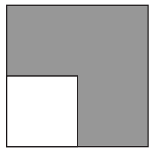
A1. It is 225 minutes until midnight. What time is it on a 24-hour digital clock?

A2. The diagram shows what I see when I look straight down on the top face of a non-standard cubical die. A positive integer is written on each face of the die. The numbers on every pair of opposite faces add up to 10. What is the sum of the numbers on the faces I cannot see?



A3. The diagram shows one square inside another. The perimeter of the shaded region has length 24 cm.

What is the area of the larger square?

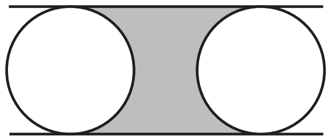


A4. My fruit basket contains apples and oranges. The ratio of apples to oranges in the basket is 3 : 8. When I remove one apple the ratio changes to 1 : 3.

How many oranges are in the basket?

A5. Two circles of radius 1 cm fit exactly between two parallel lines, as shown in the diagram. The centres of the circles are 3 cm apart.

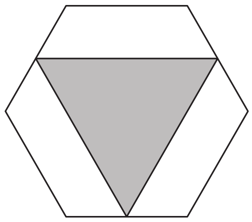
What is the area of the shaded region bounded by the circles and the lines?



A6. There are 81 players taking part in a knock-out quiz tournament. Each match in the tournament involves 3 players and only the winner of the match remains in the tournament – the other two players are knocked out. How many matches are required until there is an overall winner?

A7. The diagram shows an equilateral triangle inside a regular hexagon that has sides of length 14 cm. The vertices of the triangle are midpoints of sides of the hexagon.

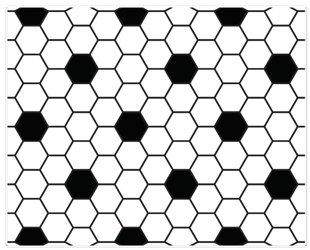
What is the length of the perimeter of the triangle?



A8. What is the units digit in the answer to the sum $9^{2015} + 9^{2016}$?

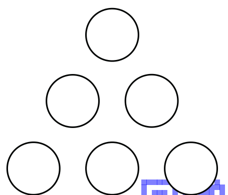
A9. The figure shows part of a tiling, which extends indefinitely in every direction across the whole plane. Each tile is a regular hexagon. Some of the tiles are white, the others are black.

What fraction of the plane is black?



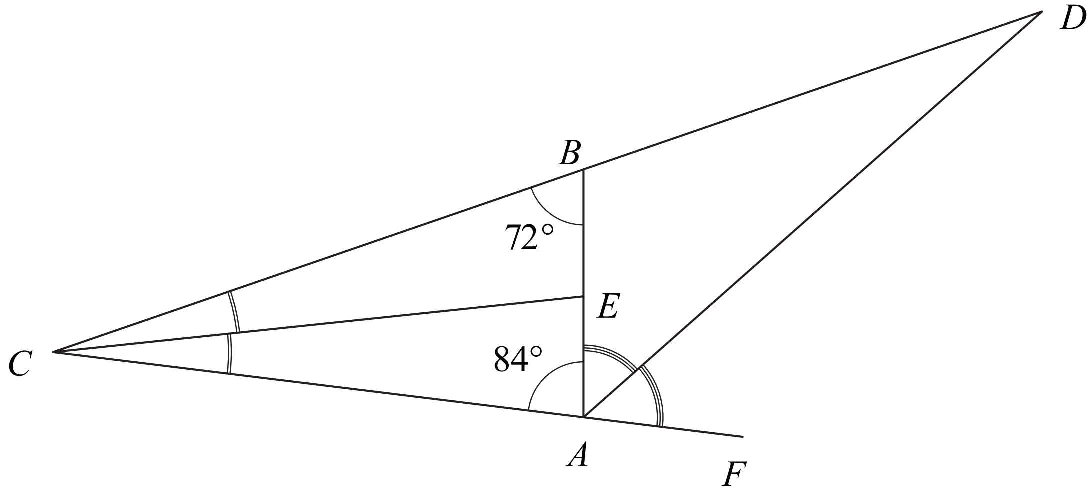
A10. Lucy wants to put the numbers 2, 3, 4, 5, 6 and 10 into the circles so that the products of the three numbers along each edge are the same, and as large as possible.

What is this product?



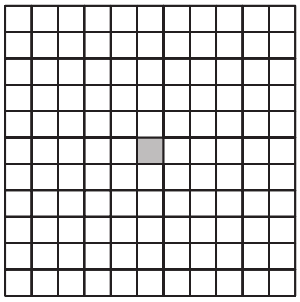
B1. Let N be the smallest positive integer whose digits add up to 2015.
What is the sum of the digits of $N + 1$?

B2. The diagram shows triangle ABC, in which $\angle ABC = 72°$ and $\angle CAB = 84°$. The point E lies on AB so that EC bisects $\angle BCA$. The point F lies on CA extended. The point D lies on CB extended so that DA bisects $\angle BAF$.



Prove that $AD = CE$.

B3. Jack starts in the small square shown shaded on the grid, and makes a sequence of moves. Each move is to a neighbouring small square, where two small squares are neighbouring if they have an edge in common. He may visit a square more than once.
Jack makes four moves. In how many different small squares could Jack finish?



B4. The point F lies inside the regular pentagon $ABCDE$ so that $ABFE$ is a rhombus. Prove that EFC is a straight line.

B5. I have two types of square tile. One type has a side length of 1 cm and the other has a side length of 2 cm.
What is the smallest square that can be made with equal numbers of each type of tile?

B6. The letters a, b, c, d, e and f represent single digits and each letter represents a different digit. They satisfy the following equations:

$$a + b = d, \qquad b + c = e \qquad \text{and} \qquad d + e = f.$$

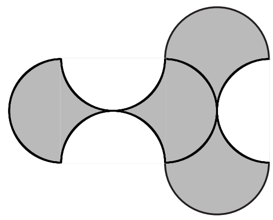
Find all possible solutions for the values of a, b, c, d, e and f.

A1. What is the largest digit that appears in the answer to the calculation $(3 \times 37)^2$?

A2. What is the sum of all fractions of the form $\frac{N}{7}$, where N is a positive integer less than 7?

A3. The six angles of two different triangles are listed in decreasing order. The list starts 115°, 85°, 75° and 35°. What is the last angle in the list?

A4. The figure shows two shapes that fit together exactly. Each shape is formed by four semicircles of radius 1. What is the total shaded area?



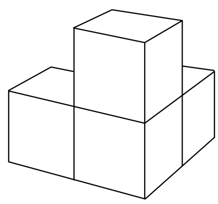
A5. The integer 113 is prime, and its 'reverse' 311 is also prime. How many two-digit primes are there between 10 and 99 which have the same property?

A6. A square of side length 1 is drawn. A larger square is drawn around it such that all parallel sides are a distance 1 apart. This process continues until the total perimeter of the squares drawn is 144.

What is the area of the largest square drawn?

A7. The time is 20:14. What is the smaller angle between the hour hand and the minute hand on an accurate analogue clock?

A8. Sam has four cubes all the same size: one blue, one red, one white and one yellow. She wants to glue the four cubes together to make the solid shape shown.



How many differently-coloured shapes can Sam make?

[Two shapes are considered to be the same if one can be picked up and turned around so that it looks identical to the other.]

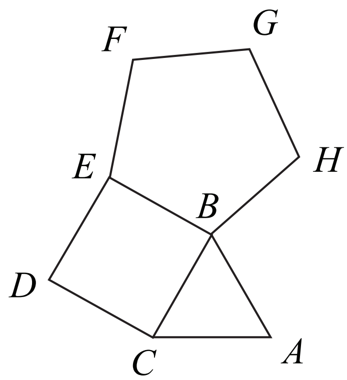
A9. A rectangle is made by placing together three smaller rectangles P, Q and R, without gaps or overlaps. Rectangle P measures 3 cm × 8 cm and Q measures 2 cm × 5 cm.

How many possibilities are there for the measurements of R?

A10. My four pet monkeys and I harvested a large pile of peanuts. Monkey A woke in the night and ate half of them; then Monkey B woke and ate one third of what remained; then Monkey C woke and ate one quarter of the rest; finally Monkey D ate one fifth of the much diminished remaining pile. What fraction of the original harvest was left in the morning?

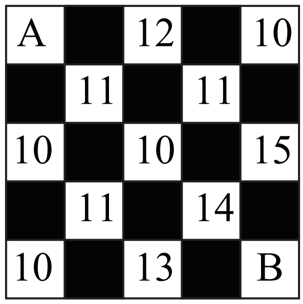
B1. The figure shows an equilateral triangle ABC, a square $BCDE$, and a regular pentagon $BEFGH$.

What is the difference between the sizes of $\angle ADE$ and $\angle AHE$?



B2. I start at the square marked A and make a succession of moves to the square marked B. Each move may only be made downward or to the right. I take the sum of all the numbers in my path and add 5 for every black square I pass through.

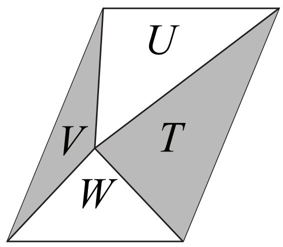
How many paths give a sum of 51?



B3. A point lying somewhere inside a parallelogram is joined to the four vertices, thus creating four triangles T, U, V and W, as shown.

Prove that

$$\text{area } T + \text{area } V = \text{area } U + \text{area } W.$$



B4. There are 20 sweets on the table. Two players take turns to eat as many sweets as they choose, but they must eat at least one, and never more than half of what remains. The loser is the player who has no valid move.

Is it possible for one of the two players to force the other to lose? If so, how?

B5. Find a fraction $\frac{m}{n}$, with m not equal to n, such that all of the fractions

$$\frac{m}{n}, \frac{m+1}{n+1}, \frac{m+2}{n+2}, \frac{m+3}{n+3}, \frac{m+4}{n+4}, \frac{m+5}{n+5}$$

can be simplified by cancelling.

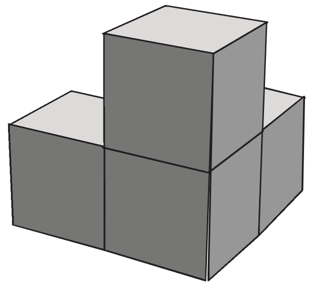
B6. The sum of four different prime numbers is a prime number. The sum of some pair of the numbers is a prime number, as is the sum of some triple of the numbers. What is the smallest possible sum of the four prime numbers?

A1 What is the value of $\sqrt{3102 - 2013}$?

A2 For how many three-digit positive integers does the product of the digits equal 20?

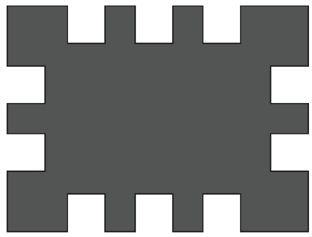
A3 The solid shown is made by gluing together four 1 cm × 1 cm × 1 cm cubes.

What is the total surface area of the solid?



A4 What percentage of $\frac{1}{4}$ is $\frac{1}{5}$?

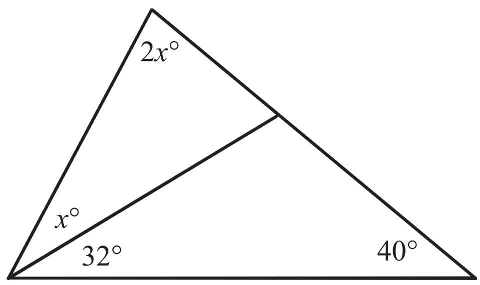
A5 Sue has a rectangular sheet of paper measuring 40 cm × 30 cm. She cuts out ten squares each measuring 5 cm × 5 cm, as shown. In each case, exactly one side of the square lies along a side of the rectangle and none of the cut-out squares overlap.



What is the perimeter of the resulting shape?

A6 I want to write a list of integers containing two square numbers, two prime numbers, and two cube numbers. What is the smallest number of integers that could be in my list?

A7 Calculate the value of x in the diagram shown.



A8 The area of a square is 0.25 m^2. What is the perimeter of the square, in metres?

A9 Each interior angle of a quadrilateral, apart from the smallest, is twice the next smaller one. What is the size of the smallest interior angle?

A10 A cube is made by gluing together a number of unit cubes face-to-face.
The number of unit cubes that are glued to exactly four other unit cubes is 96.

How many unit cubes are glued to exactly five other unit cubes?

B1 How many numbers less than 2013 are both:

(i) the sum of two consecutive positive integers; **and**

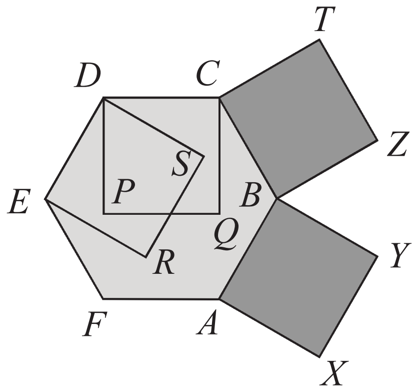
(ii) the sum of five consecutive positive integers?

B2 Pippa thinks of a number. She adds 1 to it to get a second number. She then adds 2 to the second number to get a third number, adds 3 to the third to get a fourth, and finally adds 4 to the fourth to get a fifth number.

Pippa's brother Ben also thinks of a number but he subtracts 1 to get a second. He then subtracts 2 from the second to get a third, and so on until he too has five numbers.

They discover that the sum of Pippa's five numbers is the same as the sum of Ben's five numbers. What is the difference between the two numbers of which they first thought ?

B3 Two squares *BAXY* and *CBZT* are drawn on the outside of a regular hexagon *ABCDEF*, and two squares *CDPQ* and *DERS* are drawn on the inside, as shown.



Prove that $PS = YZ$.

B4 A regular polygon P with n sides is divided into two pieces by a single straight cut. One piece is a triangle T, the other is a polygon Q with m sides.

How are m and n related?

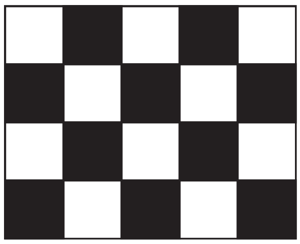
B5 Consider three-digit integers N with the two properties:

(a) N is not exactly divisible by 2, 3 or 5;

(b) no digit of N is exactly divisible by 2, 3 or 5.

How many such integers N are there?

B6 On the 4×5 grid shown, I am only allowed to move from one square to a neighbouring square by crossing an edge. So the squares I visit alternate between black and white. I have to start on a black square and visit each black square exactly once. What is the smallest number of white squares that I have to visit? Prove that your answer is indeed the smallest.



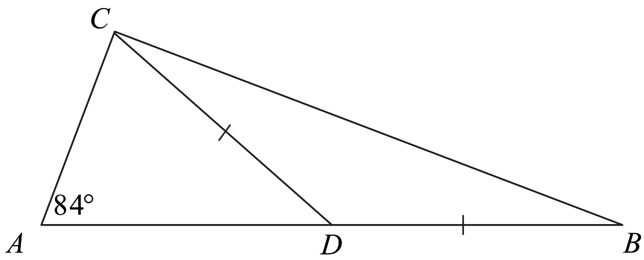
(If I visit a white square more than once, I only count it as one white square visited).

A1 What is the value of $1^1 + 2^2 + 3^3 + 4^4 - (1^4 + 2^3 + 3^2 + 4^1)$?

A2 Mike drank 60% of his glass of milk. Afterwards, 80 ml of milk remained in the glass. What volume of milk was initially in the glass?

A3 In triangle ABC, $\angle CAB = 84°$; D is a point on AB such that $\angle CDB = 3 \times \angle ACD$ and $DC = DB$.

What is the size of $\angle BCD$?

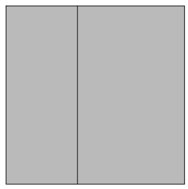


A4 A book costs £3.40 and a magazine costs £1.60. Clara spends exactly £23 on books and magazines. How many magazines does she buy?

A5 Each digit of a positive integer is 1 or 2 or 3.
Given that each of the digits 1, 2 and 3 occurs at least twice, what is the smallest such integer that is not divisible by 2 or 3?

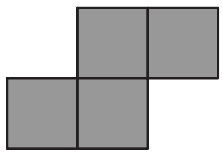
A6 A square is cut into two rectangles, as shown, so that the sum of the lengths of the perimeters of these two rectangles is 30 cm.

What is the length of a side of the square?



A7 The diagram shows a shape made from four 1×1 squares.

What is the maximum number of such shapes that can be placed inside a 5×5 square without overlapping?
(The shapes may be rotated or turned over.)



A8 An athletics club has junior (i.e. boy or girl) members and adult members. The ratio of girls to boys to adults is 3 : 4 : 9 and there are 16 more adult members than junior members. In total, how many members does the club have?

A9 What is the integer x so that $\frac{x}{9}$ lies between $\frac{71}{7}$ and $\frac{113}{11}$?

A10 A positive integer, N, has three digits and the product of its digits is also a three-digit integer. What is the smallest possible value of N?

B1 There was an old woman who lived in a shoe. She had 9 children at regular intervals of 15 months. The oldest is now six times as old as the youngest. How old is the youngest child?

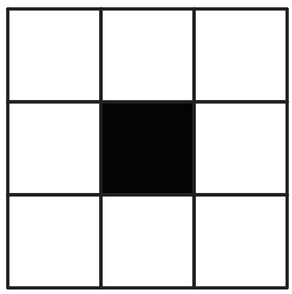
B2 Anastasia thinks of a positive integer, which Barry then doubles. Next, Charlie trebles Barry's number. Finally, Damion multiplies Charlie's number by six. Eve notices that the sum of these four numbers is a perfect square. What is the smallest number that Anastasia could have thought of?

B3 Mr Gallop has two stables which each initially housed three ponies. His prize pony, Rein Beau, is worth £250 000. Usually Rein Beau spends his day in the small stable, but when he wandered across into the large stable, Mr Gallop was surprised to find that the average value of the ponies in each stable rose by £10 000. What is the total value of all six ponies?

B4 An irregular pentagon has five different interior angles each of which measures an integer number of degrees. One angle is 76°.

The other four angles are three-digit integers which fit one digit per cell across and down into the grid on the right.

In how many different ways can the grid be completed?



B5 Three identical, non-overlapping, squares *ABCD*, *AEFG*, *AHIJ* (all labelled anticlockwise) are joined at the point *A*, and are 'equally spread' (so that $\angle JAB = \angle DAE = \angle GAH$). Calculate $\angle GBH$.

B6 The integer 23173 is such that

(a) every pair of neighbouring digits, taken in order, forms a prime number;

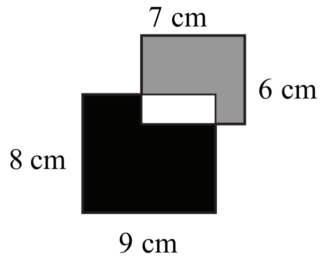
and (b) all of these prime numbers are different.

What is the largest integer which meets these conditions?

A1 What is the value of $3^3 + 3 \times 3 - 3$?

A2 Two rectangles measuring 6 cm × 7 cm and 8 cm × 9 cm overlap as shown. The region shaded grey has an area of 32 cm^2.

What is the area of the black region?



A3 Paul is 32 years old. In 10 years' time, Paul's age will be the sum of the ages of his three sons.

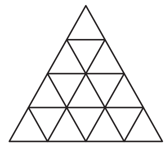
What do the ages of each of Paul's three sons add up to at present?

A4 What is the value of $\dfrac{1}{2-3} - \dfrac{4}{5-6} - \dfrac{7}{8-9}$?

A5 The base of a pyramid has n edges. In terms of n, what is the difference between the number of edges of the pyramid and the number of its faces?

A6 The diagram shows a grid of 16 identical equilateral triangles.

How many different rhombuses are there made up of two adjacent small triangles?

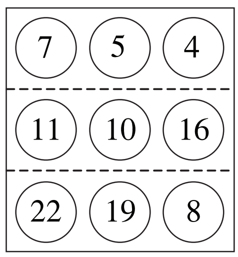


A7 Some rectangular sheets of paper, all the same size, are placed in a pile. The pile is then folded in half to form a booklet. The pages are then numbered in order 1, 2, 3, 4 ... from the first page to the last page.
On one of the sheets, the sum of the numbers on the four pages is 58.

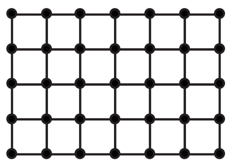
How many sheets of paper were there at the start?

A8 A puzzle starts with nine numbers placed in a grid, as shown.
On each move you are allowed to swap any two numbers.
The aim is to arrange for the total of the numbers in each row to be a multiple of 3.

What is the smallest number of moves needed?

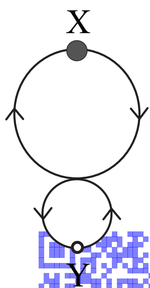


A9 The diagram represents a rectangular fishing net, made from ropes knotted together at the points shown. The net is cut several times; each cut severs precisely one section of rope between two adjacent knots.
What is the largest number of such cuts that can be made without splitting the net into two separate pieces?



A10 A 'figure of eight' track is constructed from two circles: a large circle of radius 2 units and a small circle of radius 1 unit. Two cars X, Y start out from the positions shown: X at the top of the large circular part and Y at the bottom of the small circular part. Each car travels round the complete circuit in the directions shown by the arrows.

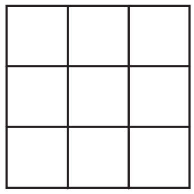
If Y travels twice as fast as X, how far must X travel before the cars collide?



B1 Every digit of a given positive integer is either a 3 or a 4 with each occurring at least once. The integer is divisible by both 3 and 4.

What is the smallest such integer?

B2 A 3×3 grid contains nine numbers, not necessarily integers, one in each cell. Each number is doubled to obtain the number on its immediate right and trebled to obtain the number immediately below it.



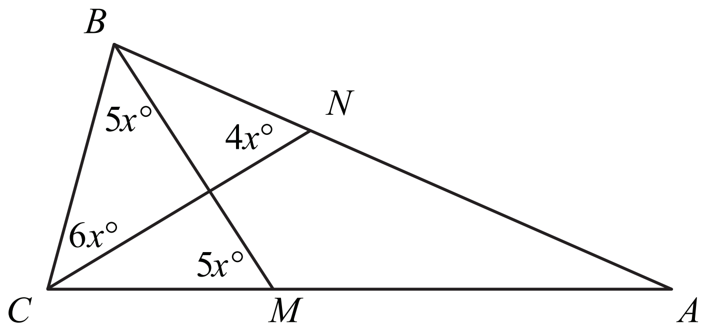
If the sum of the nine numbers is 13, what is the value of the number in the central cell?

B3 When Dad gave out the pocket money, Amy received twice as much as her first brother, three times as much as the second, four times as much as the third and five times as much as the last brother. Peter complained that he had received 30p less than Tom.

Use this information to find all the possible amounts of money that Amy could have received.

B4 In a triangle ABC, M lies on AC and N lies on AB so that $\angle BNC = 4x^\circ$, $\angle BCN = 6x^\circ$ and $\angle BMC = \angle CBM = 5x^\circ$.

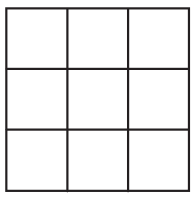
Prove that triangle ABC is isosceles.



B5 Calum and his friend cycle from A to C, passing through B.
During the trip he asks his friend how far they have cycled.
His friend replies "one third as far as it is from here to B".
Ten miles later Calum asks him how far they have to cycle to reach C.
His friend replies again "one third as far as it is from here to B".

How far from A will Calum have cycled when he reaches C?

B6 Pat has a number of counters to place into the cells of a 3×3 grid like the one shown. She may place any number of counters in each cell or leave some of the cells empty. She then finds the number of counters in each row and each column. Pat is trying to place counters in such a way that these six totals are all different.



What is the smallest total number of counters that Pat can use?

A1 What is the value of $\frac{1}{1} + \frac{2}{\frac{1}{2}} + \frac{3}{\frac{1}{3}} + \frac{4}{\frac{1}{4}} + \frac{5}{\frac{1}{5}}$?

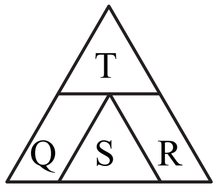
A2 Given that $x : y = 1 : 2$ and $y : z = 3 : 4$, what is $x : z$?

A3 Tom correctly works out 20^{10} and writes down his answer in full.
How many digits does he write down in his full answer?

A4 Three monkeys Barry, Harry and Larry met for tea in their favourite café, taking off their hats as they arrived. When they left, they each put on one of the hats at random. What is the probability that none of them left wearing the same hat as when they arrived?

A5 The sum of two positive integers is 97 and their difference is 37. What is their product?

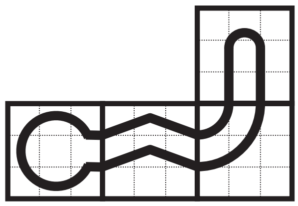
A6 In the diagram, the equilateral triangle is divided into two identical equilateral triangles S and T, and two parallelograms Q and R which are mirror images of each other.



What is the ratio of area R : area T ?

A7 What is the largest possible angle in an isosceles triangle, in which the difference between the largest and smallest angles is 6°?

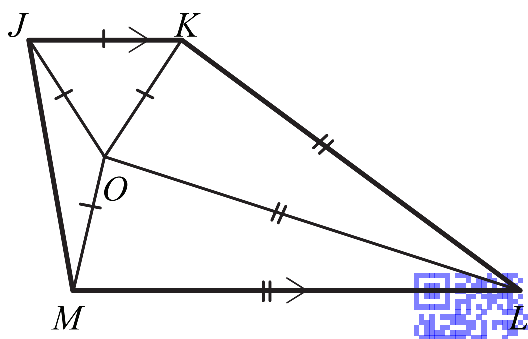
A8 The four square tiles having the designs as shown can be arranged to create a closed loop.
How many distinct closed loops, including the one shown here, can be made from the tiles?



(The tiles may be rotated, but a rotation of a loop is not considered distinct. A loop need not use all four tiles and may not use more than one of each type).

A9 Abbie, Betty and Clara write names on bookmarks sold for charity.
Abbie writes 7 names in 6 minutes, Betty writes 18 names in 10 minutes and Clara writes 23 names in 15 minutes.
If all of the girls work together at these rates, how long will it take them to write 540 names?

A10 In the diagram, JK and ML are parallel, $JK = KO = OJ = OM$ and $LM = LO = LK$.
Find the size of angle JMO.



B1 In a sequence of six numbers, every term after the second term is the sum of the previous two terms. Also, the last term is four times the first term, and the sum of all six terms is 13.

What is the first term?

B2 The eight-digit number "*ppppqqqq*", where p and q are digits, is a multiple of 45.

What are the possible values of p?

B3 Jack and Jill went up a hill. They started at the same time, but Jack arrived at the top one-and-a-half hours before Jill. On the way down, Jill calculated that, if she had walked 50% faster and Jack had walked 50% slower, then they would have arrived at the top of the hill at the same time.

How long did Jill actually take to walk up to the top of the hill?

B4 The solution to each clue of this crossnumber is a two-digit number, not beginning with zero.

In how many different ways can the crossnumber be completed correctly?

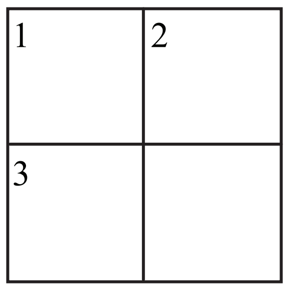
Clues

Across

1. A triangular number
3. A triangular number

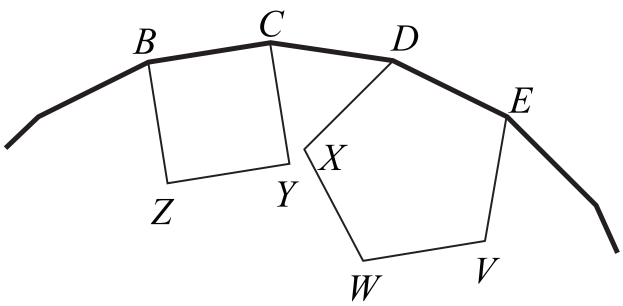
Down

1. A square number
2. A multiple of 5



B5 The diagram shows part of a regular 20-sided polygon (an icosagon) *ABCDEF*..., a square *BCYZ* and a regular pentagon *DEVWX*.

Show that the vertex X lies on the line DY.

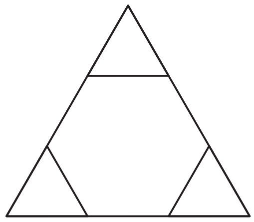


B6 Sam has put sweets in five jars in such a way that no jar is empty and no two jars contain the same number of sweets. Also, any three jars contain more sweets in total than the total of the remaining two jars.

What is the smallest possible number of sweets altogether in the five jars?

A1 What is the value of $200^2 + 9^2$?

A2 The diagram shows a regular hexagon inside an equilateral triangle. The area of the larger triangle is 60 cm^2. What is the area of the hexagon?



A3 The positive whole numbers a, b and c are all different and $a^2 + b^2 + c^2 = 121$. What is the value of $a + b + c$?

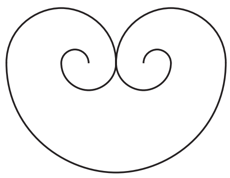
A4 The sum of three numbers is 2009. The sum of the first two numbers is 1004 and the sum of the last two is 1005. What is the product of all three numbers?

A5 Andrea's petrol tank holds up to 44 litres of fuel. She goes to the garage when her tank is a quarter full and puts more petrol in the tank until it is two-thirds full. How many litres of petrol does she put in the tank?

A6 The shorter sides of a right-angled isosceles triangle are each 10 cm long. The triangle is folded in half along its line of symmetry to form a smaller triangle. How much longer is the perimeter of the larger triangle than that of the smaller?

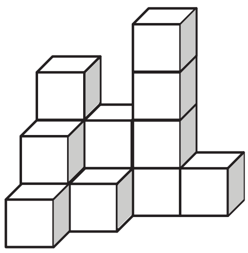
A7 Dean runs on a treadmill for thirty minutes. To keep his mind active as well as his legs, he works out what fraction of the total time has passed at each half minute and minute from the start. How many of the results of his calculations can be expressed in the form $\frac{1}{n}$, where n is an integer greater than 1?

A8 The diagram shows a curve made from seven semicircular arcs, the radius of each of which is 1 cm, 2 cm, 4 cm or 8 cm. What is the length of the curve?



A9 A book has 89 pages, but the page numbers are printed incorrectly. Every third page number has been omitted, so that the pages are numbered 1, 2, 4, 5, 7, 8, … and so on. What is the number on the last printed page?

A10 Gill piles up fourteen bricks into the shape shown in the diagram.
Each brick is a cube of side 10 cm and, from the second layer upwards, sits exactly on top of the brick below.
Including the base, what is the surface area of Gill's construction?

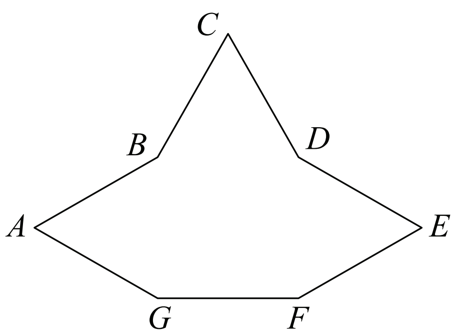


B1 In 2007 Alphonse grew twice the number of grapes that Pierre did. In 2008 Pierre grew twice the number of grapes that Alphonse did. Over the two years Alphonse grew 49 000 grapes, which was 7600 less than Pierre. How many grapes did Alphonse grow in 2007?

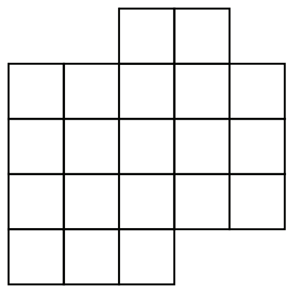
B2 *ABCD* is a square. The point *E* is outside the square so that *CDE* is an equilateral triangle. Find angle *BED*.

B3 Tom left a motorway service station and travelled towards Glasgow at a steady speed of 60 mph. Tim left the same service station 10 minutes after Tom and travelled in the same direction at a steady speed, overtaking Tom after a further 1 hour 40 minutes. At what speed did Tim travel?

B4 The diagram shows a polygon *ABCDEFG*, in which *FG* = 6 and *GA* = *AB* = *BC* = *CD* = *DE* = *EF*. Also *BDFG* is a square. The area of the whole polygon is exactly twice the area of *BDFG*. Find the length of the perimeter of the polygon.

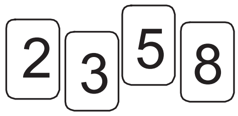


B5 An ant wishes to make a circuit of the board shown, visiting each square exactly once and returning to the starting square. At each step the ant moves to an adjacent square across an edge. Two circuits are considered to be the same if the first follows the same path as the second but either starts at a different square or follows the same path in reverse. How many such circuits are possible?



B6 I want to choose a list of *n* different numbers from the first 20 positive integers so that no two of my numbers differ by 5. What is the largest value of *n* for which this is possible? How many different lists are there with this many numbers?

A1 In how many ways is it possible to place side by side two of the cards shown to form a two-digit prime number?



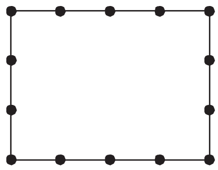
A2 Tony wants to form a square with perimeter 12 cm by folding a rectangle in half and then in half again. What is the maximum possible perimeter of the original rectangle?

A3 Given that $\frac{1}{2} + \frac{1}{3} + \frac{1}{12} + \frac{1}{18} + \frac{1}{x} = 1$, what is the value of x ?

A4 How many three-digit numbers have the product of their digits equal to 6?

A5 A 3 by 4 rectangle has 14 points equally spaced around its four sides, as shown.

In how many ways is it possible to join two of the points by a straight line so that the rectangle is divided into two parts which have areas in the ratio 1 : 3?



A6 How many positive square numbers are factors of 1600?

A7 In a *Magic Square*, the sum of the three numbers in each row, each column and each of the two main diagonals is the same.

What is the value of x in the partially completed magic square shown?

		6
x	4	5

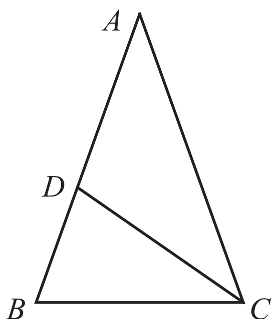
A8 Granny shares a packet of sweets between her four granddaughters. The girls, Clarrie, Lizzie, Annie and Danni, always in that order, each take 8 sweets in turn, over and over again until, finally, there are some sweets left for Danni, but there are fewer than 8. Danni takes all the sweets that are left. The other three girls then give Danni some of their sweets so that all four girls have the same number of sweets.

How many sweets does each of the other three granddaughters give to Danni?

A9 In the diagram, CD is the bisector of angle ACB.

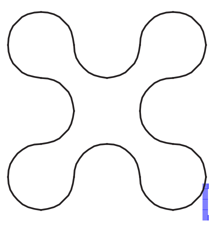
Also, $BC = CD$ and $AB = AC$.

What is the size of angle CDA?



A10 The perimeter of the shape shown on the right is made from 20 quarter-circles, each with radius 2 cm.

What is the area of the shape?

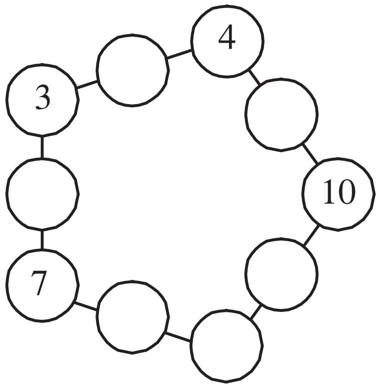


B1 Tamsin has a selection of cubical boxes whose internal dimensions are whole numbers of centimetres, that is, 1 cm × 1 cm × 1 cm, 2 cm × 2 cm × 2 cm, and so on.

What are the dimensions of the smallest of these boxes in which Tamsin could fit ten rectangular blocks each measuring 3 cm × 2 cm × 1 cm without the blocks extending outside the box?

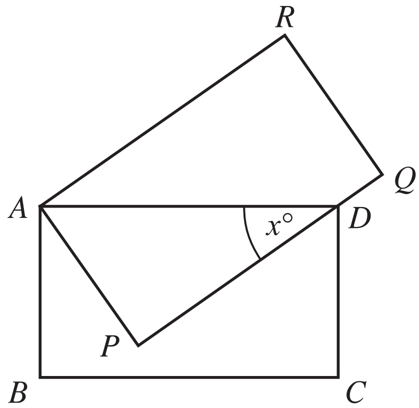
B2 Each of the numbers from 1 to 10 is to be placed in the circles so that the sum of each line of three numbers is equal to T. Four numbers have already been entered.

Find all the possible values of T.



B3 In the diagram $ABCD$ and $APQR$ are congruent rectangles. The side PQ passes through the point D and $\angle PDA = x°$.

Find an expression for $\angle DRQ$ in terms of x.

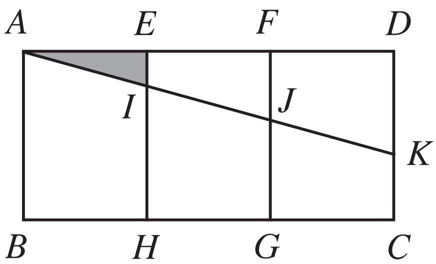


B4 For each positive two-digit number, Jack subtracts the units digit from the tens digit; for example, the number 34 gives $3 - 4 = -1$.

What is the sum of all his results?

B5 In the diagram, the rectangle $ABCD$ is divided into three congruent rectangles. The line segment JK divides $CDFG$ into two parts of equal area.

What is the area of triangle AEI as a fraction of the area of $ABCD$?



B6 In a sequence of positive integers, each term is larger than the previous term. Also, after the first two terms, each term is the sum of the previous two terms.

The eighth term of the sequence is 390. What is the ninth term?

A1 What is the value of $1^5 - 2^4 + 3^3 - 4^2 + 5^1$?

A2 What is the value of k if "$7k$ minutes past nine" is the same time as "$8k$ minutes to ten"?

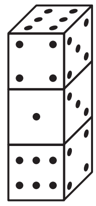
A3 Charlie boils seven eggs for his breakfast. He puts the eggs into the pan one at a time, but waits one minute after putting one egg in before putting the next egg in. If he boils each egg for three minutes, how long does the whole operation take from the moment he puts the first egg in to the moment he takes the seventh egg out?

A4 The hobbits Frodo, Sam, Pippin and Merry have breakfast at different times. Each one takes a quarter of the porridge in the pan, thinking that the other three have not yet eaten. What fraction of the porridge is left after all four hobbits have had their breakfast?

A5 The diagram shows a tower consisting of three identical dice.

On these dice, each pair of opposite faces has a total of seven dots.

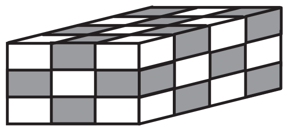
How many dots are there on the face on which the tower stands?



A6 The sizes in degrees of the interior angles of a pentagon are consecutive whole numbers. What is the size of the largest of these angles?

A7 A large cuboid is made from cuboids of equal size, coloured alternately black and white, as shown.

What fraction of the surface area of the large cuboid is black?



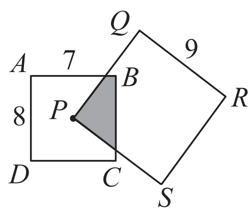
A8 Pegs numbered 1 to 50 are placed in order in a line with number 1 on the left. They are then knocked over one at a time following these rules:

- Of the pegs which are still standing, knock down alternate ones, starting with the first peg on the left.
- Each time you reach the end of the row, repeat the previous rule.

What is the number of the last peg to be knocked over?

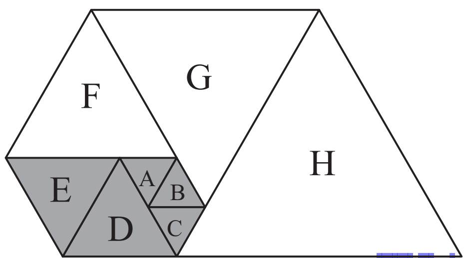
A9 The diagram shows squares $ABCD$ and $PQRS$ of side length 8 units and 9 units respectively. Point P is the centre of square $ABCD$; PQ intersects AB at a point 7 units from A.

What is the perimeter of the shaded region?



A10 The diagram shows a spiral of equilateral triangles. After the first five triangles A, B, C, D, E (shown shaded), the next triangle is always placed alongside two others: the one placed immediately before and one placed earlier. The smallest triangles have sides of length 1 unit.

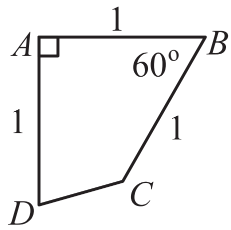
What is the length of the sides of the fifteenth triangle?



B1 Find four integers whose sum is 400 and such that the first integer is equal to twice the second integer, three times the third integer and four times the fourth integer.

B2 The diagram shows a quadrilateral $ABCD$ in which AB, BC and AD are all of length 1 unit, $\angle BAD$ is a right angle and $\angle ABC$ is $60°$.

Prove that $\angle BDC = 2 \times \angle DBC$.

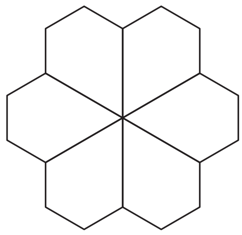


B3 (a) Yesterday evening, my journey home took 25% longer than usual.
By what percentage was my average speed reduced compared to normal?

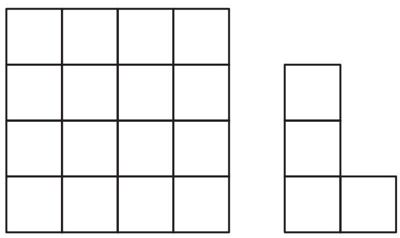
(b) By what percentage would I need to increase my usual average speed in order for the journey to take 20% less time than usual?

B4 Find a rule which predicts exactly when five consecutive integers have sum divisible by 15.

B5 A window is constructed of six identical panes of glass. Each pane is a pentagon with two adjacent sides of length two units.
The other three sides of each pentagon, which are on the perimeter of the window, form half of the boundary of a regular hexagon
Calculate the exact area of glass in the window.



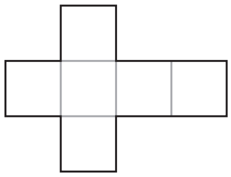
B6 We want to colour red some of the cells in the 4×4 grid shown so that wherever the L-shaped piece is placed on the grid it covers at least one red cell. The L-shaped piece may only cover complete cells, may be rotated, but may not be turned over and may not extend beyond the grid.



(a) Show that it is possible to achieve this by colouring exactly four cells red.
(b) Show that it is impossible to achieve this by colouring fewer than four cells red.

A1 What is the value of $1 + 2 \times (3 + 4^5) + 6 + 7 - 8 \times 9 + 10$?

A2 The perimeter of this net of a cube is 42cm.
What is the volume of the cube?

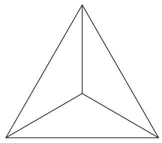


A3 Sarah writes down all the three-digit positive integers for which the product of their digits equals 36. What is the difference between the greatest and the smallest of these numbers?

A4 An equilateral triangle is drawn on a sheet of white card and divided into three identical regions as shown. Then each region is painted red or yellow or blue. More than one region may be painted in the same colour.

How many different painted triangles can be made in this way?

(Rotating a triangle does not make it different.)

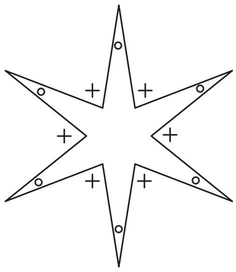


A5 A balloon seller starts the day with a certain number of balloons. He then sells one third of his balloons to boys, 20% to girls, and three times the difference between these amounts to adults. At the end of the day, he has eight balloons left. How many balloons did the seller have at the start?

A6 In the diagram the star is made up of equal line segments.

Each of the angles marked + is 70°.

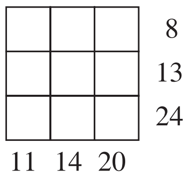
What is the size of the angles marked $\circ$?



A7 Each year on Tom's birthday, his grandfather gives him some pocket money. The amount, in pence, is calculated by multiplying together Tom's age and his grandfather's age on that day.

This year Tom received £7.81. How much did he receive last year?

A8 The numbers 1 to 9 are to be placed so that there is one number in each square and the row and column totals are as shown



What number goes in the central square?

A9 The prime number 11 may be written as the sum of three prime numbers in two different ways: 2 + 2 + 7 and 3 + 3 + 5. What is the smallest prime number which can be written in two different ways as the sum of three prime numbers **which are all different**?

A10 In the flag shown alongside, the regions shaded grey are quarter circles.
If the height of the flag is 1m, what is its breadth?

B1 In her purse, Jenny has 20 coins, with a total value of £5. There are three denominations of coin – 10p, 20p and 50p – in her purse and she has more 50p coins than 10p coins. How many of each type of coin does she have?

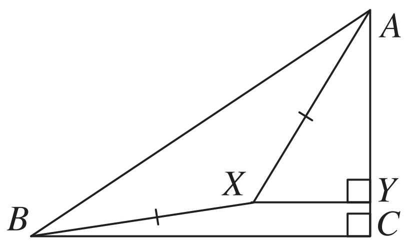
B2

$$97 \rightarrow 63 \rightarrow 18 \rightarrow 8.$$

An example of a particular type of number chain is shown above. The first number must be a positive integer. Each number after the first is the product of the digits of the previous number, so in this case $63 = 9 \times 7$; $18 = 6 \times 3$; $8 = 1 \times 8$. The chain stops when a single-digit number is reached.

Suppose that in such a chain the final number is 6. Find all possible two-digit first numbers for this chain.

B3 In this diagram, Y lies on the line AC, triangles ABC and AXY are right angled and in triangle ABX, $AX = BX$. The line segment AX bisects angle BAC and angle AXY is seven times the size of angle XBC. What is the size of angle ABC?



B4 Start with an equilateral triangle ABC of side 2 units, and construct three outward-pointing squares $ABPQ$, $BCTU$, $CARS$ on the three sides AB, BC, CA. What is the area of the hexagon $PQRSTU$?

B5 An intelligent bug starts at the point (4, 0) and follows these instructions:

(i) first face "East" and walk one unit to the point (5,0);

(ii) from then on, whenever you arrive at a point (x, y) with x and y both integers,
either turn left through 90° if $x - y$ is a multiple of 4 or is 1 more than a multiple of 4;
or turn right through 90° if $x - y$ is 2 more than a multiple of 4 or is 3 more than a multiple of 4;
and then walk one unit to the next point whose coordinates are both integers.

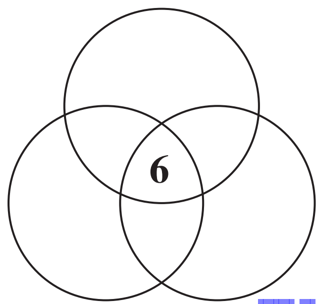
After one move, the bug is at the point (5,0).

(a) Where will the bug be after 12 moves?

(b) Where will the bug be after 50 moves?

B6 The numbers 1 to 7 are to be placed in the seven regions formed by three overlapping circles, with 6 in the central region, so that there is one number inside each region and the total of the numbers inside each circle is T.

What values of T are possible?

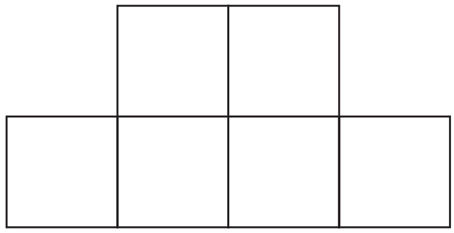


A1 How many seconds are there in one fortieth of an hour?

A2 The diagram shows a shape made from six squares, each of side 1cm.

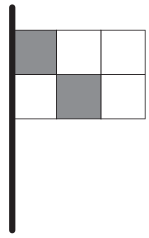
Four copies of the shape are placed together (without leaving any holes or having any overlaps) to form a rectangle.

What is the perimeter of the rectangle?



A3 Three different integers have a sum of 1 and a product of 36. What are they?

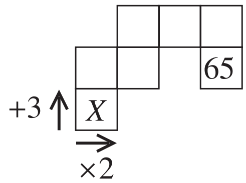
A4 A picture of a flag is to be completed by shading two squares which do not share an edge. The diagram shows one way in which this can be done.
How many different possible completed pictures are there (including the one shown)?



A5 In this puzzle, when you move up one square you **add 3**, when you move down one square you **subtract 3** and when you move to the right one square you **multiply by 2**.

The last square contains the number 65.

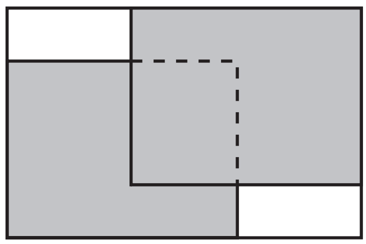
What number is in the square marked X ?



A6 Charlie's factory makes crème eggs and caramel eggs. The crème eggs are produced by a machine at the rate of 30 per minute, while the caramel eggs are produced by a different machine at the rate of 40 per minute. On a day when these two machines were in operation for a combined time of 18 hours, 36 000 eggs were produced in total. For how many hours was the crème egg machine in use?

A7 A sheet of paper is exactly the same size as a rectangular table top. The paper is cut in half and the two halves are placed on the table as shown.

What is the ratio of the area of table left uncovered (white) to the area which is covered twice?



A8 A large container holds 14 litres of a solution which is 25% antifreeze, the remainder being water. How many litres of antifreeze must be added to the container to make a solution which is 30% antifreeze?

A9 Colin has a collection of more than 24 coins. When he puts the coins in piles of 6, there are 3 coins remaining. When he puts the coins in piles of 8, there are 7 coins remaining. How many coins remain when he puts the coins in piles of 24?

A10 A closed rectangular box is a 'double cube', in which the top and bottom are squares, and the height is twice the width. The greatest distance between any two points of this box is 9 cm. What is the total surface area of the box?

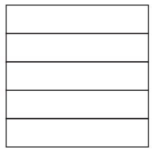
B1 The first three terms of a sequence are $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$. The fourth term is $\frac{1}{2} - \frac{1}{3} + \frac{1}{4}$; henceforth, each new term is calculated by taking the previous term, subtracting the term before that, and then adding the term before that.

(i) Write down the first six terms of the sequence, giving your answers as simplified fractions.

(ii) Find the 10th term and the 100th term, and explain why they have to be what you claim.

B2 The diagram shows a square which has been divided into five congruent rectangles. The perimeter of each rectangle is 51 cm.

What is the perimeter of the square?



B3

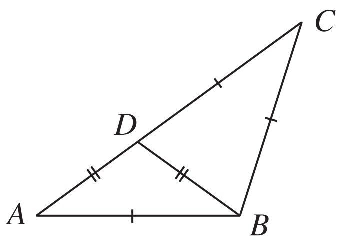
	175										70	

The diagram above is to be completed so that each box contains a whole number, the total of the numbers in the thirteen boxes is 2005 and the sum of the numbers in any three consecutive boxes is always the same.

In how many different ways is it possible to complete the diagram in this way?

B4 In this figure ADC is a straight line and $AB = BC = CD$. Also, $DA = DB$.

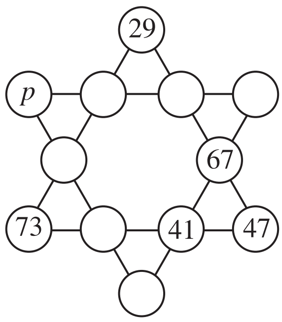
Find the size of $\angle BAC$.



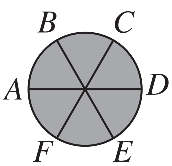
B5 In a magic hexagram, the numbers in every line of four circles have the same total. The diagram shows a magic hexagram which uses twelve different prime numbers.

Five numbers, including the smallest and the largest of the twelve primes, are shown.

Find the value of p, explaining the steps in your reasoning.



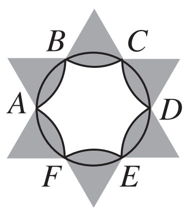
B6



Points A, B, C, D, E and F are equally spaced around a circle of radius 1. The circle is divided into six sectors as shown on the left.

The six sectors are then rearranged so that A, B, C, D, E and F lie on a new circle, also of radius 1, as shown on the right with the sectors pointing outwards.

Find the area of the curvy *unshaded* region.



A1 Write $1 + \dfrac{1}{1 + \frac{1}{3}}$ as a decimal.

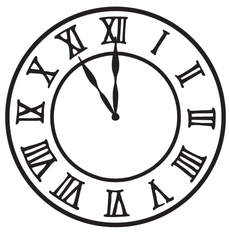
A2 Gus is older than Flora. Alessandro is older than Zara but younger than Flora. Oliver is younger than Gus but older than Zara. Yvette is younger than Gus. Alessandro is older than Oliver. Flora is younger than Yvette. Which of these six friends is the youngest?

A3 On Monday the Pied Piper caught 1000 rats in a city. On Tuesday he caught 10% fewer than on Monday. On Wednesday he caught 20% more than on Tuesday. On Thursday he caught 30% fewer than on Wednesday. On Friday he rested. How many rats did he catch in total that week?

A4 Three brothers and a sister shared a sum of money equally among themselves. If the brothers alone had shared the money, then they would have increased the amount they each received by £20. What was the original sum of money?

A5 Walking up a steep hill, I pass 10 equally spaced street lamps. I take 5 seconds to walk from the first lamp to the second lamp, 6 seconds from the second lamp to the third, and so on, with each time increasing by 1 second as I slow down. How long do I take to walk from the first lamp to the last?

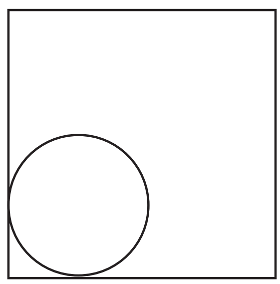
A6 Professor Brainstorm's clock gains 16 minutes every day. After he has set it to the correct time, how many days pass before it next tells the correct time?



A7 The square of a number is 12 more than the number itself. The cube of the number is 9 times the number. What is the number?

A8 A large pan contains a mixture of oil and water. After 2 litres of water are added to the original contents of the pan, the ratio of oil to water is 1:2. However, when 2 litres of oil are added to the new mixture, the ratio becomes 2:3. Find the original ratio of oil to water in the pan.

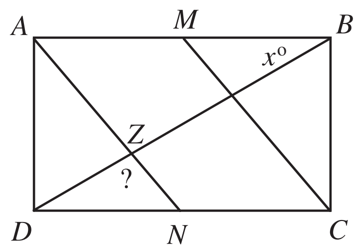
A9 A circle of radius 1 rolls without slipping round the inside of a square of side 4. Find an expression involving π for the exact number of revolutions the circle makes by the time it first returns to its original position.



A10 The Famous Five have been given 20 sweets as a reward for solving a tricky crime. They have agreed that the oldest of them must receive more than the next oldest, who must receive more than the next oldest, and so on. Assuming that each of the five gets at least one sweet, in how many different ways can they share the sweets?

B1 In the rectangle $ABCD$, M and N are the midpoints of AB and CD respectively; AB has length 2 and AD has length 1.

Given that $\angle ABD = x°$, calculate $\angle DZN$ in terms of x.



B2 Three identical rectangular cards can be placed end to end (with their short sides touching) to make rectangle A, and can be placed side by side (with their long sides touching) to make rectangle B. The perimeter of rectangle A is twice the perimeter of rectangle B.

Find the ratio of the length of a short side to the length of a long side of each card.

B3 The solution to each clue of this cross-number is a two-digit number. None of these numbers begins with zero.

Complete the crossnumber, stating the order in which you solved the clues and explaining why there is only one possibility at each stage.

Clues Across

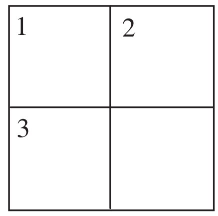
1. Multiple of 3

3. Three times a prime

Clues Down

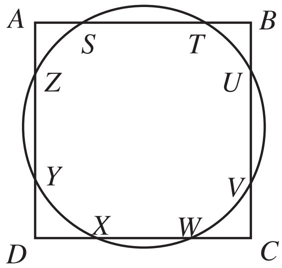
1. Multiple of 25

2. Square



B4 In the square $ABCD$, S is the point one quarter of the way from A to B and T is the point one quarter of the way from B to A. The points U, V, W, X, Y, Z are defined similarly. The eight points S, T, U, V, W, X, Y, Z lie on a circle, whose centre is at the centre of the square.

Determine which has the larger area: the square $ABCD$, or the circle.



B5 On an adventure holiday five children, called A, B, C, D, E, all take part in five competitions, called V, W, X, Y, Z. In each competition marks of 5, 4, 3, 2, 1 are awarded for coming 1st, 2nd, 3rd, 4th or 5th respectively. There are no ties for places.

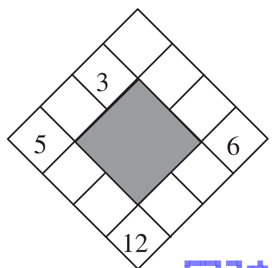
Child A scores a total of 24 marks, child C scores the same in each of four competitions, child D scores 4 in competition V, and child E scores 5 in W and 3 in X. Surprisingly, their overall positions are in alphabetical order.

Show that this information is enough to find all the scores, and that there is only one solution. Give the marks scored by each child in each competition by filling in a copy of this table.

	V	W	X	Y	Z	Total
A						
B						
C						
D						
E						

B6 Suppose that the diagram is to be completed so that each white square contains a different whole number from 1 to 12 inclusive and also so that the four numbers in the set of squares along each edge have the same total.

In how many different ways can the diagram be completed correctly?

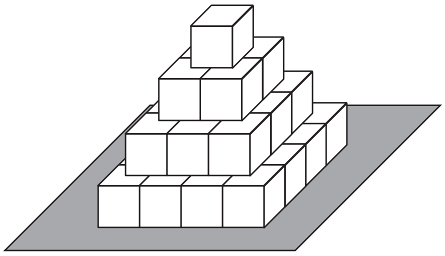


A1 Calculate $\sqrt{4 + \sqrt{16 + \sqrt{81}}}$.

A2 The diagram shows a solid tower built from identical white cubes.

The outer surface of the tower, apart from where it is in contact with the table, is now painted red.

If the tower is then broken up into individual cubes, how many cubes will still be completely white?



A3 Sam starts to list, in ascending order, every positive integer which is *not* a factor of 720.
What is the tenth number on her list?

A4 In this addition, G, N and O represent different digits, none of which is zero.

What is the value of $G + N + O$?

$$\begin{array}{r} ON \\ ON \\ ON \\ +\,ON \\ \hline GO \end{array}$$

A5 Part of a bridge spans a river 35 metres wide. One third of the length of the bridge is on one side of the river and 20% of the length of the bridge is on the other side. How long is the bridge?

A6 Given a "starting" number, you double it and add 1, then divide the answer by 1 less than the starting number to get the "final" number.
If you start with 2, your final number is 5. If you start with 4, your final number is 3.
What starting number gives the final number 4?

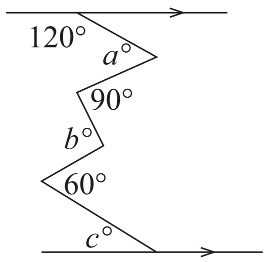
A7 What is the sum of the five numbers which must be placed in the empty cells of this magic square so that every row, every column and both diagonals all have the same total?

7		8
12		13

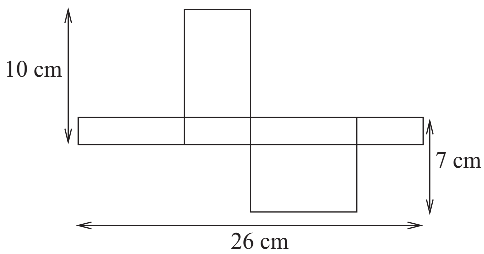
A8 The perimeter of this symmetrical creature is made from two semicircles of radius 4 cm and four semicircles of radius 2 cm.

Find the area enclosed by this perimeter.

A9 Find the value of $a + b + c$.



A10 This is the net of a cuboid. What is the volume of the cuboid?



B1 Find all of the ways in which 200 can be written in the form $p + q^2 + r^3$ where p, q and r are prime numbers.

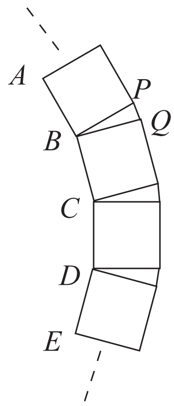
B2 Five children, boys Vince, Will, and Zac and girls Xenia and Yvonne, sit at a round table. They come from five different cities, Aberdeen, Belfast, Cardiff, Durham and Edinburgh. The child from Aberdeen sits between Zac and the child from Edinburgh. Neither of the two girls is sitting next to Will. Vince sits between Yvonne and the child from Durham. Zac writes to the child from Cardiff.

Find, giving reasons, where each child comes from.

B3 The diagram shows part of a ring of squares and triangles around a regular polygon which has vertices A, B, C, D, E, ….

(a) Suppose the polygon $ABCDE\ldots$ has 10 sides. Calculate the sizes of $\angle ABC$ and $\angle BQP$.

(b) Suppose the regular polygon has N sides.
Find the value of the ratio of the size of $\angle ABC$ to the size of $\angle BQP$.

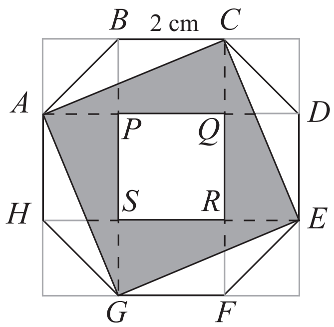


B4 (a) Find the sum of all positive three-digit integers each of whose digits is either 2 or 3.

(b) Find the sum of all positive six-digit integers each of whose digits is either 2 or 3, giving your answer as the product of prime numbers.

B5 The diagram shows two squares, $ACEG$ and $PQRS$, inside a regular octagon $ABCDEFGH$ which has sides of length 2 cm.

What fraction of the entire octagon is shaded?



B6 12 is a 2-digit number such that the number '1' formed by the first digit is divisible by 1 and the number '12' formed by the first two digits is divisible by 2.

(a) How many 3-digit numbers 'abc' are there, where a, b, c are the digits 1, 2, 3 in some order, such that 'a' is divisible by 1, 'ab' is divisible by 2 and 'abc' is divisible by 3?

(b) How many 4-digit numbers '$abcd$' are there, where a, b, c, d are the digits 1, 2, 3, 4 in some order, such that 'a' is divisible by 1, 'ab' is divisible by 2, 'abc' is divisible by 3 and '$abcd$' is divisible by 4?

(c) How many 5-digit numbers '$abcde$' are there, where a, b, c, d, e are the digits 1,2,3,4,5 in some order, such that 'a' is divisible by 1, 'ab' is divisible by 2, 'abc' is divisible by 3, '$abcd$' is divisible by 4 and '$abcde$' is divisible by 5?

(d) How many 6-digit numbers '$abcdef$' are there, where a, b, c, d, e, f are the digits 1, 2, 3, 4, 5, 6 in some order, such that 'a' is divisible by 1, 'ab' is divisible by 2, 'abc' is divisible by 3, '$abcd$' is divisible by 4, '$abcde$' is divisible by 5 and '$abcdef$' is divisible by 6?

UK Junior Mathematical Olympiad 2002

Organised by The United Kingdom Mathematics Trust

Tuesday 11th June 2002

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required.* Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first hour so as to allow at least one hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Numerical answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

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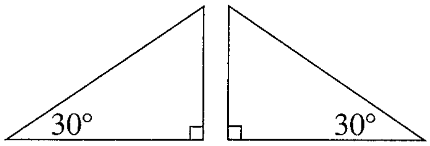
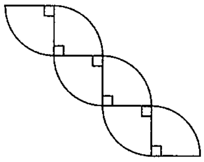
Section A

A1 Two identical right-angled triangles are made out of cardboard. How many different shapes can be produced by gluing the two triangles together, matching two equal edges?

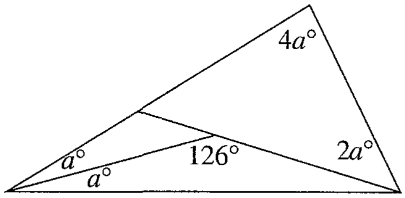


A2 Calculate $[(-1) + (-1)^2] \div [(-1) - (-1)^2]$.

A3 In the figure shown, each arc is of radius r. What is the perimeter of the figure?



A4 What is the value of a in the triangle shown?

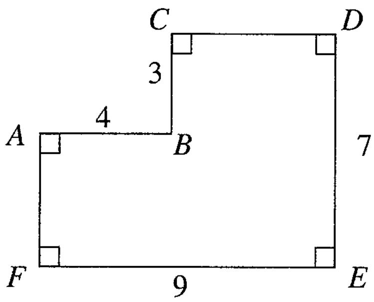


A5 The sum of seven consecutive even numbers is 2002. What is the smallest of these seven numbers?

A6 Find two whole numbers, neither of which ends in the digit zero, whose product is ten thousand.

A7 Two thirds of five sixths of a number X is the same as three quarters of four fifths of a number Y. What is the value of $\frac{X}{Y}$ as a fraction in lowest terms?

A8 Calculate the length of BE in the figure shown.



A9 Given that $\frac{1}{x+6} = 4$, find the value of $\frac{1}{x+8}$.

A10 Two boxes, P and Q, each contain 3 jewels. When a jewel worth £5000 is transferred from P to Q, the average value of the jewels in each box increases by £1000. What is the total value of all 6 jewels?

Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 A number like 4679 is called an *ascending* number because each digit in the number is larger than the preceding one.

(i) How many ascending numbers are there between 1000 and 2000?

(ii) How many ascending numbers are there between 1000 and 10000?

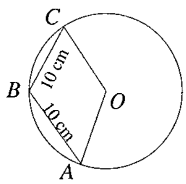
B2 Five teams played in a competition and every team played once against each of the other four teams. Each team received three points for a match it won, one point for a match it drew and no points for a match it lost. At the end of the competition the points were:

Yellows 10, Reds 9, Greens 4, Blues 3 and Pinks 1.

(i) How many of the matches resulted in a draw?

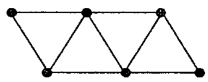
(ii) What were the results of Greens' matches against the other four teams?

B3 In the diagram, O is the centre of the circle. The lengths of AB and BC are both 10 cm. The area of quadrilateral $OABC$ is 120 cm^2. Calculate the radius of the circle.



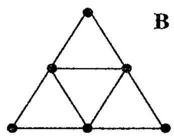
B4 (i) Network A has nine edges which meet at six nodes. The numbers 1, 2, 3, 4, 5, 6 are placed at the nodes, with a different number at each node. Is it possible to do this so that the sum of the two numbers at the ends of an edge is different for each edge?

Either show a way of doing this, or prove that it is impossible.



A

(ii) Repeat the same procedure for Network B, i.e. show that it is possible to place the six numbers so that the sum of the two numbers at the ends of an edge is different for each edge, or prove that it is impossible to do so.

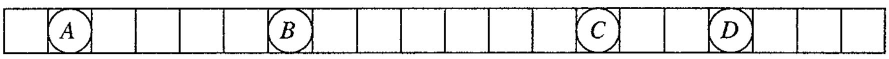


B

B5 *ABCDE* is a pentagon in which triangles *ABC*, *AED* and *CAD* are all isosceles, $AC = AD$, $\angle CAD$ is acute. Interior angles *ABC* and *AED* are both right angles.

Draw a sketch of pentagon *ABCDE*, marking all the equal sides and equal angles. Show how to fit four such identical pentagons together to form a hexagon. Explain how you know the pentagons will fit exactly.

B6 A game for two players uses four counters on a board which consists of a 20×1 rectangle. The two players take it in turns to move one counter. A turn consists of moving any one of the four counters any number of squares to the right, but the counter may not land on top of, or move past, any of the other counters. For instance, in the position shown, the next player could move D one, two or three squares to the right, or move C one or two squares to the right and so on.



The winner of the game is the player who makes the last legal move. (After this move the counters will occupy the four squares on the extreme right of the board and no further legal moves will be possible.)

In the position shown above, it is your turn. Which move should you make and what should be your strategy in subsequent moves to ensure that you will win the game?

UK Junior Mathematical Olympiad 2001

Organised by The United Kingdom Mathematics Trust

Tuesday 12th June 2001

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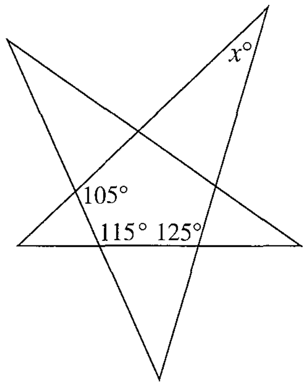
Section A

A1 In the late 18th century, a decimal clock was proposed, in which there were 100 'minutes' in one 'hour' and 10 'hours' in one day. Assuming that such a clock started from 0:00 at midnight, what time would it show when an ordinary clock showed 6 o'clock the following morning?

A2 $\square + 8 \div 2 = 10.$
Which number should replace the box to make a true statement?

A3 What is the value of x in the diagram alongside?



A4 The areas of three of the faces of a cuboid are 24 cm^2, 18 cm^2 and 12 cm^2.
What is the volume of the cuboid ?

A5 Find the 100th digit after the decimal point in the decimal representation of $\frac{3}{7}$.

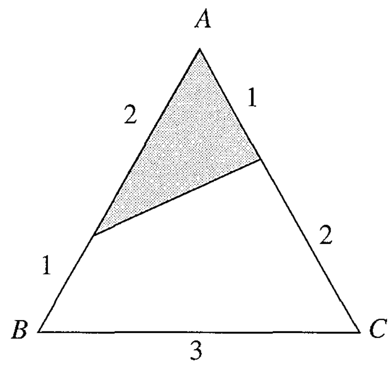
A6 'One third of the population now has access to the internet; this is 50% more than one year ago.'
What fraction of the population had access to the internet one year ago?

A7 The length of each side of a quadrilateral $ABCD$ is a whole number of centimetres. Given that $AB = 4$ cm, $BC = 5$ cm and $CD = 6$ cm, what is the maximum possible length of the fourth side DA?

A8 Find the smallest three-digit number which is neither prime nor divisible by 2, 3 or 5.

A9 Points A, B, P, C, Q, D, R, E, S and F are marked in that order around the circumference of a circle so that $ABCDEF$ is a regular hexagon and $APQRS$ is a regular pentagon.
What is the size of $\angle BAP$?

A10 What fraction of triangle ABC is shaded?



Section B

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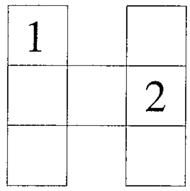
B1 The numbers from 1 to 7 inclusive are to be placed, one per square, in the diagram on the right so that the totals of the three numbers in the horizontal row and each of the two columns are the same.

In how many different ways can this be done if the numbers 1 and 2 must be in the positions shown?



B2 In a sequence, each term after the first is the sum of the squares of the digits of the previous term. Thus, if the first term were 12, the second term would be $1^2 + 2^2 = 5$, the third term $5^2 = 25$, the fourth term $2^2 + 5^2 = 29$ and so on.

(i) Find the first five terms of the sequence whose first term is 25.

(ii) Find the 2001st term of the sequence whose first term is 25.

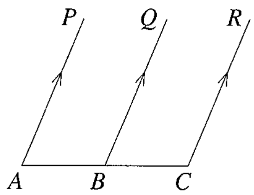
B3 In the diagram, B is the midpoint of AC and the lines AP, BQ and CR are parallel. The bisector of $\angle PAB$ meets BQ at Z.

Draw a diagram to show this, and join Z to C.

(i) Given that $\angle PAZ = x°$, find $\angle ZBC$ in terms of x.

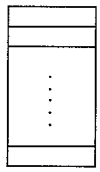
(ii) Show that CZ bisects $\angle BCR$.

(You must give full reasons to justify your answers.)



B4 The diagram shows a large rectangle whose perimeter is 300 cm. It is divided up as shown into a number of identical rectangles, each of perimeter 58 cm. Each side of these rectangles is a whole number of centimetres.

Show that there are exactly two possibilities for the number of smaller rectangles and find the size of the large rectangle in each case.

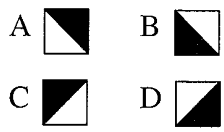


B5 Observe that $49 = 4 \times 9 + 4 + 9$.

(i) Find all other two-digit numbers which are equal to the product of their digits plus the sum of their digits.

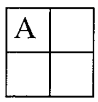
(ii) Prove that there are no three-digit numbers which are equal to the product of their digits plus the sum of their digits.

B6 This question is about ways of placing square tiles on a square grid, all the squares being the same size. Each tile is divided by a diagonal into two regions, one black and one white. Such a tile can be placed on the grid in one of four different positions as shown:



When two tiles meet along an edge (side by side or one below the other) the two regions which touch must be of different types (i.e. one black and one white).

(i) A 2×2 grid of four squares is to be covered by four tiles.



(a) If the top-left square is covered by a tile in position A, find all the possible ways in which the other three squares may be covered.

(b) In how many different ways can a 2×2 grid be covered by four tiles?

(ii) In how many different ways can a 3×3 grid be covered by nine tiles?

(iii) Explaining your reasoning, find a formula for the number of different ways in which a square grid measuring $n \times n$ can be covered by n^2 tiles.

UK Junior Mathematical Olympiad 2000

Organised by The United Kingdom Mathematics Trust

Tuesday 6th June 2000

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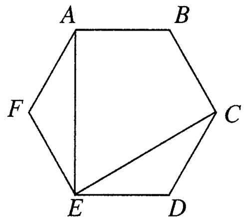
Section A

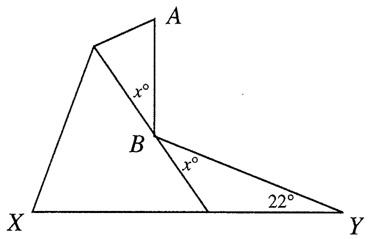
A1 What is the value of $2000 + 1999 \times 2000$?



A2 *ABCDEF* is a regular hexagon.
What fraction of the area of the hexagon is the area of the kite *ABCE*?

A3 Estimate, correct to the nearest mm, the side of a square of area $0{\cdot}5\ \text{cm}^2$.

A4 What is 20% of 30% of 40% of £50?



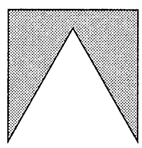
A5 The diagram shows part of a mosaic of tiles.
AB is vertical and *XY* is horizontal.

What is the value of x?

A6 The hour hand on the Mad Hatter's watch moves at the correct speed, but the minute hand moves one and a half times as fast as it should. Yesterday, it showed the correct time at 3 p.m. When did it next show the correct time?

A7 Six pupils have, between them, won three gold medals, two silver medals and a bronze medal in a painting competition. Unfortunately, their teacher, Mr. Turner, has lost all record of which medals should go to which pupils, so he allocates them by drawing names out of a hat. The first three names drawn receive the gold medals, the next two drawn have the silver medals and the bronze medal goes to the remaining pupil.
In how many different ways can the medals be allocated by this method?

A8 An equilateral triangle is cut out of a square of side 2 cm, as shown.



What area of the square remains?

A9 The machine which prints photographs at *Snippysnaps* runs for the same time every day. It prints colour photographs at a constant rate and monochrome (black and white) photographs at a different constant rate.
On Monday, the machine printed 2100 colour photographs and 2450 monochrome photographs.
On Tuesday it printed 2800 colour photographs and 1400 monochrome photographs.
On Wednesday, it printed only monochrome photographs.
How many of these were there?

A10 It takes four gardeners four hours to dig four circular flower beds, each of diameter four metres. How long will it take six gardeners to dig six circular flower beds, each of diameter six metres?

Section B

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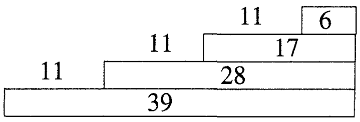
B1 Kate has 90 identical building blocks. She uses all of the blocks to build this four-step 'staircase' in which each step, apart from the top one, is the same length.



(i) Show that there are exactly two different ways in which it is possible to use all 90 blocks to build a six-step 'staircase'.

(ii) Explain fully why it is impossible to use all 90 blocks to build a seven-step 'staircase'.

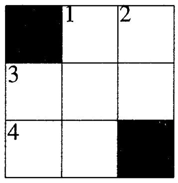
B2 A crossnumber puzzle is like a crossword puzzle – except that the answers are numbers instead of words and each square contains one single digit. None of the answers starts with the digit 0.

How many solutions are there to this crossnumber?

(You must use logic, not guesswork.)

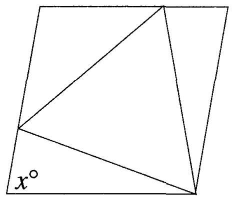
Across
1. Square
3. Square
4. Square

Down
1. Cube
2. Square
3. Cube times square



B3 The diagram shows an equilateral triangle inside a rhombus. The sides of the rhombus are equal in length to the sides of the triangle.

What is the value of x?



B4 How many different solutions are there to the letter sum on the right?

Different letters stand for different digits, and no number begins with a zero.

```
  J M C
+ J M O
-------
S U M S
```

B5 (i) Explain why the sum of three consecutive integers is always divisible by 3.

(ii) Is it true that the sum of four consecutive integers is always divisible by 4?

(iii) For which k is it true that the sum of k consecutive integers is always divisible by k?

B6 X and Y play a game in which X starts by choosing a number, which must be either 1 or 2.

Y then adds either 1 or 2 and states the total of the two numbers chosen so far. X does likewise, adding either 1 or 2 and stating the total, and so on. The winner is first player to make the total reach (or exceed) 20.

(i) Explain how X can always win.

(ii) The game is now modified so that at each stage the number chosen must be 1 or 2 or 4. Which of X or Y can now always win and how?

UK Junior Mathematical Olympiad 1999

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Tuesday 8th June 1999

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Section A

A1 What is the angle between the hands of a clock at 4.20 p.m.?

A2 In how many different ways can 50 be written as the sum of two prime numbers ?
(Note: $x + y$ and $y + x$ do not count as different.)

A3 Today's date, 8/6/99, might be read as a fraction in two different ways:

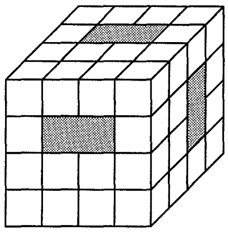
$$\frac{{}^{8}/_{6}}{99} \quad \text{or} \quad \frac{8}{{}^{6}/_{99}}.$$

Find the whole number nearest to the sum of these two fractions.

A4 Tickets for a concert cost £7 for an adult and £5 for a child.
The organiser of a group works out that the tickets for the group will cost a total of £108, and notices that the cost would be the same if the prices were changed to £8 per adult and £4 per child.
How many children are in the group?

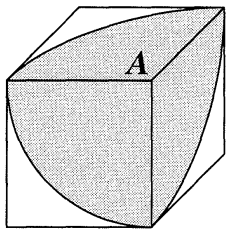
A5 UKMT, TMUK and KTUM are all different arrangements of the letters U, K, M and T. If the number of all the different arrangements of these four letters is p and the number of all the different arrangements of the letters U, K, J, M and O is q, what is the value of $\frac{q}{p}$?

A6 A cube is made of 64 small cubes. Three holes are made, with each hole perpendicular to two faces and passing right through the cube. The shape and position of each hole is shown in the diagram.
How many small cubes are in the remaining solid?



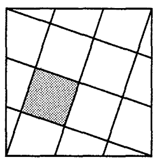
A7 Before the decimalisation of money in the UK, there were 12 pence (d) in 1 shilling (s) and 20 shillings in 1 pound (£). Thus 1 pound 3 shillings and 4 pence was written £1 3s 4d. What would have been the total cost of 7 items each costing £1 6s 8d? Write your answer in simplest £ s d form.

A8 At a corner, A, of the cube shown here, a circular arc with centre A is drawn on each of the three faces meeting at the corner A.
What fraction of the surface area of the cube is shaded?



A9 Skimmed milk contains 0.1% fat and pasteurised whole milk contains 4% fat. When 6 litres of skimmed milk are mixed with n litres of pasteurised whole milk, the fat content of the resulting mixture is 1.66%.
What is the value of n?

A10 What fraction of the whole square is occupied by the shaded square?



Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 Suppose you know that the middle two digits of a four digit integer N are '12' in that order and that N is an exact multiple of 15.

Determine all the different possibilities for the integer N.

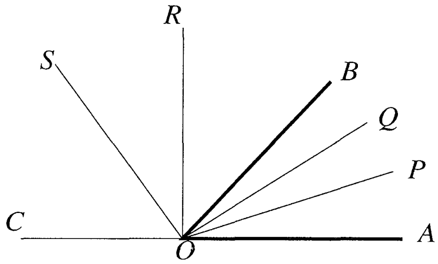
(You must explain clearly why your list is complete.)

B2 AOC is a straight line and angle $AOB = 42°$. OP and OQ trisect angle AOB (which means they divide the angle into three equal parts).

OR and OS trisect angle BOC.

(a) Showing all working, calculate angle QOR and angle POS.

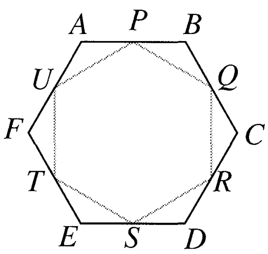
(b) Calculate QOR when angle $AOB = x°$.



B3 Alice, the March Hare and the Mock Turtle were the only three competitors at the Wonderland sports day, and all three of them competed in each event. The scoring system was exactly the same for each event: the points awarded for first, second and third places were all positive integers and (even in Wonderland) more points were awarded for first place than for second and more points for second place than for third.

Of course, the March Hare won the Sack Race. At the end of the day, Alice had scored 18 points while the Mock Turtle had 9 points and the March Hare had 8 points. Can you decide how many events there were? And can you tell who came last in 'Egg and Spoon' race?

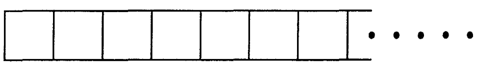
B4 The regular hexagon $ABCDEF$ has sides of length 2. The point P is the midpoint of AB, Q is the midpoint of BC, and so on. Find the area of the hexagon $PQRSTU$.

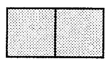


B5 (a) Find all the two-digit numbers which are increased by 75% when their digits are reversed.

(b) Find all the three-digit numbers which are increased by 75% when their digits are reversed.

B6





A counter.

Two players, X and Y, play a game on a board which consists of a narrow strip which is one square wide and n squares long. They take turns at placing counters, which are one square wide and two squares long, on unoccupied squares on the board. The first player who cannot go loses. X always plays first and both players always make the best available move.

(a) Who wins the game on a 4×1 board? Explain how they must play to win and why they are then certain to win.

(b) Who wins the game on a 5×1 board? Explain why. So who wins on a 7×1 board? Explain.

(c) Who wins the game on a 6×1 board? How?

(d) Who wins the game on an 8×1 board? How?

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