United Kingdom
Mathematics Trust

Intermediate Mathematical Challenge

Follow-up Competitions

2003 – 2019 Collection

August 18, 2020

Comments and suggestions to 89272376@QQ.com .

IMC Follow-up 2

Year 10/11 Pink Kangaroo

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Comments and suggestions to 89272376@QQ.com .

Answers:

	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	
1	C	C	C		A	D	C	D	D	D	D	C	C	B	B	C	D		1
2	A	A	C		A	C	A	D	A	C	C	E	B	A	E	C	B		2
3	A	C	B		C	B	C	C	C	A	C	A	A	B	D	E	E		3
4	E	E	B		D	B	B	D	D	C	E	C	E	D	D	A	C		4
5	D	D	D		E	C	B	E	C	E	E	E	E	C	C	B	B		5
6	D	D	B		A	C	E	C	C	B	C	C	E	D	A	B	D		6
7	B	C	E		B	B	B	A	E	D	A	C	C	C	D	A	B		7
8	B	A	D		B	D	E	C	B	D	A	B	B	E	C	D	A		8
9	C	D	E		C	E	B	D	B	C	D	D	C	D	E	B	B		9
10	D	B	C		B	C	E	B	A	B	B	A	C	A	C	D	C		10
11	B	B	B		C	B	A	B	C	B	C	E	D	E	B	C	C		11
12	D	D	D		A	A	C	C	A	C	E	E	C	C	B	E	E		12
13	B	C	B		B	D	D	D	E	B	A	C	A	B	A	B	D		13
14	C	E	C		B	A	A	B	E	A	A	D	B	D	C	A	C		14
15	D	B	E		E	B	C	C	D	E	A	C	E	C	A	D	C		15
16	A	D	D		D	D	E	A	A	D	B	D	E	B	A	E	D		16
17	E	D	C		C	D	C	B	B	E	D	D	E	A	D	B	E		17
18	E	C	B		C	E	C	B	D	A	D	D	B	C	C	A	A		18
19	A	C	D		C	E	C	B	C	C	E	B	C	B	C	C	D		19
20	B	B	A		B	A	B	A	B	B	B	C	D	E	E	E	C		20
21	C	E	C		E	B	A	A	C	D	E	C	C	A	B	B	C		21
22	B	B	E		D	C	B	D	A	C	B	B	A	C	E	D	D		22
23	D	B	E		D	A	B	E	C	C	B	E	B	A	B	B	B		23
24	C	D	D		C	E	C	A	E	B	D	B	C	B	D	D	A		24
25	C	D	A		D	D	D	D	E	B	A	C	B	D	A	D	C		25

1. What is the value of $20 \times 19 + 20 + 19$?

A 389 B 399 C 409 D 419 E 429

2. A model train takes exactly 1 minute and 11 seconds for one complete circuit of its track. How long does it take for six complete circuits?

A 6 minutes and 56 seconds B 7 minutes and 6 seconds C 7 minutes and 16 seconds
D 7 minutes and 26 seconds E 7 minutes and 36 seconds

3. A barber wants to write the word SHAVE on a board behind the client's seat in such a way that a client looking in the mirror reads the word correctly. Which of the following should the barber write on the board?

A SHAVE B SHAVƎ C ƎVAHS D EVAHƧ E ƎVAHƧ

4. How many different totals can be obtained by rolling three standard dice and adding up the scores?

A 14 B 15 C 16 D 17 E 18

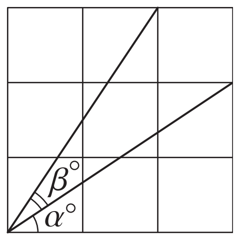
5. A park has five gates. In how many ways can Monica choose a gate to enter the park and a different gate to leave the park?

A 25 B 20 C 16 D 15 E 10

6. Pedro is asked to find three kangaroos whose weights are all whole numbers of kilograms and whose total weight is 97 kg. What is the largest possible weight of the lightest of the kangaroos Pedro could find?

A 1 kg B 30 kg C 31 kg D 32 kg E 33 kg

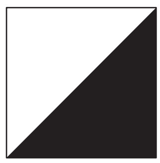
7. Two angles are marked on the 3×3 grid of squares.



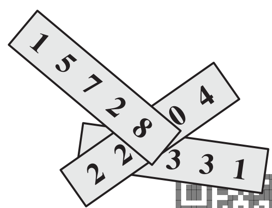
Which of the following statements about the angles is correct?

A $\alpha = \beta$ B $2\alpha + \beta = 90$ C $\alpha + \beta = 60$ D $2\beta + \alpha = 90$
E $\alpha + \beta = 45$

8. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

A  B  C  D  E 

9. On each of three pieces of paper a five-digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. What are the covered digits?



A 0, 2 and 2 B 1, 2 and 9 C 2, 4 and 9 D 2, 7 and 8
E 5, 7 and 8

10. A square has vertices P, Q, R, S labelled clockwise. An equilateral triangle is constructed with vertices P, T, R labelled clockwise. What is the size of angle RQT in degrees?

A 30 B 45 C 135 D 145 E 150

11. The numbers a, b, c and d are distinct positive integers chosen from 1 to 10 inclusive. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?

A $\frac{2}{10}$ B $\frac{3}{19}$ C $\frac{14}{45}$ D $\frac{29}{90}$ E $\frac{25}{72}$

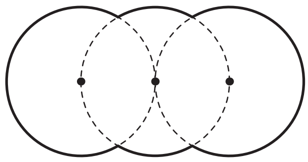
12. The flag of Kangaria is a rectangle with side-lengths in the ratio 3 : 5. The flag is divided into four rectangles of equal area as shown. What is the ratio of the length of the shorter sides of the white rectangle to the length of its longer sides?

A 1 : 3 B 1 : 4 C 2 : 7 D 3 : 10 E 4 : 15

13. The triathlon consists of swimming, cycling and running. The cycling accounts for three-quarters of the total distance, the running for one-fifth and the swimming for 2 km. What is the total distance of this triathlon?

A 10 km B 20 km C 38 km D 40 km E 60 km

14. The diagram shows a shape made of arcs of three circles, each with radius R. The centres of the circles lie on the same straight line, and the middle circle passes through the centres of the other two circles. What is the perimeter of the shape?



A $\frac{2\pi R\sqrt{3}}{3}$ B $\frac{5\pi R}{3}$ C $\frac{10\pi R}{3}$ D $2\pi R\sqrt{3}$ E $4\pi R$

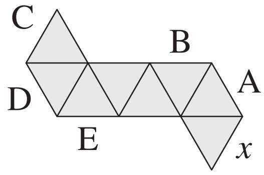
15. The sum of the seven digits of the number '$aaabbbb$' is equal to the two-digit number 'ab'. What is the value of $a + b$?

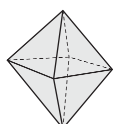
A 8 B 9 C 10 D 11 E 12

16. Sixty apples and sixty pears are to be packed into boxes so that each box contains the same number of apples, and no two boxes contain the same number of pears. What is the largest possible number of boxes that can be packed in this way?

A 20 B 15 C 12 D 10 E 6

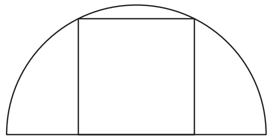
17. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment labelled x?





A B C D E

18. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown. The radius of the circle is 1. What is the area of the square?



A $\frac{4}{5}$ B $\frac{\pi}{4}$ C 1 D $\frac{4}{3}$ E $\frac{2}{\sqrt{3}}$

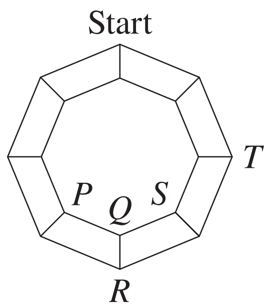
19. The integers from 1 to 99 are written in ascending order without spaces. The sequence of digits is then grouped into triples of digits:

$$123456789101112\ldots979899 \to (123)(456)(789)(101)(112)\ldots(979)(899).$$

Which of the following is not one of the triples?

A (222) B (434) C (464) D (777) E (888)

20. A network consists of 16 vertices and 24 edges that connect them, as shown. An ant begins at the vertex labelled Start. Every minute, it walks from one vertex to a neighbouring vertex, crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 minutes?



A only P, R or S, B not Q C only Q
D only T E all of the vertices are possible

21. Each of the positive integers a, b, and c has three digits, and for each of these integers the first digit is the same as its last digit. Also $b = 2a + 1$ and $c = 2b + 1$. How many possibilities are there for the integer a?

A 0 B 1 C 2 D 3 E more than 3

22. A positive integer is to be placed on each vertex of a square. For each pair of these integers joined by an edge, one should be a multiple of the other. However, for each pair of diagonally opposite integers, neither should be a multiple of the other. What is the smallest possible sum of the four integers?

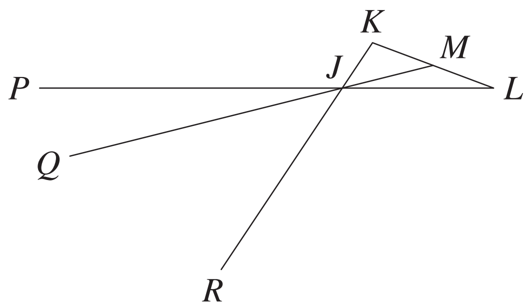
A 12 B 24 C 30 D 35 E 60

23. Rhona wrote down a list of nine multiples of ten: 10, 20, 30, 40, 50, 60, 70, 80, 90. She then deleted some of the nine multiples so that the product of the remaining multiples was a square number. What is the least number of multiples that she could have deleted?

A 1 B 2 C 3 D 4 E 5

24. The diagram shows triangle JKL of area S. The point M is the midpoint of KL. The points P, Q, R lie on the extended lines LJ, MJ, KJ, respectively, such that $JP = 2 \times JL$, $JQ = 3 \times JM$ and $JR = 4 \times JK$.

What is the area of triangle PQR?



A S B $2S$ C $3S$ D $\frac{1}{2}S$ E $\frac{1}{3}S$

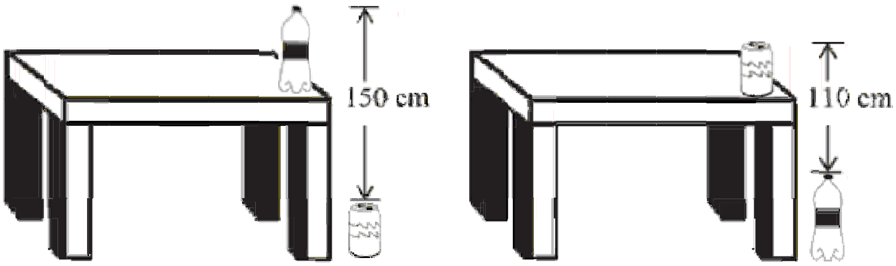
25. How many four-digit numbers have the following property? "For each of its digits, when this digit is deleted the resulting three-digit number is a factor of the original number."

A 5 B 9 C 14 D 19 E 23

1. The lengths of two sides of a triangle are 5 cm and 2 cm. The length of the third side in cm is an odd integer. What is the length of the third side?

A 1 cm B 3 cm C 5 cm D 7 cm E 9 cm

2. The distance from the top of the can on the floor to the top of the bottle on the table is 150 cm. The distance from the top of the bottle on the floor to the top of the can on the table is 110 cm. What is the height of the table?



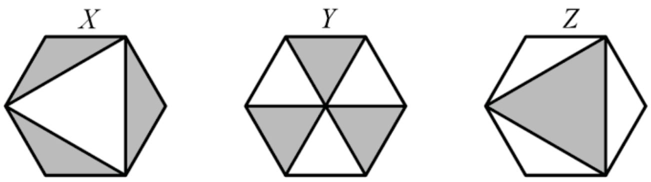
A 110 cm B 120 cm C 130 cm D 140 cm E 150 cm

3. The sum of five consecutive integers is 10^{2018}. What is the middle number?

A 10^{2013} B 5^{2017} C 10^{2017} D 2^{2018} E 2×10^{2017}

4. The diagram shows three congruent regular hexagons. Some diagonals have been drawn, and some regions then shaded. The total shaded areas of the hexagons are X, Y, Z as shown. Which of the following statements is true?

A X, Y and Z are all the same
B Y and Z are equal, but X is different
C X and Z are equal, but Y is different
D X and Y are equal, but Z is different
E X, Y, Z are all different



5. Marta has collected 42 apples, 60 apricots and 90 cherries. She wants to divide them into identical piles using all of the fruit and then give a pile to some of her friends. What is the largest number of piles she can make?

A 3 B 6 C 10 D 14 E 42

6. Some of the digits in the following correct addition have been replaced by the letters P, Q, R and S, as shown. What is the value of $P + Q + R + S$?

$$\begin{array}{rccc} & P & 4 & 5 \\ + & Q & R & S \\ \hline & 6 & 5 & 4 \end{array}$$

A 14 B 15 C 16 D 17 E 24

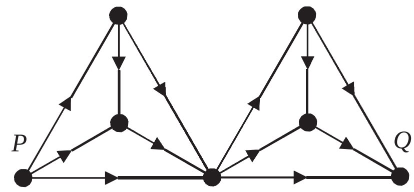
7. What is the sum of 25% of 2018 and 2018% of 25?

A 1009 B 2016 C 2018 D 3027 E 5045

8. Two buildings are located on one street at a distance of 250 metres from each other. There are 100 students living in the first building. There are 150 students living in the second building. Where should a bus stop be built so that the total distance that all residents of both buildings have to walk from their buildings to this bus stop would be the least possible?

A In front of the first building
B 100 metres from the first building
C 100 metres from the second building
D In front of the second building
E Anywhere between the buildings

9. Monika plans to travel across the network in the diagram from point P to point Q, travelling only in the direction of the arrows. How many different routes are possible?

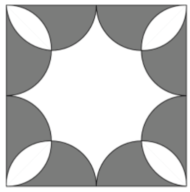


A 20 B 16 C 12 D 9 E 6

10. A sequence of positive integers starts with one 1, followed by two 2s, three 3s, and so on. (Each positive integer n occurs n times.) How many of the first 105 numbers in this sequence are divisible by 3?

A 4 B 12 C 21 D 30 E 45

11. Eight congruent semicircles are drawn inside a square of side-length 4. Each semicircle begins at a vertex of the square and ends at a midpoint of an edge of the square. What is the area of the non-shaded part of the square?



A 2π B $3\pi + 2$ C 8 D $6 + \pi$ E 3π

12. In a certain region are five towns, Freiburg, Göttingen, Hamburg, Ingolstadt and Jena.

On a certain day 40 trains each made a journey, leaving one of these towns and arriving at one of the other towns.

Ten trains travelled either from or to Freiburg. Ten trains travelled either from or to Göttingen.

Ten trains travelled either from or to Hamburg. Ten trains travelled either from or to Ingolstadt.

How many trains travelled from or to Jena?

A 0 B 10 C 20 D 30 E 40

13. At the University of Bugelstein you can study Languages, History and Philosophy.

35% of students that study a language study English.

13% of all the university students study a language other than English.

No student studies more than one language.

What percentage of the university students study Languages?

A 13 % B 20 % C 22 % D 48 % E 65 %

14. Peter wanted to buy a book, but he didn't have any money. He bought it with the help of his father and his two brothers. His father gave him half of the amount given by his brothers. His elder brother gave him one third of what the others gave. The younger brother gave him 10 euros. What was the price of the book?

A 24 euros B 26 euros C 28 euros D 30 euros E 32 euros

15. How many 3-digit numbers are there with the property that the 2-digit number obtained by deleting the middle digit is equal to one ninth of the original 3-digit number?

A 1 B 2 C 3 D 4 E 5

16. In the calculation shown, how many times does the term 2018^2 appear inside the square root to make the calculation correct?

$$\sqrt{2018^2 + 2018^2 + \dots + 2018^2} = 2018^{10}$$

A 5 B 8 C 18 D 2018^8 E 2018^{18}

17. A list of integers has a sum of 2018, a product of 2018, and includes the number 2018 in the list. Which of the following could be the number of integers in the list?

A 2016 B 2017 C 2018 D 2019 E 2020

18. Lonneke drew a regular polygon with 2018 vertices, which she labelled from 1 to 2018, in a clockwise direction. She then drew a diagonal from the vertex labelled 18 to the vertex labelled 1018. She also drew the diagonal from the vertex labelled 1018 to the vertex labelled 2000. This divided the original polygon into three new polygons. How many vertices did each of the resulting three polygons have?

A 38, 983, 1001 B 37, 983, 1001 C 38, 982, 1001 D 37, 982, 1000 E 37, 983, 1002

19. Abdul wrote down four positive numbers. He chose one of them and added it to the mean of the other three. He repeated this for each of the four numbers in turn.

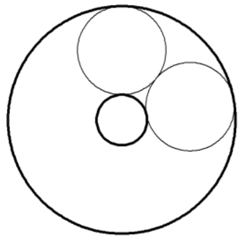
The results were 17, 21, 23 and 29. What was the largest of Abdul's numbers?

A 12 B 15 C 21 D 24 E 29

20. Omar marks a sequence of 12 points on a straight line beginning with a point O, followed by a point P with $OP = 1$. He chooses the points so that each point is the midpoint of the two immediately following points. For example O is the midpoint of PQ, where Q is the third point he marks. What is the distance between the first point O and the 12th point Z?

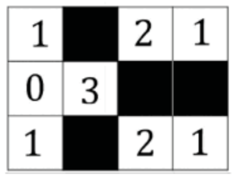
A 171 B 341 C 512 D 587 E 683

21. An annulus is a shape made from two concentric circles. The diagram shows an annulus consisting of two concentric circles of radii 2 and 9. Inside this annulus two circles are drawn without overlapping, each being tangent to both of the concentric circles that make the annulus. In a different annulus made by concentric circles of radii 1 and 9, what would be the largest possible number of non-overlapping circles that could be drawn in this way?



A 2 B 3 C 4 D 5 E 6

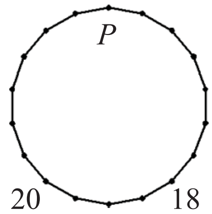
22. Diana drew a rectangular grid of 12 squares on squared paper. Some of the squares were then painted black. In each white square she wrote the number of black squares that shared an edge with it (a whole edge, not just a vertex). The figure shows the result. Then she did the same with a rectangular grid of 2 by 1009 squares. What is the maximum value that she could obtain as the result of the sum of all the numbers in this grid?



A 1262 B 2016 C 2018 D 3025 E 3027

23. At each vertex of the 18-gon in the picture a number should be written which is equal to the sum of the numbers at the two adjacent vertices.

Two of the numbers are given. What number should be written at the vertex P?



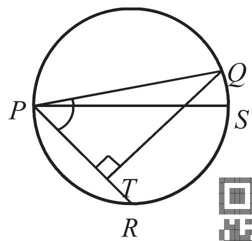
A 2018 B 38 C 18 D −20 E −38

24. Each of the numbers 1, 2, 3, 4, 5, 6 is to be placed in the cells of a 2×3 table, with one number in each cell. In how many ways can this be done so that in each row and in each column the sum of the numbers is divisible by 3?

A 36 B 42 C 45 D 48 E another number

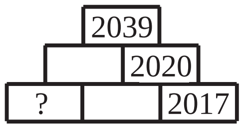
25. Two chords PQ and PR are drawn in a circle with diameter PS.

The point T lies on PR and QT is perpendicular to PR. The angle $QPR = 60°$, $PQ = 24$ cm, $RT = 3$ cm. What is the length of the chord QS in cm?



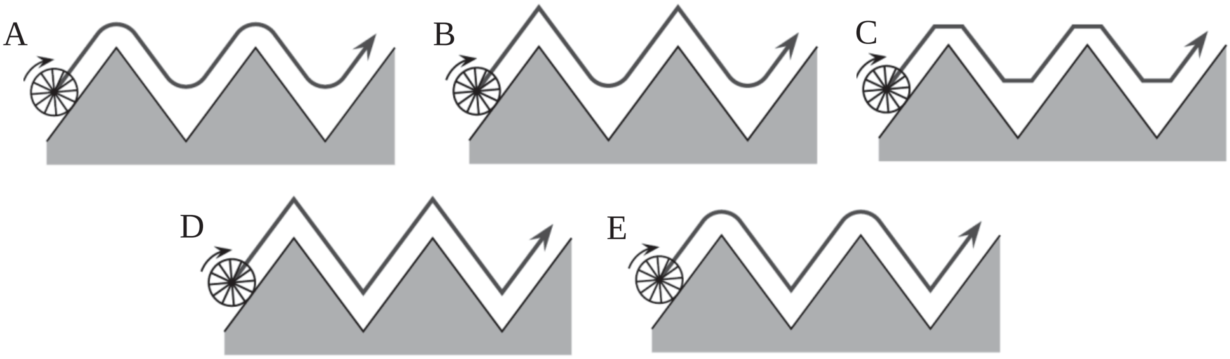
A $\sqrt{3}$ B 2 C 3 D $2\sqrt{3}$ E $3\sqrt{2}$

1. In the number pyramid shown each number is the sum of the two numbers immediately below. What number should appear in the left-hand cell of the bottom row?



A 15 B 16 C 17 D 18 E 19

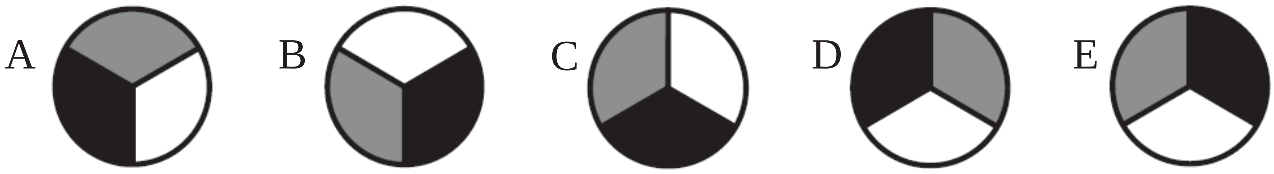
2. Which of the following diagrams shows the locus of the midpoint of the wheel when the wheel rolls along the zig-zag curve shown?



3. Some girls were dancing in a circle. Antonia was the fifth to the left from Bianca and the eighth to the right from Bianca. How many girls were in the group?

A 10 B 11 C 12 D 13 E 14

4. A circle of radius 1 rolls along a straight line from the point K to the point L, where $KL = 11\pi$. Which of the following pictures shows the correct appearance of the circle when it reaches L?



5. Martina plays chess. She has played 15 games this season, out of which she has won nine. She has five more games to play. What will her success rate be in this season if she wins all five remaining games?

A 60% B 65% C 70% D 75% E 80%

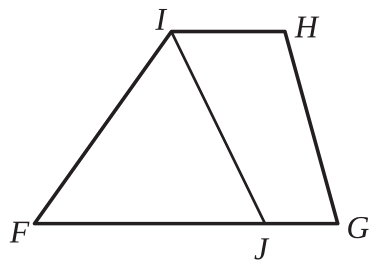
6. One-eighth of the guests at a wedding were children. Three-sevenths of the adult guests were men. What fraction of the wedding guests were adult women?

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{5}$ D $\frac{1}{7}$ E $\frac{3}{7}$

7. A certain maths teacher has a box containing buttons of three different colours. There are 203 red buttons, 117 white buttons and 28 blue buttons. A student is blindfolded and takes some buttons from the box at random. How many buttons does the student need to take before he can be sure that he has taken at least 3 buttons of the same colour?

A 3 B 4 C 6 D 7 E 28

8. As shown in the diagram, $FGHI$ is a trapezium with side GF parallel to HI. The lengths of FG and HI are 50 and 20 respectively. The point J is on the side FG such that the segment IJ divides the trapezium into two parts of equal area. What is the length of FJ?



A 25 B 30 C 35 D 40 E 45

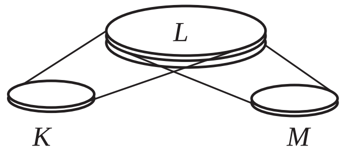
9. How many positive integers N possess the property that exactly one of the numbers N and $(N + 20)$ is a 4-digit number?

A 19 B 20 C 38 D 39 E 40

10. The sum of the squares of three consecutive positive integers is 770. What is the largest of these integers?

A 15 B 16 C 17 D 18 E 19

11. A belt drive system consists of the wheels K, L and M, which rotate without any slippage. The wheel L makes 4 full turns when K makes 5 full turns; also L makes 6 full turns when M makes 7 full turns. The perimeter of wheel M is 30 cm. What is the perimeter of wheel K?



A 27 cm B 28 cm C 29 cm D 30 cm E 31 cm

12. Tycho wants to prepare a schedule for his jogging for the next few months. He wants to jog three times per week. Every week, he wants to jog on the same days of the week. He never wants to jog on two consecutive days. How many schedules can he choose from?

A 6 B 7 C 9 D 10 E 35

13. Four brothers have different heights. Tobias is shorter than Victor by the same amount by which he is taller than Peter. Oscar is shorter than Peter by the same amount as well. Tobias is 184 cm tall and the average height of all the four brothers is 178 cm. How tall is Oscar?

A 160 cm B 166 cm C 172 cm D 184 cm E 190 cm

14. Johannes told me that it rained seven times during his holiday. When it rained in the morning, it was sunny in the afternoon; when it rained in the afternoon, it was sunny in the morning. There were 5 sunny mornings and 6 sunny afternoons. Without more information, what is the least number of days that I can conclude that the holiday lasted?

A 7 B 8 C 9 D 10 E 11

15. Maja decided to enter numbers into the cells of a 3×3 grid. She wanted to do this in such a way that the numbers in each of the four 2×2 grids that form part of the 3×3 grid have the same totals. She has already written numbers in three of the corner cells, as shown in the diagram. Which number does she need to write in the bottom right corner?

3		1
2		?

A 0 B 1 C 4 D 5 E impossible to determine

16. Seven positive integers a, b, c, d, e, f, g are written in a row. Every number differs by one from its neighbours. The total of the seven numbers is 2017. Which of the numbers can be equal to 286?

A only a or g B only b or f C only c or e D only d E any of them

17. Niall's four children have different integer ages under 18. The product of their ages is 882. What is the sum of their ages?

A 23 B 25 C 27 D 31 E 33

18. Ivana has two identical dice and on the faces of each are the numbers −3, −2, −1, 0, 1, 2. If she throws her dice and multiplies the results, what is the probability that their product is negative?

A $\frac{1}{4}$ B $\frac{11}{36}$ C $\frac{1}{3}$ D $\frac{13}{36}$ E $\frac{1}{2}$

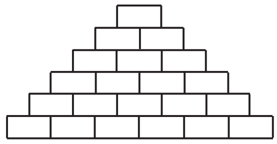
19. Maria chooses two digits a and b and uses them to make a six-digit number $ababab$. Which of the following is always a factor of numbers formed in this way?

A 2 B 5 C 7 D 9 E 11

20. Frederik wants to make a special seven-digit password. Each digit of his password occurs exactly as many times as its digit value. The digits with equal values always occur consecutively, e.g. 4444333 or 1666666. How many possible passwords can he make?

A 6 B 7 C 10 D 12 E 13

21. Carlos wants to put numbers in the number pyramid shown in such a way that each number above the bottom row is the sum of the two numbers immediately below it. What is the largest number of *odd* numbers that Carlos could put in the pyramid?

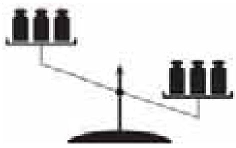


A 13 B 14 C 15 D 16 E 17

22. Liza found the total of the interior angles of a convex polygon. She missed one of the angles and obtained the result 2017°. Which of the following was the angle she missed?

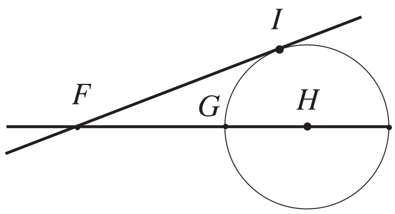
A 37° B 53° C 97° D 127° E 143°

23. On a balance scale, three different masses were put at random on each pan and the result is shown in the picture. The masses are of 101, 102, 103, 104, 105 and 106 grams. What is the probability that the 106 gram mass stands on the heavier pan?



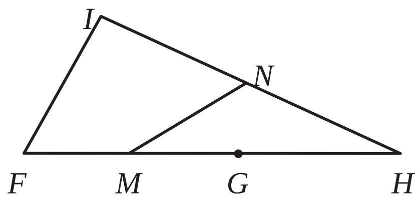
A 75% B 80% C 90% D 95% E 100%

24. The points G and I are on the circle with centre H, and FI is tangent to the circle at I. The distances FG and HI are integers, and $FI = FG + 6$. The point G lies on the straight line through F and H. How many possible values are there for HI?



A 0 B 2 C 4 D 6 E 8

25. The diagram shows a triangle FHI, and a point G on FH such that $GH = FI$. The points M and N are the midpoints of FG and HI respectively. Angle $NMH = \alpha°$. Which of the following gives an expression for $\angle IFH$?



A $2\alpha°$ B $(90 - \alpha)°$ C $45 + \alpha°$ D $(90 - \frac{1}{2}\alpha)°$ E $60°$

1. Which of the following numbers is the closest to the value of $\frac{17 \times 0.3 \times 20.16}{999}$?

A 0.01 B 0.1 C 1 D 10 E 100

2. Four of the following points are vertices of the same square. Which point is not a vertex of this square?

A $(-1, 3)$ B $(0, -4)$ C $(-2, -1)$ D $(1, 1)$ E $(3, -2)$

3. When the positive integer x is divided by 6, the remainder is 3. What is the remainder when $3x$ is divided by 6?

A 4 B 3 C 2 D 1 E 0

4. How many weeks are equivalent to 2016 hours?

A 6 B 8 C 10 D 12 E 16

5. Little Lucas invented his own way to write down negative numbers before he learned the usual way with the minus sign in front. Counting backwards, he would write: 3, 2, 1, 0, 00, 000, 0000, …. What is the result of 000 + 0000 in his notation?

A 1 B 00000 C 000000 D 0000000 E 00000000

6. Marie changed her dice by replacing 1, 3, and 5 with -1, -3 and -5 respectively. She left the even numbers unchanged. If she throws two such dice, which of the following totals cannot be achieved?

A 3 B 4 C 5 D 7 E 8

7. Angelo wrote down the word TEAM. He then swapped two adjacent letters around and wrote down the new order of the letters. He proceeded in this way until he obtained the word MATE. What is the least number of swaps that Angelo could have used?

A 3 B 4 C 5 D 6 E 7

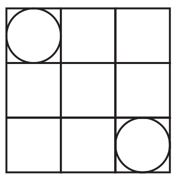
8. Sven wrote five different one-digit positive integers on a blackboard. He discovered that none of the sums of two different numbers on the board equalled 10. Which of the following numbers did Sven definitely write on the blackboard?

A 1 B 2 C 3 D 4 E 5

9. Four numbers a, b, c, d are such that $a + 5 = b^2 - 1 = c^2 + 3 = d - 4$. Which of them is the largest?

A a B b C c D d E more information required

10. A square is split into nine identical squares, each with sides of length one unit. Circles are inscribed in two of these squares, as shown. What is the shortest distance between the two circles?

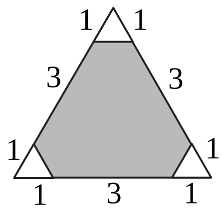


A $2\sqrt{2} - 1$ B $\sqrt{2} + 1$ C $2\sqrt{2}$ D 2 E 3

11. A tennis tournament was played on a knock-out basis. The following list is of all but one of the last seven matches (the quarter-finals, the semi-finals and the final), although not correctly ordered: Bella beat Ann; Celine beat Donna; Gina beat Holly; Gina beat Celine; Celine beat Bella; and Emma beat Farah. Which result is missing?

A Gina beat Bella B Celine beat Ann C Emma beat Celine
D Bella beat Holly E Gina beat Emma

12. The large triangle shown has sides of length 5 units. What percentage of the area of the triangle is shaded?



A 80% B 85% C 88% D 90%
E impossible to determine

13. Sepideh is making a magic multiplication square using the numbers 1, 2, 4, 5, 10, 20, 25, 50 and 100. The products of the numbers in each row, in each column and in the two diagonals should all be the same. In the figure you can see how she has started. Which number should Sepideh place in the cell with the question mark?

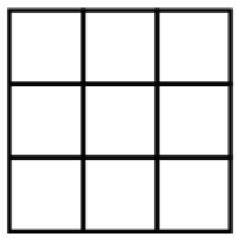
20	1	
		?

A 2 B 4 C 5 D 10 E 25

14. Eight unmarked envelopes contain the numbers: 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses a few envelopes randomly. Alie takes the rest. Both sum up their numbers. Eve's sum is 31 more than Alie's. How many envelopes did Eve take?

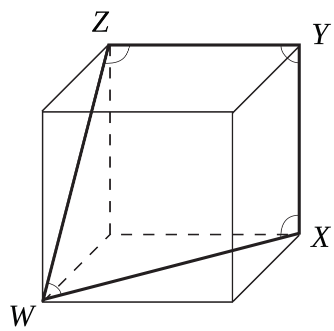
A 2 B 3 C 4 D 5 E 6

15. Peter wants to colour the cells of a 3×3 square in such a way that each of the rows, each of the columns and both diagonals have cells of three different colours. What is the least number of colours Peter could use?



A 3 B 4 C 5 D 6 E 7

16. The picture shows a cube with four marked angles, $\angle WXY$, $\angle XYZ$, $\angle YZW$ and $\angle ZWX$. What is the sum of these angles?



A 315° B 330° C 345° D 360° E 375°

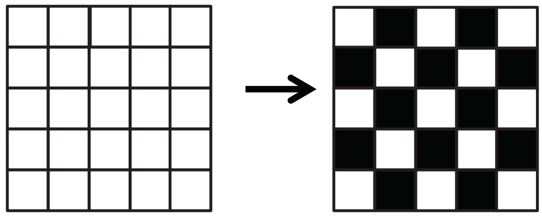
17. There are 2016 kangaroos in a zoo. Each of them is either grey or pink, and at least one of them is grey and at least one is pink. For every kangaroo, we calculate this fraction: the number of kangaroos of the other colour divided by the number of kangaroos of the same colour as this kangaroo (including himself). Find the sum of all the 2016 fractions calculated.

A 2016 B 1344 C 1008 D 672 E more information required

18. What is the largest possible remainder that is obtained when a two-digit number is divided by the sum of its digits?

A 13 B 14 C 15 D 16 E 17

19. A 5 × 5 square is divided into 25 cells. Initially all its cells are white, as shown. Neighbouring cells are those that share a common edge. On each move two neighbouring cells have their colours changed to the opposite colour (white cells become black and black ones become white).



What is the minimum number of moves required in order to obtain the chess-like colouring shown on the right?

A 11 B 12 C 13 D 14 E 15

20. It takes 4 hours for a motorboat to travel downstream from X to Y. To return upstream from Y to X it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from X to Y by the current, assuming it is unhindered by any obstacles? [Assume that the current flows at a constant rate, and that the motorboat moves at a constant speed relative to the water.]

A 5 B 10 C 12 D 20 E 24

21. In the Kangaroo republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

A 1 B 2 C 3 D 4 E 5

22. Jakob wrote down four consecutive positive integers. He then calculated the four possible totals made by taking three of the integers at a time. None of these totals was a prime. What is the smallest integer Jakob could have written?

A 12 B 10 C 7 D 6 E 3

23. Two sportsmen (Ben and Filip) and two sportswomen (Eva and Andrea) − a speed skater, a skier, a hockey player and a snowboarder − had dinner at a square table, with one person on each edge of the square. The skier sat at Andrea's left hand. The speed skater sat opposite Ben. Eva and Filip sat next to each other. A woman sat at the hockey player's left hand. Which sport did Eva do?

A speed skating B skiing C hockey D snowboarding E more information required

24. Dates can be written in the form DD.MM.YYYY. For example, today's date is 17.03.2016. A date is called ‘surprising’ if all 8 digits in its written form are different. In what month will the next surprising date occur?

A March B June C July D August E December

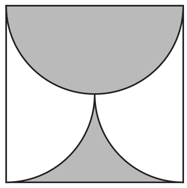
25. At a conference, the 2016 participants were registered from P1 to P2016. Each participant from P1 to P2015 shook hands with exactly the same number of participants as the number on their registration form. How many hands did the 2016th participant shake?

A 1 B 504 C 672 D 1008 E 2015

1. What is the units digit of the number $2015^2 + 2015^0 + 2015^1 + 2015^5$?

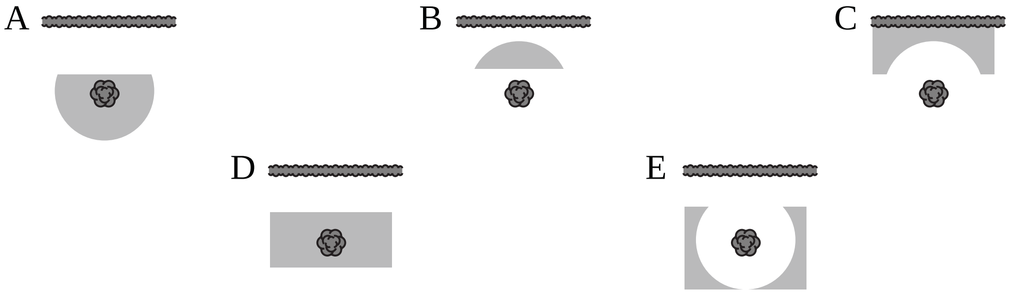
A 1 B 5 C 6 D 7 E 9

2. The diagram shows a square with sides of length a. The shaded part of the square is bounded by a semicircle and two quarter-circle arcs. What is the shaded area?



A $\frac{\pi a^2}{8}$ B $\frac{a^2}{2}$ C $\frac{\pi a^2}{2}$ D $\frac{a^2}{4}$ E $\frac{\pi a^2}{4}$

3. Mr Hyde can't remember exactly where he has hidden his treasure. He knows it is at least 5 m from his hedge, and at most 5 m from his tree. Which of the following shaded areas could represent the largest region where his treasure could lie?



4. Three sisters bought a packet of biscuits for £1.50 and divided them equally among them, each receiving 10 biscuits. However, Anya paid 80 pence, Berini paid 50 pence and Carla paid 20 pence. If the biscuits had been divided in the same ratios as the amounts each sister had paid, how many more biscuits would Anya have received than she did originally?

A 10 B 9 C 8 D 7 E 6

5. Each of the children in a class of 33 children likes either PE or Computing, and 3 of them like both. The number who like only PE is half as many as like only Computing. How many students like Computing?

A 15 B 18 C 20 D 22 E 23

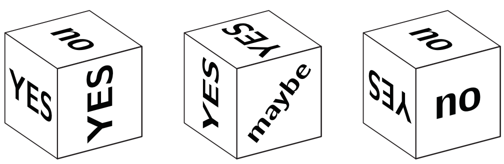
6. Which of the following is neither a square nor a cube?

A 2^9 B 3^{10} C 4^{11} D 5^{12} E 6^{13}

7. Martha draws some pentagons, and counts the number of right-angles in each of her pentagons. No two of her pentagons have the same number of right-angles. Which of the following is the complete list of possible numbers of right-angles that could occur in Martha's pentagons?

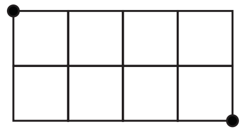
A 1, 2, 3 B 0, 1, 2, 3, 4 C 0, 1, 2, 3 D 0, 1, 2 E 1, 2

8. The picture shows the same die in three different positions. When the die is rolled, what is the probability of rolling a 'YES' ?



A $\frac{1}{3}$ B $\frac{1}{2}$ C $\frac{5}{9}$ D $\frac{2}{3}$ E $\frac{5}{6}$

9. In the grid, each small square has side of length 1. What is the minimum distance from 'Start' to 'Finish' travelling only on edges or diagonals of the squares?

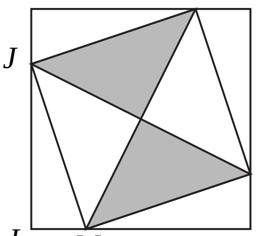


A $2\sqrt{2}$ B $\sqrt{10}+\sqrt{2}$ C $2+2\sqrt{2}$ D $4\sqrt{2}$ E 6

10. Three inhabitants of the planet Zog met in a crater and counted each other's ears. Imi said, "I can see exactly 8 ears"; Dimi said, "I can see exactly 7 ears"; Timi said, "I can see exactly 5 ears". None of them could see their own ears. How many ears does Timi have?

A 2 B 4 C 5 D 6 E 7

11. The square $FGHI$ has area 80. Points J, K, L, M are marked on the sides of the square so that $FK = GL = HM = IJ$ and $FK = 3KG$. What is the area of the shaded region?

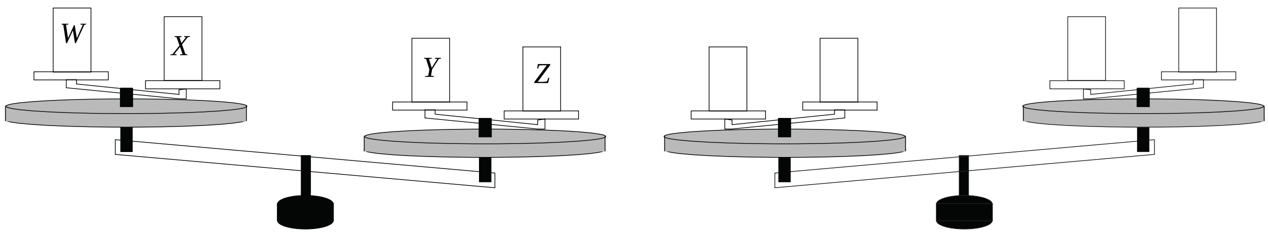


A 40 B 35 C 30 D 25 E 20

12. The product of the ages of a father and his son is 2015. What is the difference between their ages?

A 29 B 31 C 34 D 36 E None of these

13. A large set of weighing scales has two identical sets of scales placed on it, one on each pan. Four weights W, X, Y, Z are placed on the weighing scales as shown in the left diagram.



Then two of these weights are swapped, and the pans now appear as shown in the diagram on the right. Which two weights were swapped?

A W and Z B W and Y C W and X D X and Z E X and Y

14. The two roots of the quadratic equation

$$x^2 - 85x + c = 0$$

are both prime numbers. What is the sum of the digits of c?

A 12 B 13 C 14 D 15 E 21

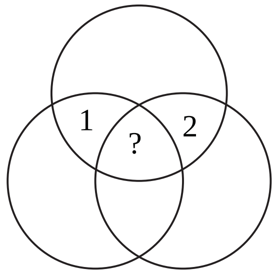
15. How many three-digit numbers are there in which any two adjacent digits differ by 3?

A 12 B 14 C 16 D 18 E 20

16. Which of the following values of n is a counterexample to the statement, 'If n is a prime number, then exactly one of $n - 2$ and $n + 2$ is prime' ?

A 11 B 19 C 21 D 29 E 37

17. The figure shows seven regions enclosed by three circles. We call two regions neighbouring if their boundaries have more than one common point. In each region a number is written. The number in any region is equal to the sum of the numbers of its neighbouring regions. Two of the numbers are shown. What number is written in the central region?



A -6 B 6 C -3 D 3 E 0

18. Petra has three different dictionaries and two different novels on a shelf. How many ways are there to arrange the books if she wants to keep the dictionaries together and the novels together?

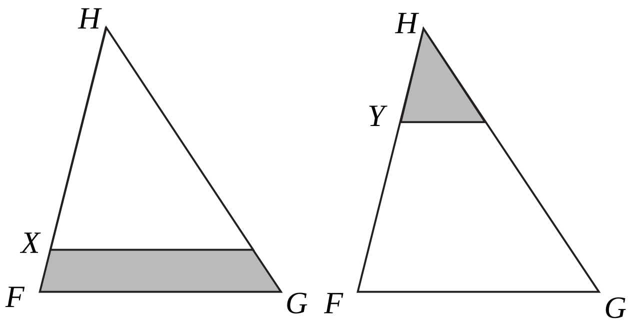
A 12 B 24 C 30 D 60 E 120

19. How many 2-digit numbers can be written as the sum of exactly six different powers of 2, including 2^0 ?

A 0 B 1 C 2 D 3 E 4

20. In the triangle FGH, we can draw a line parallel to its base FG, through point X or Y. The areas of the shaded regions are the same. The ratio $HX : XF = 4 : 1$. What is the ratio $HY : YF$?

A $1:1$ B $2:1$ C $3:1$ D $3:2$ E $4:3$



21. In a right-angled triangle, the angle bisector of an acute angle divides the opposite side into segments of length 1 and 2. What is the length of the bisector?

A $\sqrt{2}$ B $\sqrt{3}$ C $\sqrt{4}$ D $\sqrt{5}$ E $\sqrt{6}$

22. We use the notation $\overline{ab}$ for the two-digit number with digits a and b. Let a, b, c be different digits. How many ways can you choose the digits a, b, c such that $\overline{ab} < \overline{bc} < \overline{ca}$?

A 84 B 96 C 504 D 729 E 1000

23. When one number was removed from the set of positive integers from 1 to n, inclusive, the mean of the remaining numbers was 4.75. What number was eliminated?

A 5 B 7 C 8 D 9 E impossible to determine

24. Ten different numbers (not necessarily integers) are written down. Any number that is equal to the product of the other nine numbers is then underlined. At most, how many numbers can be underlined?

A 0 B 1 C 2 D 9 E 10

25. Several different points are marked on a line, and all possible line segments are constructed between pairs of these points. One of these points lies on exactly 80 of these segments (not including any segments of which this point is an endpoint). Another one of these points lies on exactly 90 segments (not including any segments of which it is an endpoint). How many points are marked on the line?

A 20 B 22 C 80 D 85 E 90

[This page is intentionally left blank.]