- 1 Write down an equation of the circle with the given centre and radius in each case.
 - **a** centre (0, 0)

- radius 5 **b** centre (1, 3) radius 2 **c** centre (4, -6)
- radius 1

- **d** centre (-1, -8) radius 3 **e** centre $(-\frac{1}{2}, \frac{1}{2})$ radius $\frac{1}{2}$ **f** centre (-3, 9)
- radius $2\sqrt{3}$
- Write down the coordinates of the centre and the radius of each of the following circles. 2
 - **a** $x^2 + v^2 = 16$
- **b** $(x-6)^2 + (y-1)^2 = 81$ **c** $(x+1)^2 + (y-4)^2 = 121$
- **d** $(x-7)^2 + v^2 = 0.09$
- e $(x+2)^2 + (y+5)^2 = 32$ f $(x-8)^2 + (y+9)^2 = 108$
- Find the coordinates of the centre and the radius of each of the following circles. 3
 - **a** $x^2 + y^2 4y + 3 = 0$

- **b** $x^2 + y^2 2x 10y 23 = 0$
- $x^2 + y^2 + 12x 8y + 36 = 0$
- **d** $x^2 + y^2 2x + 16y = 35$

 $e^{-x^2+v^2}=8x-6v$

- $\mathbf{f} \quad x^2 + v^2 + 10x 2v 19 = 0$
- $\mathbf{g} \quad 4x^2 + 4y^2 4x 24y + 1 = 0$
- **h** $9x^2 + 9y^2 + 6x 24y + 8 = 0$

- 4 Find an equation of the circle
 - a with centre (1, -2) which passes through the point (4, 2),
 - **b** with centre (-5, 7) which passes through the point (0, 5).
- 5 Find an equation of the circle in which AB is a diameter in each case.
- **a** A(1,-2) B(3,-2) **b** A(-7,2) B(1,8)
- c A(1,1) B(4,0)

- 6 The points P(0, 1), Q(3, 10) and R(6, 9) all lie on circle C.
 - a Show that $\angle PQR$ is a right-angle.
 - **b** Hence, show that C has the equation $x^2 + y^2 6x 10y + 9 = 0$.
- 7 Find in each case whether the given point lies inside, outside or on the given circle.

 - **a** (0, -9) $x^2 + y^2 = 64$

- **b** (4, 7) $x^2 + y^2 2x 6y 26 = 0$

- **c** (7, -3) $x^2 + y^2 + 10x 4y = 140$ **d** (-4, 1) $x^2 + y^2 + 2x + 8y 13 = 0$
- The point P lies on the circle with equation $x^2 + y^2 + 12x 6y + 27 = 0$ and the point Q has 8 coordinates (8, 1). Find the minimum length of PQ giving your answer in the form $k\sqrt{2}$.
- 9 Find an equation of the circle which crosses the x-axis at the points (2, 0) and (8, 0) and touches the y-axis at the point (0, 4).
- Given that the circle with equation $x^2 + y^2 + 8x 12y + k = 0$ does not touch or cross either of 10 the coordinate axes, find the set of possible values of the constant k.
- 11 The circle C passes through the points P, Q and R with coordinates (-2, -2), (2, -4) and (7, 1)respectively.
 - a Find an equation of the perpendicular bisector of the points P and Q.
 - **b** Find the coordinates of the centre of C.
 - **c** Find an equation of *C*.



- 12 The circle C has the equation $x^2 + y^2 4x 4y 28 = 0$.
 - a Find the distance of the point A (10, 8) from the centre of C.

The tangent to C at the point B passes through A.

- **b** Find the length AB.
- 13 A circle has the equation $x^2 + y^2 + 6x 2y = 0$ and passes through the point P.

Given that the tangent to the circle at P passes through the point Q(2, 6), find the exact length PQ in its simplest form.

- 14 The circle C has the equation $x^2 + y^2 6x 10y + 16 = 0$ and passes through the point A (6, 2).
 - **a** Find the coordinates of the centre of C.
 - **b** Find the gradient of the normal to the circle at A.
 - **c** Find an equation of the normal to the circle at A.
- 15 Find an equation of
 - **a** the normal to the circle with equation $x^2 + y^2 + 4x = 13$ at the point (-1, 4),
 - **b** the tangent to the circle with equation $x^2 + y^2 + 2x + 4y 40 = 0$ at the point (5, 1),
 - c the tangent to the circle with equation $x^2 + y^2 10x + 4y + 4 = 0$ at the point (2, 2).
- Find the coordinates of the points where the circle with equation $x^2 + y^2 6x + 6y 16 = 0$ intersects the coordinate axes.
- 17 Find in each case the coordinates of the points where the line *l* intersects the circle *C*.
 - **a** l: y = x 4 $C: x^2 + y^2 = 10$
 - **b** l: 3x + y = 17 $C: x^2 + y^2 4x 2y 15 = 0$
 - **c** l: y = 2x + 2 $C: 4x^2 + 4y^2 + 4x 8y 15 = 0$
- The line with equation y = 1 x intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B.

Find the length of the chord AB, giving your answer in the form $k\sqrt{2}$.

- Show that the line with equation y = 2x + 1 is a tangent to the circle with equation $x^2 + y^2 8x 8y + 27 = 0$ and find the coordinates of the point where they touch.
- The line with equation y = x + k is a tangent to the circle with equation $x^2 + y^2 + 6x 8y + 17 = 0$ Find the two possible values of k.
- The line with equation y = mx is a tangent to the circle with equation $x^2 + y^2 8x 16y + 72 = 0$ Find the two possible values of m.
- The line with equation 2x + 3y = k is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y = 0$. Find the two possible values of k.
- The circle with equation $x^2 + y^2 4x 6y = 7$ crosses the y-axis at the points A and B.
 - **a** Find the coordinates of the points A and B.
 - **b** Find the coordinates of the point where the tangent to the circle at A intersects the tangent to the circle at B.



C2

COORDINATE GEOMETRY

- 1 The circle C has centre (3, -2) and radius 5.
 - **a** Write down an equation of *C* in cartesian form.

The line y = 2x - 3 intersects C at the points A and B.

- **b** Show that $AB = 4\sqrt{5}$.
- The line AB is a diameter of circle C.

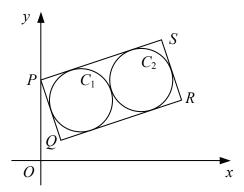
Given that A has coordinates (-5, 6) and B has coordinates (3, 8), find

- **a** the coordinates of the centre of C,
- **b** a cartesian equation for C,
- \mathbf{c} an equation of the tangent to C at A.
- 3 The circle C has equation $x^2 + y^2 + 8x 16y + 62 = 0$.
 - **a** Find the coordinates of the centre of C and the exact radius of C.

The line *l* has equation y = 2x + 1.

b Show that the minimum distance between l and C is $3(\sqrt{5} - \sqrt{2})$.

4



The diagram shows rectangle *PQRS* and circles C_1 and C_2 .

Each circle touches the other circle and three sides of the rectangle. The coordinates of the corners of the rectangle are P(0, 4), Q(1, 1), R(7, 3) and S(6, 6).

- **a** Find the radius of C_1 .
- **b** Find the coordinates of the point where the two circles touch.
- **c** Show that C_1 has equation $2x^2 + 2y^2 8x 12y + 21 = 0$.
- The circle C touches the y-axis at the point A(0, 3) and passes through the point B(2, 7).
 - **a** Find an equation of the perpendicular bisector of AB.
 - **b** Find an equation for C.
 - **c** Show that the tangent to C at B has equation

$$3x - 4y + 22 = 0$$
.

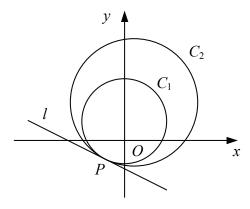
6 The point P(x, y) moves such that its distance from the point A(-3, 4) is twice its distance from the point B(0, -2).

Show that the locus of *P* is a circle and find the coordinates of the centre and the exact radius of this circle.



- 7 The points P(-4, 9) and Q(-2, -5) are such that PQ is a diameter of circle C.
 - **a** Find the coordinates of the centre of C.
 - **b** Find an equation for *C*.
 - **c** Show that the point R(2, 7) lies on C.
 - **d** Hence, state the size of $\angle PRQ$, giving a reason for your answer.

8



The diagram shows circles C_1 and C_2 , which both pass through the point P, and the common tangent to the circles at P, the line l.

Circle C_1 has the equation $x^2 + y^2 - 4y - 16 = 0$.

a Find the coordinates of the centre of C_1 .

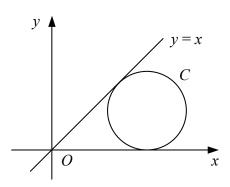
Circle C_2 has the equation $x^2 + y^2 - 2x - 8y - 60 = 0$.

- **b** Find an equation of the straight line passing through the centre of C_1 and the centre of C_2 .
- **c** Find an equation of line *l*.
- 9 The circle C has equation $x^2 + y^2 8x + 4y + 12 = 0$.
 - **a** Find the coordinates of the centre of C and the radius of C.

The point P has coordinates (3, 5) and the point Q lies on C.

- **b** Find the largest and smallest values of the length PQ, giving your answers in the form $k\sqrt{2}$.
- **c** Find the length of PQ correct to 3 significant figures when the line PQ is a tangent to C.

10



The diagram shows the circle C and the line y = x.

Given that circle C has centre (a, b), where a and b are positive constants, and that C touches the x-axis,

a find a cartesian equation for C in terms of a and b.

Given also that the line y = x is a tangent to C,

b show that $a = (1 + \sqrt{2})b$.