

# Solomon Practice Paper

## Pure Mathematics 6H

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

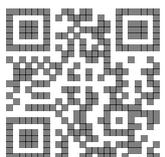
Question	Points	Score
1	5	
2	8	
3	8	
4	12	
5	13	
6	14	
7	15	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023



1. Given that

$$t_{n+1} = t_n - 4 \quad n \geq 1, \quad t_1 = 3,$$

prove by induction that  $t_n = 7 - 4n$  for all integers  $n, n \geq 1$ .

2. (a) On the same Argand diagram sketch the locus of the points defined by the equations

i.  $z + z^* = 2,$

ii.  $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4},$  where  $\text{Im}(z) \geq 0.$

The region  $R$  of the complex  $z$ -plane is defined by the inequalities

$$z + z^* \leq 2, \quad \arg\left(\frac{z-2}{z+2}\right) \geq \frac{\pi}{4} \quad \text{Im}(z) \geq 0.$$

(b) Shade the region  $R$  on the Argand diagram.

3. The points  $A, B$  and  $C$  with coordinates  $(x_{-1}, y_{-1}), (x_0, y_0)$  and  $(x_1, y_1)$  respectively lie on the curve  $y = f(x)$  where  $x_1 - x_0 = x_0 - x_{-1} = h$  and  $y_n = f(x_n).$

(a) By drawing a sketch, or otherwise, show that

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0 - h)}{2h}.$$

Given that

$$f'(x) = \sqrt{2x + f(x)}, \quad f(0) = 1, \quad f(0.2) = 1.25,$$

(b) use two applications of the approximation in (a) with a step length of 0.2 to find an estimate for  $f(0.6).$

4. The points  $A, B$  and  $C$  have position vectors  $\mathbf{a}, \mathbf{b}$  and  $\mathbf{c}$  respectively such that

$$\begin{aligned} \mathbf{a} &= 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \\ \mathbf{b} &= \mathbf{i} + q\mathbf{j} - 3\mathbf{k}, \\ \mathbf{c} &= 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}, \end{aligned}$$

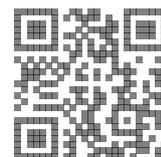
where  $q$  is a constant and  $q \neq 2.$

(a) Find  $\overrightarrow{AB} \times \overrightarrow{AC},$  giving your answer in terms of  $q.$

(b) Hence show that the vector  $\mathbf{n} = 4\mathbf{i} - \mathbf{k}$  is perpendicular to the plane  $\Pi$  containing  $A, B$  and  $C$  for all real values of  $q.$

(c) Find an equation of the plane  $\Pi,$  giving your answer in the form  $\mathbf{r} \cdot \mathbf{n} = p.$

Given that  $q = -1,$



[5]

[6]

[2]

Total: 8

[3]

[5]

Total: 8

[5]

[2]

[2]

(d) find the volume of the tetrahedron  $OABC$ . [3]

Total: 12

5. (a) Use De Moivre's theorem to show that [6]

$$\cos(5\theta) \equiv \cos(\theta) (16 \cos^4(\theta) - 20 \cos^2(\theta) + 5).$$

(b) By solving the equation  $\cos^5(\theta) = 0$ , deduce that [7]

$$\cos^2\left(\frac{3\pi}{10}\right) = \frac{5 - \sqrt{5}}{8}.$$

Total: 13

6. (a) Find the first three derivatives of  $\ln\left(\frac{1+x}{1-2x}\right)$ . [6]

(b) Hence, or otherwise, find the expansion of  $\ln\left(\frac{1+x}{1-2x}\right)$  in ascending powers of  $x$  up to and including the term in  $x^3$ . [4]

(c) State the values of  $x$  for which this expansion is valid. [1]

(d) Use this expansion to find an approximate value for  $\ln\left(\frac{4}{3}\right)$ , giving your answer to 3 decimal places. [3]

Total: 14

7.

$$\mathbf{A} = \begin{pmatrix} 2 & a & 2 \\ -1 & b & -2 \\ 0 & 0 & c \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 6 & 5 & 2 \\ -1 & 0 & -2 \\ 0 & 0 & 5 \end{pmatrix}$$

and

$$(\mathbf{B} - 2\mathbf{I})\mathbf{A} = 3\mathbf{I} \quad (\star)$$

where  $a, b$  and  $c$  are constants and  $I$  is the  $3 \times 3$  identity matrix.

(a) Find the values of  $a, b$  and  $c$ . [6]

(b) Using equation  $\star$ , or otherwise, find  $\mathbf{A}^{-1}$ , showing your working clearly. [2]

The transformation  $T: \mathbb{R}^3 \mapsto \mathbb{R}^3$  is represented by the matrix  $\mathbf{A}$ .

(c) Find an equation satisfied by all the points which remain invariant under  $T$ . [4]

$T$  maps the vector  $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$  onto the vector  $\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$

(d) Find the values of  $p, q$  and  $r$ . [3]

Total: 15

