

# Solomon Practice Paper

## Pure Mathematics 6E

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

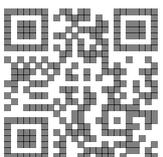
Question	Points	Score
1	5	
2	6	
3	6	
4	8	
5	11	
6	13	
7	13	
8	13	
Total:	75	

How I can achieve better:

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1. The point  $P$  represents a variable point  $z = x + iy$  in an Argand diagram where  $x, y \in \mathbb{R}$ .

Given that the locus of  $P$  is a circle with centre  $-1 + i$  and radius 2, find

- (a) an equation of the circle in terms of  $z$ , [2]  
 (b) the points on the locus of  $P$  which represent real numbers. [3]

Total: 5

2. Prove by induction that  $2^n > 2n$  for all integers  $n, n \geq 3$ . [6]

3. (a) By using the series expansion for  $\ln(1 + 2x)$  and the series expansion for  $e^x$ , or otherwise, [4]  
 and given that  $x$  is small, show that

$$\ln(1 + 2x) - 2xe^{-x} \approx Ax^3,$$

and find the value of  $A$ .

- (b) Hence find [2]

$$\lim_{x \rightarrow 0} \left( \frac{\ln(1 + 2x) - 2xe^{-x}}{x^3} \right).$$

Total: 6

4.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ -3 & 3 & 1 \end{pmatrix}$$

- (a) Show that  $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$  is an eigenvector of  $\mathbf{A}$  and find the corresponding eigenvalue. [2]

- (b) Prove that  $\mathbf{A}$  has only one real eigenvalue, showing your working clearly. [6]

Total: 8

5. A transformation  $T$  from the  $z$ -plane to the  $w$ -plane is defined by

$$w = z^2$$

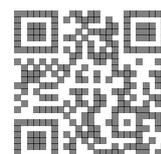
where  $z = x + iy, w = u + iv$  and  $x, y, u$  and  $v$  are real.

- (a) Show that  $T$  transforms the line  $\text{Im}(z) = 2$  in the  $z$ -plane onto a parabola in the  $w$ -plane [5]  
 and find an equation of the parabola, giving your answer in terms of  $u$  and  $v$ .

The image in the  $w$ -plane of the half-line  $\arg(z) = \frac{\pi}{4}$  is the half-line  $l$ .

- (b) Find an equation of  $l$ . [2]

The parabola and the half-line in the  $w$ -plane are represented on the same Argand diagram. Their point of intersection is represented by  $P$ .



- (c) Find the complex number which is represented by  $P$ , giving your answer in the form  $a + ib$  [4]  
 where  $a$  and  $b$  are real.

Total: 11

6. It is given that  $y$  satisfies the differential equation

$$\frac{dy}{dx} = x^2 + y \cos(x) \quad \text{and} \quad y = 1 \quad \text{at} \quad x = 0.$$

- (a) i. Use the differential equation to find expressions for  $\frac{d^2y}{dx^2}$  and  $\frac{d^3y}{dx^3}$ . [10]  
 ii. Hence, or otherwise, find  $y$  as a series in ascending powers of  $x$  up to and including the term in  $x^3$ .  
 iii. Use your series to estimate the value of  $y$  at  $x = -0.1$ .

- (b) Use the approximation  $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$  to estimate the value of  $y$  at  $x = 0.1$ . [3]

Total: 13

7. Referred to an origin  $O$ , the points  $A, B, C$  and  $D$  have coordinates  $(1, 1, 0)$ ,  $(3, 2, 5)$ ,  $(0, -1, -4)$  and  $(-2, -5, 0)$  respectively.

- (a) Find, in the form  $\mathbf{r} \cdot \mathbf{n} = p$ , an equation of the plane  $\Pi$  passing through  $A, B$  and  $C$ . [6]

The line  $l$  passes through  $D$  and is perpendicular to  $\Pi$ .

- (b) Find a vector equation of  $l$ . [1]

The line  $l$  meets the plane  $\Pi$  at the point  $E$ .

- (c) Find the coordinates of  $E$ . [4]

The point  $F$  is the reflection of  $D$  in  $\Pi$ .

- (d) Find the coordinates of  $F$ . [2]

Total: 13

8. The transformation  $T: \mathbb{R}^3 \mapsto \mathbb{R}^3$  is represented by the matrix  $\mathbf{M}$  where

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 1 \\ 2 & 2 & 0 \end{pmatrix}.$$

- (a) Find  $\mathbf{M}^{-1}$ , showing your working clearly. [6]

- (b) Find the Cartesian equations of the line mapped by the transformation  $T$  onto the line with equations [7]

$$\frac{x-1}{3} = \frac{y+1}{-3} = \frac{z}{4}.$$

Total: 13

