

Solomon Practice Paper

Pure Mathematics 5B

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

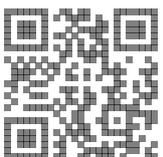
Question	Points	Score
1	7	
2	8	
3	9	
4	11	
5	12	
6	13	
7	15	
Total:	75	

How I can achieve better:

-
-
-



Last updated: May 5, 2023



1. Given that

$$y \arccos(x) - \frac{x}{\pi} e^{2x} - 1 = 0, \quad [7]$$

find the value of y at the point where $x = 0$, giving your answer in terms of π .

2.

$$f(x) = 5 \cosh(x) + 3 \sinh(x). \quad [8]$$

The minimum value of $f(x)$ occurs at the point $(p \ln(q), r)$ where p, q and r are integers.

Find the values of p, q and r .

3. The line $y = mx + c$ is a tangent to the rectangular hyperbola with equation $xy = -9$.

(a) Show that $c = \pm 6\sqrt{m}$. [4]

(b) Hence, or otherwise, find the equations of the tangents from the point $(4, -2)$ to the rectangular hyperbola $xy = -9$. [5]

Total: 9

4. The curve C is defined by

$$y^2 = x, \quad x \geq 0, \quad y \geq 0. \quad [11]$$

The region between C , the x -axis and the line $x = 1$ is rotated through 2π about the x -axis.

Show that the area of the surface generated is

$$\frac{\pi}{6} (5\sqrt{5} - 1).$$

5. (a) Using the definition of $\cosh(x)$ in terms of exponential functions, express $\operatorname{sech}(x)$ in terms of e^x and e^{-x} . [1]

(b) Sketch the graph of $y = \operatorname{sech}(x)$. [2]

(c) Show that [4]

$$\int \operatorname{sech}(x) \, dx = 2 \arctan(e^x) + c.$$

The curve C has equation $y = \operatorname{sech}(x)$. The region between C , the x -axis and the lines $x = -a$ and $x = a$, where a is a positive constant, is rotated through 2π about the x -axis.

(d) Find the volume of revolution of the solid generated. [4]

(e) Find the limit of the volume of revolution as $a \rightarrow \infty$. [1]

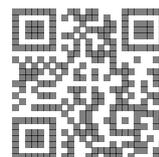
Total: 12

6.

$$I_n = \int_0^{\sqrt{2}} (2 - x^2)^n \, dx, \quad n \geq 0.$$

(a) Show that [9]

$$I_n = \frac{4n}{2n+1} I_{n-1}, \quad n \geq 1.$$



(b) Hence evaluate I_3 , leaving your answer in surd form.

[4]

Total: 13

7. The curve C has intrinsic equation

$$s = \ln \left(\tan \left(\frac{1}{2} \psi \right) \right), \quad 0 < \psi \leq \frac{\pi}{2}.$$

(a) Show that radius of curvature of C is given by $\rho \csc(\psi)$.

[4]

Given that $y = \psi = \frac{\pi}{2}$ when $x = 0$,

(b) show that $y = \psi$,

[4]

(c) use integration to show that a Cartesian equation of C is $x = \ln(\sin(y))$.

[7]

Total: 15

