Solomon Practice Paper

Pure Mathematics 4A

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	6	
3	6	
4	7	
5	9	
6	12	
7	14	
8	15	
Total:	75	

How I can achieve better:

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1.
$$f(z) \equiv z^3 - 5z^2 + 17z - 13.$$

(a) Show that $(z-1)$ is a factor of $f(z)$. [1]

(b) Hence find all the roots of the equation $f(z) = 0$, giving your answers in the form $a + \mathbf{i}b$ where a and b are integers. [5]

Total: 6

2. Find the general solution of the differential equation [6]

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{e^x}{x^2},$$

giving your answer in the form $y = f(x)$.

3. (a) Express $\dfrac{1}{r(r+1)}$ in partial fractions. [2]

(b) Hence, or otherwise, find [4]

$$\sum_{r=3}^{35} \frac{1}{r(r+1)},$$

giving your answer as a fraction in its lowest terms.

Total: 6

4. Find the set of values of x for which [7]

$$\frac{(x+3)^2}{x+1} < 2.$$

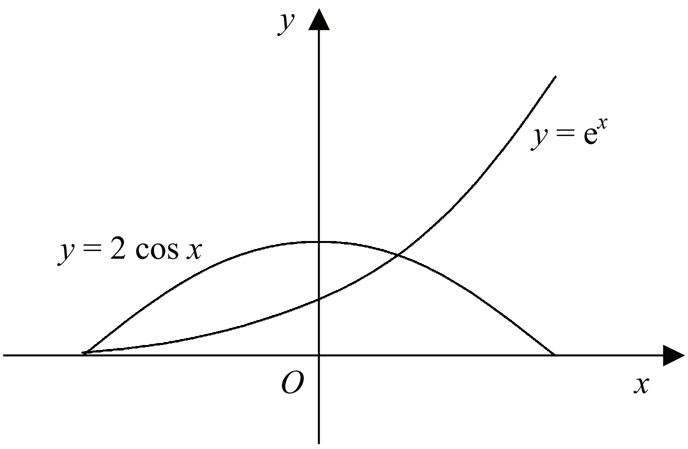
5. (a) Sketch the curve with polar equation [3]

$$r = a\cos(3\theta), \quad a > 0, \quad \text{for} \quad 0 \le \theta \le \pi.$$

(b) Show that the total area enclosed by the curve $r = a\cos(3\theta)$ is $\dfrac{\pi a^2}{4}$. [6]

Total: 9

6. Figure shows the curves $y = 2\cos(x)$ and $y = e^x$ in the interval $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.



Given that $f(x) \equiv e^x - 2\cos(x)$,

(a) write down the number of solutions of the equation $f(x) = 0$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [1]

(b) Show that the equation $f(x) = 0$ has a solution, α, in the interval $[0, 1]$. [2]

(c) Using 0.5 as a first approximation to α, use the Newton-Raphson process once to find an improved estimate for α, giving your answer correct to 2 decimal places. [4]

(d) Show that the estimate of α obtained in part (c) is accurate to 2 decimal places. [2]

There is another root, β, of the equation $f(x) = 0$ in the interval $[-2, -1]$.

(e) Use linear interpolation once on this interval to estimate the value of β, giving your answer correct to 2 decimal places. [3]

Total: 12

7. The complex numbers z and w are such that

$$z = \frac{A}{1-i} \quad \text{and} \quad w = \frac{B}{2+i},$$

where A and B are real.

Given that $z + w = 6$,

(a) find A and B. [6]

z and w are represented by the points P and Q respectively on an Argand diagram.

(b) Show P and Q on the same Argand diagram. [5]

(c) Find the distance PQ in the form $a\sqrt{5}$. [3]

Total: 14

8. (a) Find the values of p and q such that [6]

$$x = p\cos(t) + q\sin(t)$$

satisfies the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = \sin(t).$$

(b) Hence find the solution of this differential equation for which $x = 1$ and $\frac{dx}{dt} = 12$ at $t = 0$. [9]

Total: 15