

# Solomon Practice Paper

## Pure Mathematics 3K

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

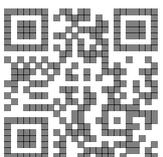
Question	Points	Score
1	7	
2	8	
3	10	
4	12	
5	12	
6	12	
7	14	
Total:	75	

How I can achieve better:

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1.

$$f(x) \equiv 3x^3 + ax^2 + 8x + b.$$

Given that when  $f(x)$  is divided by  $(3x - 1)$  the remainder is 1,

(a) show that  $a + 9b + 16 = 0$ . [3]

Given also that when  $f(x)$  is divided by  $(x - 2)$  the remainder is 11,

(b) find the values of  $a$  and  $b$ . [4]

Total: 7

2. (a) Find

$$\int \sin^2(y) \, dy.$$

[3]

(b) Given that  $y = \frac{\pi}{4}$  when  $x = 0$ , solve the differential equation

[5]

$$e^{4x} \frac{dy}{dx} = \csc^2(y).$$

Total: 8

3. A curve has the equation

$$xy - x^2 + 2y^2 = 36.$$

(a) Find an expression for  $\frac{dy}{dx}$ . [3]

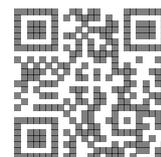
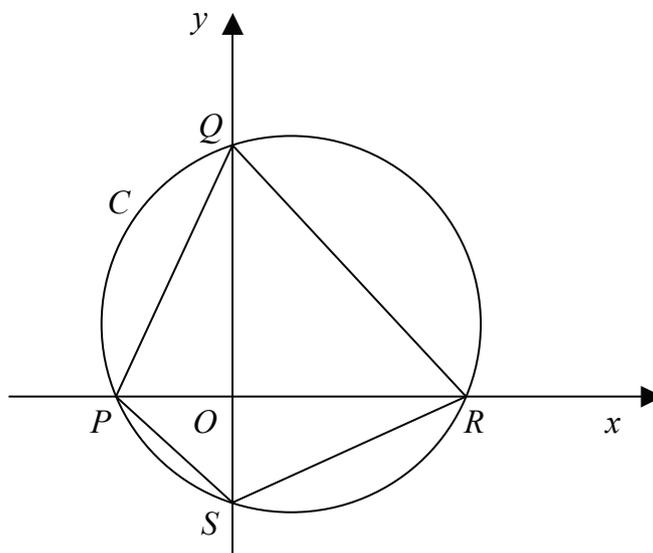
(b) Find the gradient of the curve at the point with coordinates  $(4\sqrt{2}, 6 - \sqrt{2})$ . [2]

The curve has two stationary points.

(c) Show that one of these has coordinates  $(2, 4)$  and find the coordinates of the other. [5]

Total: 10

4. Figure shows the circle  $C$  with centre  $(4, 5)$  and radius 13.



$C$  meets the  $x$ -axis at the points  $P$  and  $R$ , and the  $y$ -axis at the points  $Q$  and  $S$ .

- (a) Write down an equation for  $C$ . [2]  
 (b) Show that  $QS = 6\sqrt{17}$ . [5]  
 (c) Find the area of quadrilateral  $PQRS$ . [5]

Total: 12

5.

$$f(x) = \frac{2x^2 - 15x + 15}{(x - 1)^2(x - 3)}.$$

- (a) Find the values of  $A$ ,  $B$  and  $C$  for which [4]

$$f(x) \equiv \frac{A}{(x - 1)^2} + \frac{B}{x - 1} + \frac{C}{x - 3}.$$

The point  $P$  lies on the curve  $y = f(x)$  and has coordinates  $\left(\frac{3}{2}, p\right)$ .

- (b) Find the value of  $p$ . [2]  
 (c) Show that the tangent to the curve at  $P$  has the equation  $8x + 3y - 36 = 0$ . [6]

Total: 12

6. Algae is growing in on the surface of a large stagnant pond. A botanist records the area,  $A$  m, of the algae at the same time each day. She believes that  $t$  days after she began keeping records the area of the algae is given by

$$A = 3 \times 1.2^t.$$

- (a) Find the area of algae on the surface of the pond when the botanist began keeping records. [2]  
 (b) Show that one week later the area of the algae is growing at the rate of  $1.96 \text{ m}^2$  per day, correct to 3 significant figures. [4]  
 (c) Prove that according to the botanist's model the time taken for the area of algae to double is constant and find how long this takes correct to the nearest day. [6]

Total: 12

7. Relative to a fixed origin,  $O$ , the points  $P$  and  $Q$  have position vectors  $(4\mathbf{i} + 10\mathbf{j} - \mathbf{k})$  and  $(4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$  respectively.

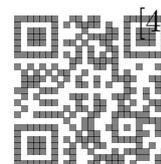
- (a) Find, in the form  $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ , an equation of the line  $l_1$  which passes through  $P$  and  $Q$ . [3]

The line  $l_2$  is given by the equation

$$\mathbf{r} = 7\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

where  $\mu$  is a parameter.

- (b) Show that  $l_1$  and  $l_2$  intersect and find the coordinates of their point of intersection,  $R$ . [4]



(c) Show that  $\angle ORQ = \cos^{-1}\left(\frac{3}{5}\right)$ . [5]

(d) Find the area of triangle  $OQR$ . [2]

Total: 14

