Solomon Practice Paper

Pure Mathematics 3G

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	8	
3	8	
4	9	
5	10	
6	10	
7	12	
8	13	
Total:	75	

How I can achieve better:

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1. Given that [5]

$$y = 2e^x(x-1),$$

show that

$$\frac{dy}{dx} = \frac{xy}{x-1}.$$

2. (a) Find [3]

$$\int \frac{x}{x^2+3}\,dx.$$

(b) Given that $y = 1$ when $x = 1$, solve the differential equation [5]

$$\left(x^2+3\right)\frac{dy}{dx} = xy,$$

giving your answer in the form $y^2 = f(x)$.

Total: 8

3. [8]

$$f(x) \equiv x^3 - x^2 - 8x + 14.$$

When $f(x)$ is divided by $(x-a)$ the remainder is 2.

By forming and factorising a cubic equation, find all possible values of a.

4. A curve has the equation

$$\cos(2x)\tan(y) = 1.$$

(a) Show that [4]

$$\frac{dy}{dx} = \tan(2x)\sin(2y).$$

The curve is stationary at the point with coordinates $\left(0, \frac{\pi}{4}\right)$.

(b) By evaluating $\frac{d^2y}{dx^2}$ at this stationary point, determine its nature. [5]

Total: 9

5. (a) Expand $(1+x)^{-1}, |x| < 1$, in ascending powers of x as far as the term in x^3. [2]

(b) Find the values of A, B and C for which [3]

$$\frac{1-3x}{(x^2+1)(x+1)} \equiv \frac{Ax+B}{x^2+1} + \frac{C}{x+1}.$$

(c) Hence, find the series expansion of [5]

$$\frac{1-3x}{(x^2+1)(x+1)}$$

as far as the term in x^3 and state the set of values of x for which it is valid.

Total: 10

6. The circle C has the equation

$$x^2 + y^2 + 2x - 8y + 15 = 0.$$

(a) Find the coordinates of the centre of C and write down its radius. [4]

P is the point with coordinates $(6, 3)$.

(b) Find the minimum distance of P from C. [3]

T is a point on C such that the line PT is a tangent to C.

(c) Find the length of the line PT in the form $k\sqrt{3}$. [3]

Total: 10

7. The lines l and m have the vector equations

$$l \; : \; \mathbf{r} = 12\mathbf{i} - 9\mathbf{j} + 8\mathbf{k} + \lambda(14\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}),$$
$$m \; : \; \mathbf{r} = 4\mathbf{i} + 8\mathbf{j} - 6\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - 4\mathbf{k}),$$

where λ and μ are parameters and a and b are constants.

Given that l and m are perpendicular,

(a) find an equation connecting a and b. [2]

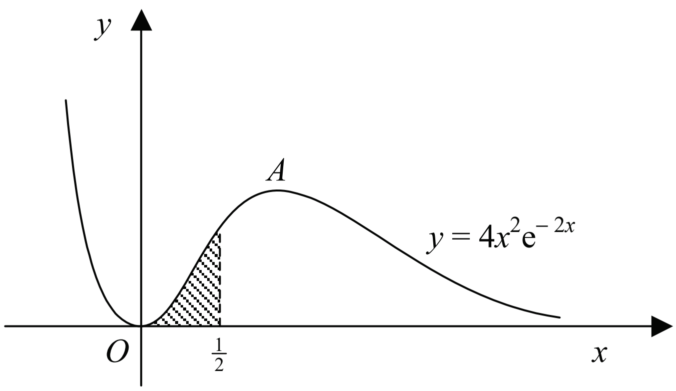
Given also that m passes through the z-axis,

(b) show that $a = 2$ and find the value of b, [5]

(c) show that the lines l and m intersect and find the coordinates of their point of intersection. [5]

Total: 12

8. Figure shows the curve with equation $y = 4x^2e^{-2x}$.



The curve is stationary at the origin, O, and at the point A.

(a) Find the coordinates of point A. [4]

The shaded region is bounded by the curve, the x-axis, and the line $x = \frac{1}{2}$.

(b) Show that the area of the shaded region is $\left(1 - \frac{5}{2}\mathrm{e}^{-1}\right)$. [9]

Total: 13