Solomon Practice Paper

Pure Mathematics 2L

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	7	
4	9	
5	9	
6	12	
7	12	
8	15	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. (a) Sketch the following graphs on separate diagrams, labelling the coordinates of any points where each graph meets the coordinate axes. [4]
   i. $y = |x+1|$.
   ii. $y = |x| + 1$.

   (b) Hence, write down the set of values of x for which [1]
   $$|x| + 1 > |x+1|.$$

Total: 5

2. (a) Prove by counter-example that $a > b$ does not imply that $(a+1)^2 > (b+1)^2$ for all integers a and b. [2]

   (b) Use proof by contradiction to show that $(4n - 3)$ is odd for all positive integers n. [4]

Total: 6

3. (a) Expand $\left(1 + \frac{1}{4}x\right)^8$ in ascending powers of x as far as the term in x^3, simplifying the coefficient in each term. [4]

   (b) Use your series to estimate the value of $\left(\frac{41}{40}\right)^8$, correct to 4 significant figures. [3]

Total: 7

4. (a) Show that for all values of x, where x is measured in degrees, [5]
   $$\cos(x + 60°) - \sqrt{3}\sin(x - 60°) \equiv 2\cos(x) - \sqrt{3}\sin(x).$$

   (b) Hence, find the values of x in the interval $-180° \leq x \leq 180°$, for which [4]
   $$\cos(x + 60°) - \sqrt{3}\sin(x - 60°) = 0,$$
   giving your answers to an appropriate degree of accuracy.

Total: 9

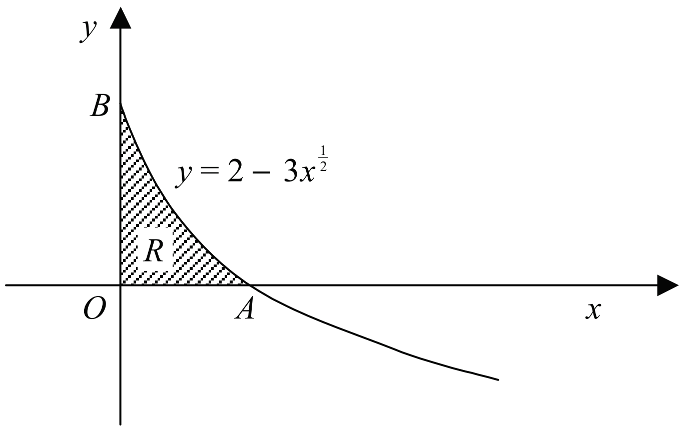
5.
$$\mathrm{f}(x) \equiv 2 - \frac{3}{x}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

   (a) Find and simplify an expression for $\mathrm{ff}(x)$ and state its domain. [5]

   (b) Show that $\mathrm{fff}(x) = \dfrac{4x+3}{6-x}$. [4]

Total: 9

6. Figure shows the curve with equation $y = 2 - 3x^{\frac{1}{2}}$.



The curve meets the x-axis at the point A and the y-axis at the point B.

(a) Find the coordinates of the points A and B. [3]

The shaded region, R, is bounded by the curve and the positive coordinate axes.

(b) Show that the volume generated when R is rotated through 360° about the x-axis is $\frac{8}{27}\pi$. [7]

(c) State, with a reason, whether the volume generated when R is rotated through 360° about the y-axis is more, less or the same as your answer to part (b). [2]

Total: 12

7.

$$f(x) \equiv \arccos(x), \quad x \in \mathbb{R}, \quad |x| \leq 1.$$

(a) State the exact value of x for which $f(x) = \frac{3}{4}\pi$. [2]

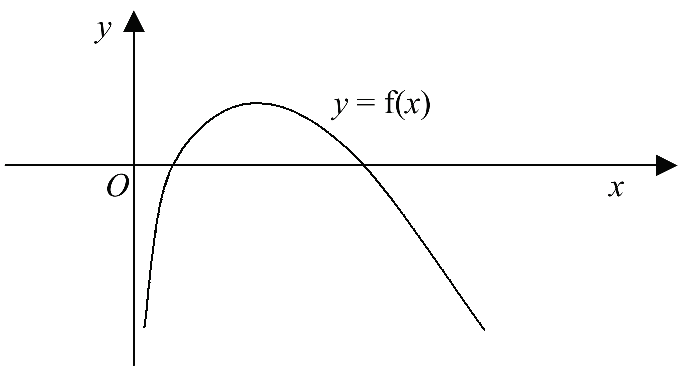
(b) Sketch the curve $y = f(x)$ and state its range. [3]

(c) Use the trapezium rule with 3 equally spaced ordinates to estimate the area enclosed by the curve $y = f(x)$ and the positive coordinate axes. Give your answer in the form $k\pi$ where k is an exact fraction. [5]

(d) Explain, with reference to the curve's symmetry, why the total area enclosed by the curve, the x-axis and the ordinate $x = -1$ is π. [2]

Total: 12

8. Figure shows the curve $y = f(x)$ where $f(x) \equiv \ln(5x) - 2x^2, x > 0$.



(a) Show that the maximum value of $\mathrm{f}(x)$ is $\ln\left(\frac{5}{2}\right) - \frac{1}{2}$. [5]

The point A lies on the curve and has x-coordinate $\frac{1}{5}$.

(b) Show that the equation of the tangent to the curve at A is $105x - 25y - 23 = 0$. [5]

(c) Show that that the equation $\mathrm{f}(x) = 0$ can be rearranged to give $x = \frac{1}{4}\left(\mathrm{e}^{2x^2} - x\right)$. [2]

(d) Use the iteration formula [3]

$$x_{n+1} = \frac{1}{4}\left(\mathrm{e}^{2x_n^2} - x\right)$$

with $x_0 = 0.25$ to find x_1, x_2 and x_3.

Hence, write down one root of the equation $\mathrm{f}(x) = 0$ correct to an appropriate degree of accuracy.

Total: 15