Solomon Practice Paper

Pure Mathematics 2H

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	6	
3	7	
4	9	
5	10	
6	12	
7	12	
8	13	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. Find, to an appropriate degree of accuracy, the values of x and y for which

   (a) $5^x = 10$, [3]

   (b) $\log_2(4^{2y}) = \log_3(27^{y+1})$. [3]

Total: 6

2. Use the trapezium rule with 5 equally spaced ordinates to estimate the value of [6]

$$\int_{-2}^{2} e^{\frac{1}{2}x+1}\,dx,$$

giving your answer correct to 3 significant figures.

3. (a) [4]

$$f(x) \equiv 4x^2 - 4x + 3, \quad x \in \mathbb{R}.$$

   Prove that $|f(x)| = f(x)$ for all values of x.

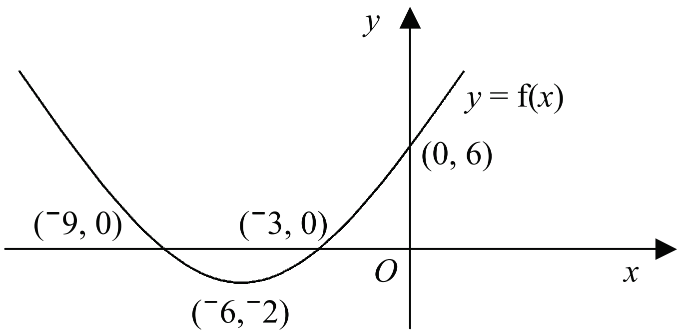
   (b) [3]

$$g(x) \equiv x^2 + 6x + 4, \quad x \in \mathbb{R}.$$

   Prove that there are no real solutions to the equation $g(|x|) = 0$.

Total: 7

4. (a) Figure shows the curve $y = f(x)$ which has a minimum point with coordinates $(-6, -2)$. The curve meets the coordinate axes at the points $(-9, 0), (-3, 0)$ and $(0, 6)$. [6]

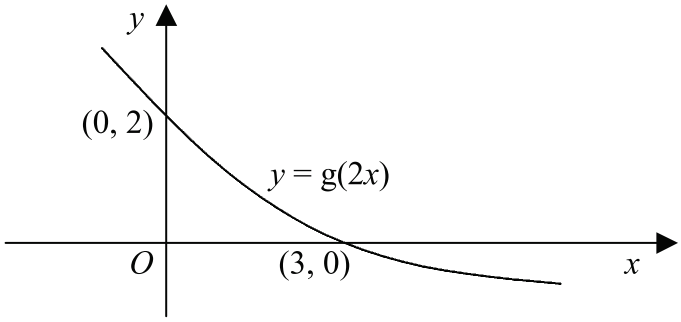


   Showing the coordinates of any turning points and any points where each curve meets the coordinate axes, sketch on separate diagrams graphs of

   i. $y = f(x - 3)$,

   ii. $y = 2 + \frac{1}{2}f(x)$.

   (b) Figure shows the curve $y = g(2x)$ which meets the coordinate axes at the points with coordinates $(3, 0)$ and $(0, 2)$. [3]



Showing the coordinates of any points where the curve meets the coordinate axes, sketch the graph $y = \mathrm{g}(x)$.

Total: 9

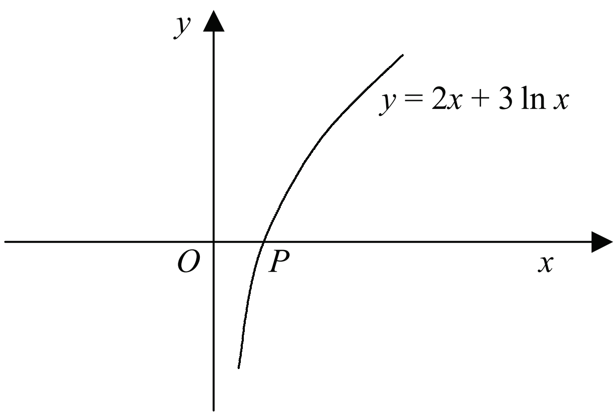
5. (a) Expand $(1+4x)^6$ in ascending powers of x as far as the term in x^3, simplifying the coefficient in each term. [4]

   (b) Use your series to estimate the value of $(1.04)^6$ correct to 4 significant figures. [3]

   (c) Find the coefficient of x^2 in the expansion of $(2 + x)(1 + 4x)^6$. [3]

Total: 10

6. Figure shows part of the curve with equation $y = 2x + 3\ln(x)$.



The curve crosses the x-axis at the point P with coordinates $(p, 0)$.

   (a) Show that $0.5 < p < 1$. [2]

   (b) Using the iteration [3]

$$x_{n+1} = \sqrt{\frac{x_n^{\frac{1}{2}}}{\mathrm{e}^{x_n}}}$$

   and $x_1 = 0.5$, find the value of x_4 correct to 3 significant figures.

   (c) Show that your answer to part (b) gives the value of p correct to 3 significant figures. [2]

The point Q with coordinates $(1, 2)$ lies on the curve.

   (d) Find an equation of the tangent to the curve at Q. [5]

Total: 12

7. The function f is given by

$$f\colon x \mapsto 2\cos(x) + \sin(x), \quad x \in \mathbb{R}.$$

Given that f(x) can be written as $R\cos(x - \alpha)$, where x is measured in degrees, $R > 0$ and $0 \leq \alpha \leq 90°$,

(a) show that $R = \sqrt{5}$ and find the value of α correct to 1 decimal place, [5]

(b) state the range of f(x). [1]

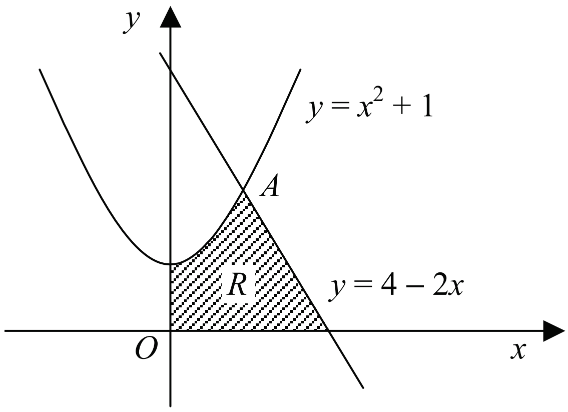
The function g is given by

$$g\colon x \mapsto \frac{8}{3+x}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(c) Find the range of gf(x), giving the minimum and maximum values in the form $a + b\sqrt{5}$. [6]

Total: 12

8. Figure shows the curve $y = x^2 + 1$ and the line $y = 4 - 2x$.



A is the point of intersection of the curve and line with a positive x-coordinate.

(a) Show that the point A has coordinates $(1, 2)$. [3]

The shaded region, R, is enclosed by the curve, the line and the positive coordinate axes.

(b) Show that the volume of the solid generated when R is rotated through 2π radians about the x-axis is $\dfrac{16}{5}\pi$. [10]

Total: 13