Solomon Practice Paper

Pure Mathematics 2A

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	7	
3	7	
4	8	
5	10	
6	11	
7	12	
8	15	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. By letting $y = 2^x$, or otherwise, solve the equation [5]

$$2^{2x} - 2^x - 6 = 0$$

giving any answers correct to 3 significant figures.

2. (a) Expand $(1-3x)^6$ in ascending powers of x as far as the term in x^3, simplifying the coefficient in each term. [4]

   (b) Using your series, together with a suitable value of x which you should state, estimate the value of $(0.997)^6$ correct to 6 significant figures. [3]

Total: 7

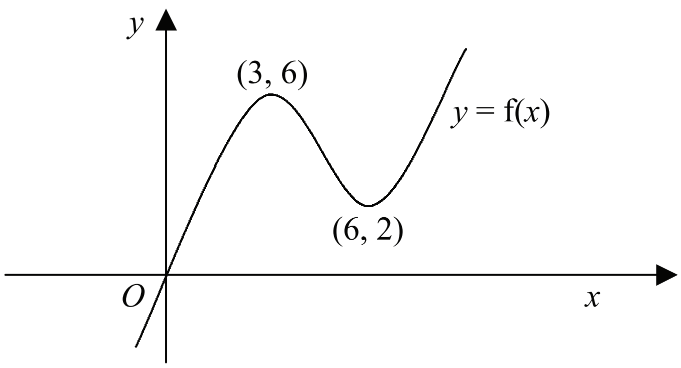
3. (a) Show that $(x+2)$ is a factor of $(x^3 - 2x^2 - 5x + 6)$. [2]

   (b) Hence, simplify the expression [5]

$$\frac{x^3 - 2x^2 - 5x + 6}{2x^2 - 5x - 3}$$

Total: 7

4. Figure shows part of the curve $y = f(x)$ which passes through the origin, O.



The curve has a maximum point with coordinates $(3, 6)$ and a minimum point with coordinates $(6, 2)$.

Showing the coordinates of any stationary points, sketch on separate diagrams the curves

   (a) $y = f(x+3)$, [2]

   (b) $y = f(2x)$, [2]

   (c) $y = f(|x|)$. [4]

Total: 8

5.

$$f(x) \equiv 1 + \frac{3}{x}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

   (a) Show that $ff(x) = \dfrac{4x+3}{x+3}$. [4]

(b) Prove that the equation $f(x) = kx + 2k$ will only have real solutions if $4k^2 + 8k + 1 \geq 0$. [4]

(c) Prove by counter-example that the equation $f(x) = kx + 2k$ does not have real solutions for all values of k. [2]

Total: 10

6. (a) Prove that for all values of x [5]

$$2\tan(x) - \sin(2x) \equiv 2\sin^2(x)\tan(x).$$

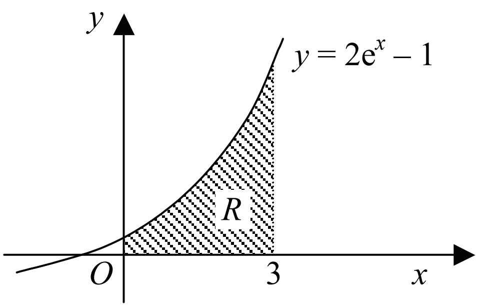
(b) Hence find the values of x in the interval $0 \leq x \leq 360°$, for which [6]

$$2\tan(x) - \sin(2x) = \sin^2(x),$$

giving your answers to an appropriate degree of accuracy.

Total: 11

7. Figure shows part of the curve with equation $y = 2e^x - 1$.



The shaded region, R, is enclosed by the curve, the positive coordinate axes and the ordinate $x = 3$.

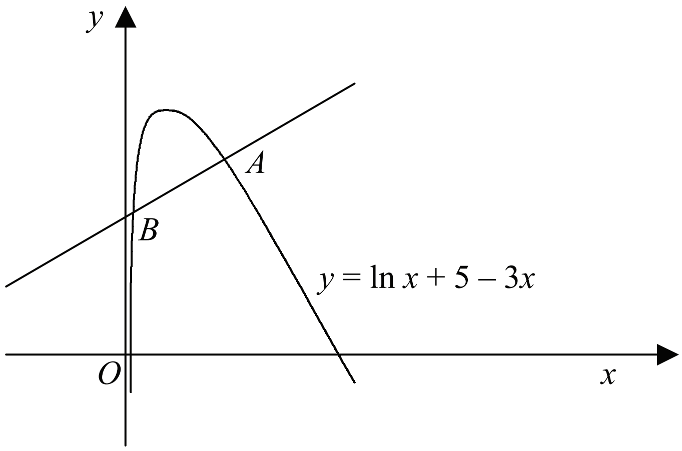
(a) Use the trapezium rule with 4 equally spaced ordinates to estimate the area of R, giving your answer in terms of e. [5]

(b) Use integration to show that the exact area of R is $2e^3 - 5$. [4]

(c) Find correct to 2 significant figures the percentage error in your estimate in part (a). [3]

Total: 12

8. Figure shows part of the curve $y = \ln(x) + 5 - 3x, x > 0$, and the normal to the curve at the point A.



The x-coordinate of the point A is 1.

(a) Find the equation of the normal to the curve at A in the form $ax + by + c = 0$. [7]

(b) Show that the x-coordinate of the point B, where the normal again intersects the curve is given by a solution of the equation $2\ln(x) + 7 - 7x = 0$. [3]

(c) Using an iteration of the form [5]

$$x_{n+1} = \mathrm{e}^{k(x_n - 1)},$$

with a starting value of $x_1 = 0.1$, find the x-coordinate of the point B giving your answer correct to 3 decimal places.

Total: 15