Solomon Practice Paper

Pure Mathematics 1E

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	7	
4	7	
5	9	
6	13	
7	13	
8	15	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. A cylinder has base radius $(\sqrt{3}-1)$ metres and height $\left(\dfrac{1}{2+\sqrt{3}}\right)$ metres. [5]

   Show that the volume of the cylinder is given by $(14-8\sqrt{3})\pi$ m^3.

2. $$f(x) \equiv x^2 + 2kx + k + 6.$$

   (a) Prove that the equation $f(x) = 0$ has repeated roots if $k^2 - k - 6 = 0$. [3]

   (b) Hence, or otherwise, find the values of k for which $f(x)$ is a perfect square. [3]

   Total: 6

3. $$y = 2x^{\frac{1}{3}} - 3x^{-\frac{1}{3}}.$$

   Given that $u = x^{\frac{1}{3}}$,

   (a) express y as a function of u. [2]

   (b) Hence, or otherwise, find the values of x for which $y = -5$. [5]

   Total: 7

4. (a) Sketch the curve $y = 2\sin(x/2) - 1$ for x in the interval $0 \le x \le 360°$. [3]

   (b) Find the values of x for which $y = 0$. [4]

   Total: 7

5. $$f(x) \equiv x^4 + 5x^3 - 26x^2 + 12x - 3.$$

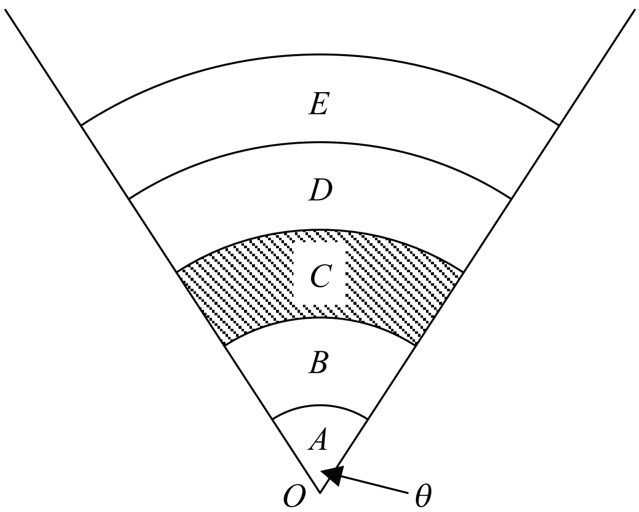
   (a) Find $f'(x)$. [2]

   (b) Show that $f'(x)$ has a factor $(x-2)$. [2]

   (c) Express $f'(x)$ as a product of 3 linear factors. [5]

   Total: 9

6. Figure shows a grid used to help spectators estimate distances at an athletics meeting.



The grid consists of circular sectors, each with centre O and angle θ.

The radius of the smallest sector is 5 m and each of the other sectors has a radius 5 m more than the previous one.

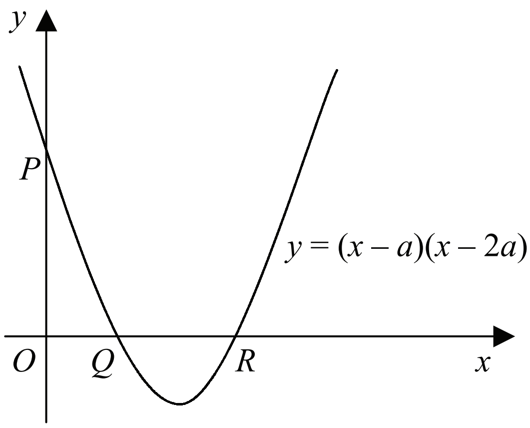
(a) Show that the perimeter, in metres, of the shaded region, C, is $25\theta + 10$. [3]

(b) Show that the perimeters of the regions A, B, C, D and E, are the terms of an arithmetic series. [5]

(c) Find the ratio of the area of the shaded region, C, to the area of the smallest sector, A, in the form $k\colon 1$. [5]

Total: 13

7. Figure shows part the graph of $y = (x - a)(x - 2a)$ which intersects the coordinate axes at P, Q, and R.



(a) Write down the coordinates of the points P, Q and R in terms of a. [3]

Given that $a = 2$,

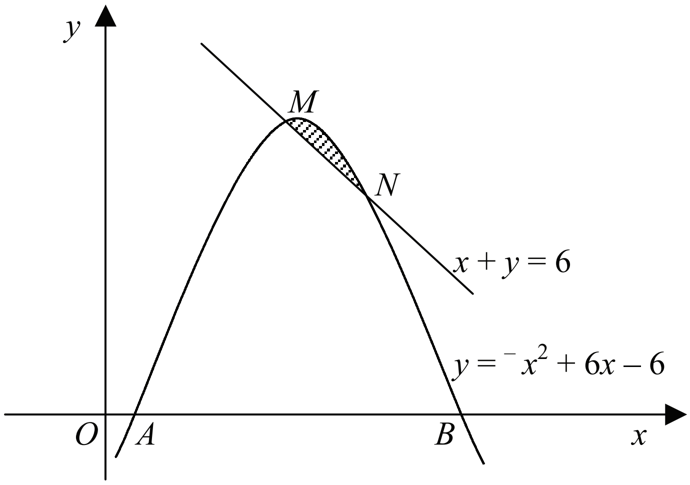
(b) show that the equation of the tangent to the curve at the point R is $y = 2x - 8$. [5]

The normal to the curve at R meets the curve again at S.

(c) Find the x-coordinate of S. [5]

Total: 13

8. Figure shows part of the curve $y = -x^2 + 6x - 6$ and the line $x + y = 6$.



The curve crosses the line at the points M and N and cuts the x-axis at the points A and B.

(a) Find the x-coordinates of the points A and B, giving your answers correct to 2 decimal places. [3]

(b) Find the coordinates of the points M and N. [5]

(c) Calculate the area of the shaded region enclosed by the curve and the line MN. [7]

Total: 15