Solomon Practice Paper

Pure Mathematics 5C

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	7	
3	10	
4	12	
5	12	
6	13	
7	16	
Total:	75	

How I can achieve better:

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Last updated: July 14, 2025

1. The curve C has intrinsic equation [5]

$$s = 4\sec^3(\psi), \quad 0 \leq \psi < \frac{\pi}{2}.$$

Find the radius of curvature of C at the point where $\psi = \frac{\pi}{4}$.

2. Solve the equation [7]

$$5\coth(x) + 1 = 7\,\text{cosech}(x),$$

giving your answer in terms of natural logarithms.

3. (a) Show that [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}.$$

(b) The curve with equation [7]

$$y = \arccos(x) - \frac{1}{2}\ln\left(1-x^2\right), \qquad -1 < x < 1,$$

has a stationary point in the interval $0 < x < 1$.

Find the exact coordinates of this stationary point.

Total: 10

4. (a) Express $3 - 6x - 9x^2$ in the form $a - (bx + c)^2$ where a, b and c are constants. [2]

Hence, or otherwise, find

(b) [4]

$$\int \frac{1}{\sqrt{3 - 6x - 9x^2}}\,\mathrm{d}x,$$

(c) expressing your answer to part (c) in terms of natural logarithms. [6]

$$\int_{-\frac{1}{3}}^{0} \frac{1}{\sqrt{3 - 6x - 9x^2}}\,\mathrm{d}x,$$

Total: 12

5.

$$f(x) = \operatorname{arctanh}\left(\frac{x^2 - 1}{x^2 + 1}\right), \quad x > 0.$$

(a) Using the definitions of $\sinh(x)$ and $\cosh(x)$ in terms of exponentials, express $\tanh(x)$ in terms of e^x and e^{-x}. [1]

(b) Hence prove that $f(x) = \ln(x)$. [6]

(c) Hence, or otherwise, show that the area bounded by the curve $y = \operatorname{arctanh}\left(\frac{x^2 - 1}{x^2 + 1}\right)$, the positive x-axis and the line $x = 2e$ is $2e\ln(2) + 1$. [5]

Total: 12

6. The ellipse C has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

(a) Find an equation of the normal to C at the point $P(5\cos(\theta), 3\sin(\theta))$. [5]

The normal to C at P meets the coordinate axes at Q and R.

Given that $ORSQ$ is a rectangle, where O is the origin,

(b) show that, as θ varies, the locus of S is an ellipse and find its equation in Cartesian form. [8]

Total: 13

7.

$$I_n(x) = \int_0^x \cos^n(2t)\,\mathrm{d}t, \qquad n \geq 0.$$

(a) Show that [7]

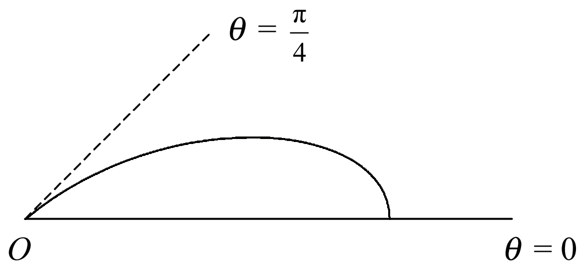
$$nI_n(x) = \frac{1}{2}\sin(2x)\cos^{n-1}(2x) + (n-1)I_{n-2}(x), \quad n \neq 2.$$

(b) Find $I_0\left(\frac{\pi}{4}\right)$ in terms of π. [2]

Figure shows the curve with polar equation

$$r = a\cos^2(2\theta), \qquad 0 \leq \theta \leq \frac{\pi}{4},$$

where a is a positive constant.



(c) Using your answers to parts (a) and (b), or otherwise, calculate the area bounded by the curve and the half-lines $\theta = 0$ and $\theta = \frac{\pi}{4}$. [7]

Total: 16