Solomon Practice Paper

Pure Mathematics 3K

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	7	
2	8	
3	10	
4	12	
5	12	
6	12	
7	14	
Total:	75	

How I can achieve better:

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1.

$$f(x) \equiv 3x^3 + ax^2 + 8x + b.$$

Given that when f(x) is divided by (3x-1) the remainder is 1,

(a) show that a + 9b + 16 = 0.

[3]

Given also that when f(x) is divided by (x-2) the remainder is 11,

(b) find the values of a and b.

[4]

Total: 7



2.	(a)	Find

$$\int \sin^2(y) \, \mathrm{d}y.$$

(b) Given that $y = \frac{\pi}{4}$ when x = 0, solve the differential equation

$$e^{4x} \frac{\mathrm{d}y}{\mathrm{d}x} = \csc^2(y).$$

Τc	ota.	l:	8

[3]





Total: 10

3. A curve has the equation

$$xy - x^2 + 2y^2 = 36.$$

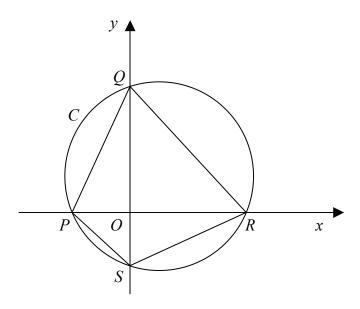
- (a) Find an expression for $\frac{dy}{dx}$. [3]
- (b) Find the gradient of the curve at the point with coordinates $(4\sqrt{2}, 6 \sqrt{2})$. [2]

The curve has two stationary points.

(c) Show that one of these has coordinates (2,4) and find the coordinates of the other. [5]



4. Figure shows the circle C with centre (4,5) and radius 13.



C meets the x-axis at the points P and R, and the y-axis at the points Q and S.

(a) Write down an equation for C.

[2]

(b) Show that
$$QS = 6\sqrt{17}$$
.

[5] [5]

(c) Find the area of quadrilateral PQRS.

Total: 12

5.

$$f(x) = \frac{2x^2 - 15x + 15}{(x-1)^2(x-3)}.$$

(a) Find the values of A, B and C for which

[4]

$$f(x) \equiv \frac{A}{(x-1)^2} + \frac{B}{x-1} + \frac{C}{x-3}.$$

The point P lies on the curve $y=\mathrm{f}(x)$ and has coordinates $\left(\frac{3}{2},p\right)$.

(b) Find the value of p. [2]

(c) Show that the tangent to the curve at P has the equation 8x + 3y - 36 = 0. [6]

Total: 12



[4]

6. Algae is growing in on the surface of a large stagnant pond. A botanist records the area, A m, of the algae at the same time each day. She believes that t days after she began keeping records the area of the algae is given by

$$A = 3 \times 1.2^t.$$

- (a) Find the area of algae on the surface of the pond when the botanist began keeping records. [2]
- (b) Show that one week later the area of the algae is growing at the rate of 1.96 m² per day, correct to 3 significant figures.
- (c) Prove that according to the botanist's model the time taken for the area of algae to double is constant and find how long this takes correct to the nearest day.

Total: 12



- 7. Relative to a fixed origin, O, the points P and Q have position vectors $(4\mathbf{i} + 10\mathbf{j} \mathbf{k})$ and $(4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$ respectively.
 - (a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$, an equation of the line l_1 which passes through P and Q. The line l_2 is given by the equation

$$\mathbf{r} = 7\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

where μ is a parameter.

- (b) Show that l_1 and l_2 intersect and find the coordinates of their point of intersection, R.
- (c) Show that $\angle ORQ = \cos^{-1}\left(\frac{3}{5}\right)$.

[5] [2]

[3]

[4]

(d) Find the area of triangle OQR.

Total: 14