Solomon Practice Paper

Pure Mathematics 3I

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	7	
3	9	
4	10	
5	11	
6	16	
7	17	
Total:	75	

How I can achieve better:

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Last updated: July 14, 2025

1. [5]

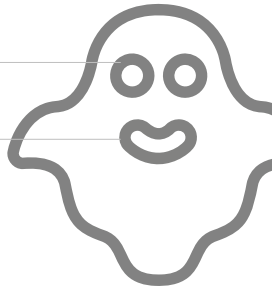
$$f(x) \equiv x^2 + 5x + 26.$$

The remainder when $f(x)$ is divided by $(x - a)$ is twice the remainder when $f(x)$ is divided by $(x + a)$.

Find the possible values of a.

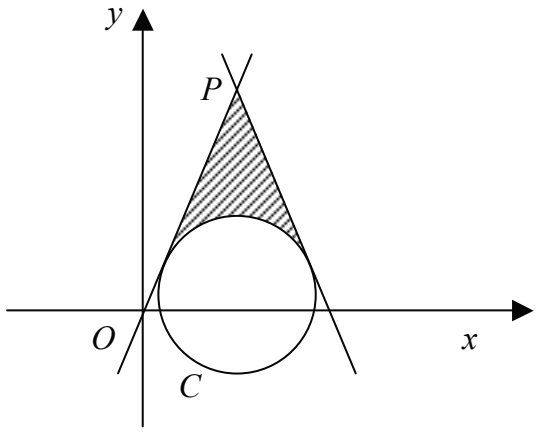
2. Given that $y = \frac{\pi}{8}$ when $x = \frac{\pi}{4}$. Solve the differential equation [7]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(2x)\cot^2(2y).$$



3. Figure shows the circle C with equation

$$x^2 + y^2 - 12x - 2y + 12 = 0.$$



(a) Find the coordinates of the centre of C and write down its radius. [4]

The shaded region in the diagram is enclosed by C and the two tangents to C which pass through the point P with coordinates $(6, 14)$.

(b) Show that the area of the shaded region is 30.6, correct to 3 significant figures. [5]

Total: 9

4. Using the substitution $u = \mathrm{e}^x - 1$, show that [10]

$$\int_{\ln(2)}^{\ln(5)} \frac{\mathrm{e}^{2x}}{\sqrt{\mathrm{e}^x - 1}}\,\mathrm{d}x = \frac{20}{3}.$$

5. Relative to a fixed origin, O, the points A, B and C have position vectors $(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$, $(10\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$ and $(4\mathbf{i} + \mathbf{j} - 5\mathbf{k})$ respectively.

   (a) Show that $\overrightarrow{AC} = k\overrightarrow{AB}$, where k is an exact fraction. [3]

   (b) Find $\cos(\angle OAC)$. [5]

   (c) Find the area of triangle OAC correct to 4 significant figures. [2]

   (d) Hence, write down the area of triangle OAB correct to 3 significant figures. [1]

Total: 11

6. (a) Find the values of A and B for which [3]

$$\frac{x-7}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}.$$

(b) Show that [5]

$$\int_2^4 \frac{x-7}{(x+2)(x-1)}\,\mathrm{d}x = \ln\left(\frac{3}{8}\right).$$

(c) Find, for $|x| < 1$, the series expansion of [8]

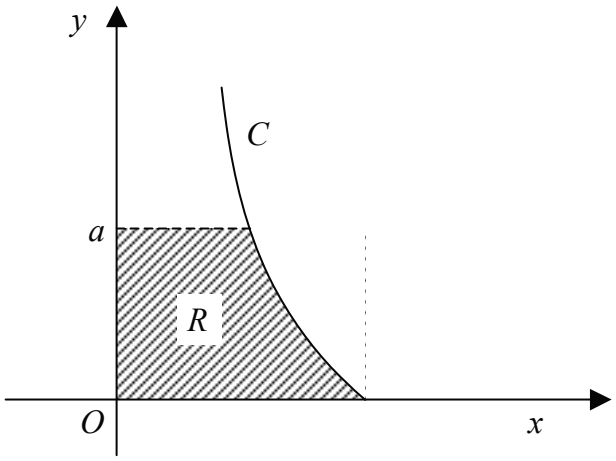
$$\frac{x-7}{(x+2)(x-1)}$$

up to and including the term in x^2, giving the coefficients as exact fractions.

Total: 16

7. Figure shows the curve C which is defined by the parametric equations

$$x = 2a\cos(t), \quad \text{and} \quad y = a\tan^2(t), \quad 0 \leq t \leq \frac{\pi}{2},$$



where t is a parameter and a is a positive constant.

(a) Find and simplify an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [5]

The point P on C has parameter $t = \dfrac{\pi}{3}$

(b) Find an equation of the tangent to C at the point P. [4]

(c) Show that a Cartesian equation of C is [3]

$$x^2 = \frac{4a^3}{a+y}.$$

The shaded region R is bounded by C, the positive coordinate axes, and the line $y = a$.

(d) Find the volume of the solid generated when the region R is rotated through 2π radians about the y-axis. [5]

Total: 17