Solomon Practice Paper

Pure Mathematics 3E

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	7	
4	9	
5	10	
6	11	
7	12	
8	15	
Total:	75	

How I can achieve better:

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1. (a) Expand [4]

$$\left(1 + \frac{2}{3}x\right)^{-2}$$

in ascending powers of x as far as the term in x^3.

(b) State the set of values of x for which your expansion is valid. [1]

Total: 5

2. Given that $y = -1$ when $x = 1$, solve the differential equation [6]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^2}{x^3},$$

giving your answer in the form $y = \mathrm{f}(x)$.

3.

$$f(x) \equiv x^3 + ax^2 + bx - 3.$$

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 2,

(a) find a linear relationship between a and b. [2]

Given also that $(3x - 2)$ is a factor of $f'(x)$,

(b) find the values of a and b. [5]

Total: 7

4.

$$\mathrm{f}(x) \equiv \frac{3x^2 - 4x - 1}{(x-2)(x+1)}.$$

(a) Express $\mathrm{f}(x)$ in the form [4]

$$A + \frac{B}{x-2} + \frac{C}{x+1}.$$

(b) Show that [5]

$$\int_3^5 \mathrm{f}(x)\,\mathrm{d}x = 6 + \ln\left(\frac{4}{3}\right).$$

Total: 9

5. A circle has the equation

$$x^2 + y^2 + 3x - 6y + 5 = 0.$$

(a) Find the distance of the centre of the circle from the origin in the form $k\sqrt{5}$ where k is an exact fraction. [5]

(b) Show that the line with equation [5]

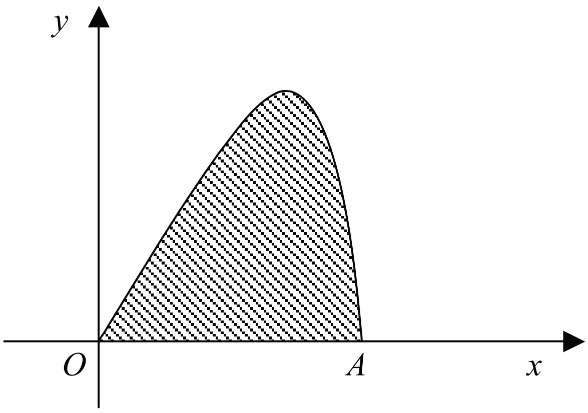
$$3x - 4y + 4 = 0$$

is a tangent to the circle.

Total: 10

6. Figure shows the curve with parametric equations

$$x = \cos(t), \quad \text{and} \quad y = 3\sin(2t), \quad 0 \leq t \leq \frac{\pi}{2}.$$



The curve meets the x-axis at the origin, O, and at the point A with coordinates $(1, 0)$.

(a) Find the value of the parameter t at the points O and A. [3]

(b) Find the area of the shaded region enclosed by the curve and the x-axis. [8]

Total: 11

7. A curve is given by the equation

$$y = 4e^{2x} + e^{-x}.$$

   (a) Find in exact form the coordinates of the stationary point on the curve. [9]

   (b) Sketch the curve, labelling the coordinates of any points of intersection with the coordinate axes. [3]

Total: 12

8. The lines l_1 and l_2 are given by

$$\begin{aligned} l_1 &: \quad \mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 9\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}), \\ l_2 &: \quad \mathbf{r} = 9\mathbf{i} + 2\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}). \end{aligned}$$

(a) Show that the lines l_1 and l_2 intersect and find the position vector of their point of intersection, P. [5]

(b) Show that the acute angle, θ, between lines l_1 and l_2 satisfies [4]

$$\cos(\theta) = \frac{1}{3}\sqrt{6}.$$

The point Q lies in the plane containing lines l_1 and l_2 and has position vector $(4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$.

(c) Find $\cos(\alpha)$, where α is the acute angle between PQ and line l_1. [3]

(d) By finding $\cos(2\theta)$, prove that $\alpha = 2\theta$. [3]

Total: 15