**Solomon Practice Paper**

**Pure Mathematics 6G**

**Time allowed:** 90 **minutes**


**Centre:** www.CasperYC.club

**Name:**

**Teacher:**

| Question | Points | Score |
|----------|--------|-------|
| 1 | 3 | |
| 2 | 6 | |
| 3 | 7 | |
| 4 | 9 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 13 | |
| 8 | 17 | |
| Total: | 75 | |


**How I can achieve better:**

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Last updated: May 5, 2023

1.   [3]

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -4 \\ 1 & 2 & -1 \\ 2 & k & 0 \end{pmatrix}$$

Find the value of the constant $k$ for which $\mathbf{A}$ is a singular matrix.

2. Solve the equation [6]
$$z^3 = -4 + 4\sqrt{3}\mathbf{i},$$

giving your answers in the form $r(\cos(\theta) + \mathbf{i}\sin(\theta))$ where $r > 0$ and $0 \leq \theta < 2\pi$.

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3. Prove by induction that $n(n^2 + 5)$ is divisible by 6 for all positive integers $n$. [7]

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4. The point $P$ represents the complex number $z$ in an Argand diagram. Given that

$$|z - 1 + 2\mathbf{i}| = 3,$$

(a) sketch the locus of $P$ in an Argand diagram. [3]

$T, U$ and $V$ are transformations from the $z$-plane to the $w$-plane where

$$
\begin{aligned}
T &: \quad w = 4z, \\
U &: \quad w = z + 5 - \mathbf{i}, \\
V &: \quad w = z\mathrm{e}^{\mathbf{i}\frac{\pi}{2}}.
\end{aligned}
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(b) Describe exactly the locus of the image of $P$ under each of these transformations. [6]

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5. (a) By finding the first four derivatives of $f(x) = \cos(x)$, find the Taylor series expansion of $f(x)$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^3$. [5]

   (b) Use this expansion to find an estimate of $\cos\left(\dfrac{\pi}{4}\right)$, giving your answer to 4 decimal places. [3]

   (c) Find the percentage error in your answer to part (b), giving your answer to 2 significant figures. [2]

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6. Given that $y$ satisfies the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = x^2 + xy - y^2, \quad y = \frac{1}{2} \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -1 \quad \text{at} \quad x = 0,$$

(a) use the Taylor series method to obtain a series for $y$ in ascending powers of $x$ up to and including the term in $x^3$.  [6]

(b) Use your series to estimate the value of $y$ at $x = -0.1$.  [1]

(c) Use the approximation  [3]

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

with a step length of 0.1 and your answer to part (b) to estimate the value of $y$ when $x = 0.1$.

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7. Referred to a fixed origin, the straight lines $l_1, l_2$ and $l_3$ have equations

$$l_1 \quad : \quad \mathbf{r} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}),$$
$$l_2 \quad : \quad \mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + t(4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}),$$
$$l_3 \quad : \quad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + u(2\mathbf{j} + \mathbf{k}).$$

The acute angle between $l_1$ and $l_2$ is $\theta$.

(a) Find the exact value of $\sin(\theta)$. [5]

The plane $\Pi$ contains the lines $l_1$ and $l_2$.

(b) Find an equation of $\Pi$, giving your answer in the form $ax + by + cz + d = 0$. [4]

(c) Show that the line $l_3$ lies on the plane $\Pi$. [4]

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8. (a) $A$ and $B$ are non-singular square matrices. Prove that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. [4]

   The transformations $S \colon \mathbb{R}^2 \mapsto \mathbb{R}^2$ and $T \colon \mathbb{R}^2 \mapsto \mathbb{R}^2$ are defined by

   $$S \colon \mapsto \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y - x \\ 2x + y \end{pmatrix} \quad \text{and} \quad T \colon \mapsto \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 3x \\ x + y \end{pmatrix}.$$

   (b) Show that $S$ represents a linear transformation. [7]

   (c) Using your result in (a), or otherwise, find the matrix that represents the transformation $(ST)^{-1}$. [6]

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