## Solomon Practice Paper

Pure Mathematics 6F

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	11	
4	11	
5	11	
6	13	
7	15	
Total:	75	

## How I can achieve better:

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[6]

1.	Prove by induction that, for all $n \in \mathbb{Z}^+$ ,
	$\sum_{r=1}^{n} \ln \left( \frac{r+1}{r} \right) = \ln(n+1).$



2.

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$$

(a) Find the eigenvalues of $\mathbf{M}$ .	[4]
(b) Find eigenvectors corresponding to each eigenvalue found in part (a).	[4]
	Total: 8
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[5]

3. A transformation T from the z-plane to the w-plane is defined by

$$w = \frac{z + 2\mathbf{i}}{z - \mathbf{i}}, \quad z \neq \mathbf{i},$$

where  $z = x + \mathbf{i}y, w = u + \mathbf{i}v$  and x, y, u and v are real.

(a) Show that the circle |z|=1 is mapped onto a straight line in the w-plane under T and find an equation of the line.

The circle |z - (a + ib)| = r in the z-plane is mapped under T onto the circle |w| = 2 in the w-plane, where a, b and r are real.

(b) Find the values of $a, b$ and $r$ .		[6]
	Total:	11
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- 4. The points A, B and C with coordinates  $(x_{-1}, y_{-1}), (x_0, y_0)$  and  $(x_1, y_1)$  respectively lie on the curve y = f(x) with  $x_1 x_0 = x_0 x_{-1} = h$ .
  - (a) Use the first three terms of the Taylor series expansion in ascending powers of  $(x x_0)$  to show that  $\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right) \approx \frac{y_1 2y_0 + y_{-1}}{h^2}.$

The variable y satisfies the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (x+2)\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0$$

with y = 1 at x = 0 and y = 1.2 at x = 0.1.

(b) Use the approximations

 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$  and  $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ 

with a step length of 0.1 to estimate the value of y at x = 0.2.

Total	al: 11



5.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & q & 1 \\ 1 & 2 & -1 \end{pmatrix}, \quad q \neq 4\frac{1}{4}.$$

- (a) Find  $\mathbf{A}^{-1}$  in terms of q.
- (b) Hence, or otherwise, solve the simultaneous equations

[4]

Total: 11

$$x - y + 3z = 1$$

$$4x + y + z = 2$$

$$x + 2y - z = 5$$

showing your working clearly.




C	Given	41 4
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$$y = \sqrt{1 - x^2} \arccos(x),$$

(a) show that		[5]
	$(1 - x^2)\frac{\mathrm{d}y}{\mathrm{d}x} + xy - x^2 + 1 = 0.$	(*)

(b) By differentiating equation $\star$ twice, or otherwise, obtain the Maclaurin expansion of $y =$	[8]
$\sqrt{1-x^2}\arccos(x)$ up to and including the term in $x^3$ .	
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7. The plane  $\Pi_1$  has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + \mathbf{k}).$$

<ul> <li>(a) Find a vector n which is normal to Π<sub>1</sub>.</li> <li>(b) Hence find a vector equation of Π<sub>1</sub> in the form r.n = p.</li> <li>(c) Find the perpendicular distance between Π<sub>1</sub> and the point A with position vector 2i+j+4k, giving your answer in the form a√6, where a ∈ Q.</li> <li>The plane Π<sub>2</sub> has equation r.(i + bj) = -4. The angle between Π<sub>1</sub> and Π<sub>2</sub> is 30°.</li> </ul>	
(d) Find the possible values of the constant $b$ .	
To	tal:

Last updated: May 5, 2023

