Solomon Practice Paper

Pure Mathematics 5G

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	7	
2	7	
3	8	
4	9	
5	12	
6	14	
7	18	
Total:	75	

How I can achieve better:

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1. Given that $y = \mathrm{e}^{\arctan(x)}$,

   (a) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [4]

   The curve $y = \mathrm{e}^{\arctan(x)}$ has a point of inflexion.

   (b) Find the coordinates of this point of inflexion. [3]

Total: 7

2. (a) Prove that [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arcosh}(x) = \frac{1}{\sqrt{x^2-1}}.$$

(b) Find [4]

$$\int \operatorname{arcosh}(x)\,\mathrm{d}x.$$

Total: 7

3. Find [8]

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin(2x)}\,\mathrm{d}x.$$

4. (a) Find [6]

$$\int \frac{1}{\sqrt{4x^2 - 4x + 10}} \, \mathrm{d}x.$$

(b) Hence evaluate [3]

$$\int_{\frac{1}{2}}^{2} \frac{1}{\sqrt{4x^2 - 4x + 10}} \, \mathrm{d}x.$$

giving your answer in terms of natural logarithms.

Total: 9

5. (a) On the same axes sketch the curves with equations $y = 2 - \tanh(x)$ and $y = 3\,\mathrm{sech}(x)$, giving the coordinates of the points of intersection of the curves with the coordinate axes and the equations of the asymptotes. [5]

   (b) Solve the equation [7]

$$2 - \tanh(x) = 3\,\mathrm{sech}(x),$$

   giving your answers to 2 decimal places.

Total: 12

6.

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,\mathrm{d}x, \quad n \geq 0.$$

(a) Show that [7]

$$I_n = \frac{n-1}{n} I_{n-2}, \quad n \geq 2.$$

The curve C is defined by $y = \sin^2(x), 0 \leq x \leq \pi$.

The area bounded by C and the positive x-axis is rotated through 2π radians about the x-axis.

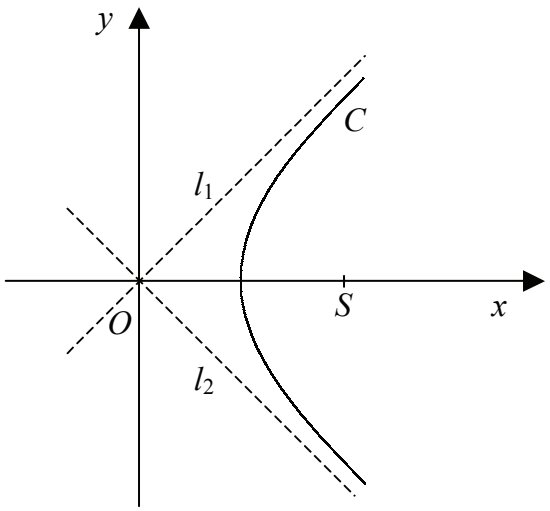
(b) Find the volume of the solid generated giving your answer in terms of π. [7]

Total: 14

7. Figure shows the curve C which is part of the hyperbola with parametric equations

$$x = a\cosh(t), \quad \text{and} \quad y = 2a\sinh(t),$$

where a is a positive constant and $x \geq a$.



The lines l_1 and l_2 are asymptotes to C.

(a) Show that the radius of curvature of C at its vertex is $4a$. [6]

(b) Show that an equation of the tangent to C at the point $P(\cosh(p), 2a\sinh(p))$ is [4]

$$2x\cosh(p) - y\sinh(p) = 2a.$$

The tangent to the curve C at P meets the asymptote l_1 at Q.

Given that QS is parallel to the y-axis, where S is the focus,

(c) show that $p = \frac{1}{2}\ln(5)$. [8]

Total: 18