# Solomon Practice Paper

## Pure Mathematics 5E

**Time allowed:** 90 **minutes**

**Centre:** www.CasperYC.club

**Name:**

**Teacher:**

| Question | Points | Score |
|---|---|---|
| 1 | 4 | |
| 2 | 6 | |
| 3 | 9 | |
| 4 | 9 | |
| 5 | 11 | |
| 6 | 11 | |
| 7 | 11 | |
| 8 | 14 | |
| Total: | 75 | |

**How I can achieve better:**

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1. A student without a calculator must find the value of $x$ given that $\operatorname{arctanh}(x) = \ln(3)$. With clear working, show how the student could find $x$ and state the value he should obtain. [4]

2.

$$\mathrm{f}(x) = \sin(2x) - x\cosh^2(x).$$

(a) Find $\mathrm{f}'(x)$. [3]

(b) Show that the curve with equation $y = \mathrm{f}(x)$ has a stationary point in the interval $0.3 < x < 0.4$. [3]

Total: 6

3. Given that [9]

$$\int_0^{\frac{2\pi}{3}} \frac{1}{5+4\cos(x)}\,\mathrm{d}x = a\pi, \qquad a \in \mathbb{Q},$$

use the substitution $t = \tan\left(\frac{1}{2}x\right)$ to find the value of $a$.

4. The curve $C$ has equation [9]

$$y = a\cosh\left(\frac{x}{a}\right),$$

where $a$ is a positive constant.

The area bounded by the curve $C$, the $x$-axis and the lines $x = -a$ and $x = a$ is rotated through $2\pi$ radians about the $x$-axis.

Show that the curved surface area of the solid generated is $\pi a^2(\sinh(2) + 2)$.

5. The intrinsic equation of the curve $C$ is $s = 2\psi$.

   Given that $s$ is measured from the origin,

   (a) find a Cartesian equation of $C$, [9]

   (b) sketch $C$. [2]

Total: 11

6. (a) Using the definitions of hyperbolic functions in terms of exponential functions, prove that [4]

$$\cosh(x+y) \equiv \cosh(x)\cosh(y) + \sinh(x)\sinh(y).$$

Given that

$$5\cosh(x) + 4\sinh(x) \equiv R\cosh(x+\alpha),$$

find

(b) the value of $R$, [3]

(c) the value of $\alpha$, giving your answer in terms of natural logarithms. [3]

(d) Hence, or otherwise, state the minimum value of $5\cosh(x) + 4\sinh(x)$. [1]

Total: 11

7.

$$I_n = \int_0^1 x^n \mathrm{e}^{x^2}\,\mathrm{d}x, \quad n \geq 0.$$

(a) Show that [5]

$$I_n = \frac{1}{2}\mathrm{e} - \frac{1}{2}(n-1)I_{n-2}, \quad n \geq 2.$$

(b) Hence find [6]

$$I_n = \int_0^1 x^5 \mathrm{e}^{x^2}\,\mathrm{d}x,$$

giving your answer in terms of e.

Total: 11

8. The line with equation $y = mx + c$ is a tangent to the parabola with equation $y^2 = 8x$.

   (a) Show that $mc = 2$. [5]

   The lines $l_1$ and $l_2$ are tangents to both the parabola with equation $y^2 = 8x$ and the circle with equation $x^2 + y^2 = 2$.

   (b) Find the equations of $l_1$ and $l_2$. [9]

Total: 14