Solomon Practice Paper

Pure Mathematics 5C

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	7	
3	10	
4	12	
5	12	
6	13	
7	16	
Total:	75	

How I can achieve better:

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[5]

1.	The curve C has intrinsic equation
	$s = 4\sec^3(\psi), 0 \le \psi < \frac{\pi}{2}.$
	Find the radius of curvature of C at the point where $\psi = \frac{\pi}{4}$.



[7]

۷.	Solve the equation $5 \coth(x) + 1 = 7 \operatorname{cosech}(x),$
	giving your answer in terms of natural logarithms.



2	(a) Show that		[3]
ა.	(a) Show that		၁
		$\frac{\mathrm{d}}{\mathrm{d}x}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}.$	
		$\frac{dr}{dr} \operatorname{arccos}(x) = \frac{1}{\sqrt{1 - m^2}}$	

(b) The curve with equation [7]

$$y = \arccos(x) - \frac{1}{2}\ln(1 - x^2), \quad -1 < x < 1,$$

has a stationary point in the interval 0 < x < 1.

Find the exact coordinates of this stationary point.

T	otal: 10



1.	(a) Express $3 - 6x - 9x^2$ in the form $a - (bx + c)^2$ where a, b and c are constants. Hence, or otherwise, find		[2]
	$\int \frac{1}{\sqrt{3 - 6x - 9x^2}} \mathrm{d}x,$		[4]
	(c) expressing your answer to part (c) in terms of natural logarithms.		[6]
	$\int_{-\frac{1}{3}}^{0} \frac{1}{\sqrt{3 - 6x - 9x^2}} \mathrm{d}x,$		
		Total:	12
		•	

5.

$$f(x) = \operatorname{arctanh}\left(\frac{x^2 - 1}{x^2 + 1}\right), \quad x > 0.$$

- (a) Using the definitions of $\sinh(x)$ and $\cosh(x)$ in terms of exponentials, express $\tanh(x)$ in terms of e^x and e^{-x} .
- (b) Hence prove that $f(x) = \ln(x)$. [6]
- (c) Hence, or otherwise, show that the area bounded by the curve $y = \operatorname{arctanh}\left(\frac{x^2 1}{x^2 + 1}\right)$, the positive x-axis and the line x = 2e is $2e \ln(2) + 1$.

Total: 12
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. The ellipse C has equation	
$x^2 - y^2$	
$\frac{x^2}{25} + \frac{y^2}{9} = 1.$	
(a) Find an equation of the normal to C at the point $P(5\cos(\theta), 3\sin(\theta))$)).
The normal to C at P meets the coordinate axes at Q and R .	
Given that $ORSQ$ is a rectangle, where O is the origin,	
(b) show that, as θ varies, the locus of S is an ellipse and find its equation	ion in Cartesian form.
	Total

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[5]

[8]

13

[7]

7.

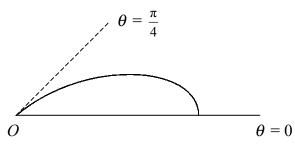
$$I_n(x) = \int_0^x \cos^n(2t) \, \mathrm{d}t, \qquad n \ge 0.$$

- (a) Show that $nI_n(x) = \frac{1}{2}\sin(2x)\cos^{n-1}(2x) + (n-1)I_{n-2}(x), \quad n \neq 2.$
- (b) Find $I_0\left(\frac{\pi}{4}\right)$ in terms of π .

Figure shows the curve with polar equation

$$r = a\cos^2(2\theta), \qquad 0 \le \theta \le \frac{\pi}{4},$$

where a is a positive constant.



(c) Using your answers to parts (a) and (b), or otherwise, calculate the area bounded by the curve and the half-lines $\theta = 0$ and $\theta = \frac{\pi}{4}$.	[7]
	al: 16

