

Solomon Practice Paper

Pure Mathematics 5C

Time allowed: 90 minutes

Centre: www.CasperYC.club

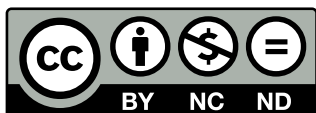
Name:

Teacher:

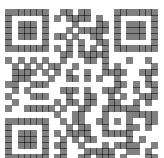
Question	Points	Score
1	5	
2	7	
3	10	
4	12	
5	12	
6	13	
7	16	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023



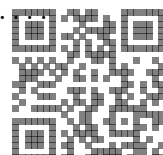
2. Solve the equation

$$5 \operatorname{coth}(x) + 1 = 7 \operatorname{cosech}(x),$$

[7]

giving your answer in terms of natural logarithms.

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5.

$$f(x) = \operatorname{arctanh}\left(\frac{x^2 - 1}{x^2 + 1}\right), \quad x > 0.$$

- (a) Using the definitions of $\sinh(x)$ and $\cosh(x)$ in terms of exponentials, express $\tanh(x)$ in terms of e^x and e^{-x} . [1]
- (b) Hence prove that $f(x) = \ln(x)$. [6]
- (c) Hence, or otherwise, show that the area bounded by the curve $y = \operatorname{arctanh}\left(\frac{x^2 - 1}{x^2 + 1}\right)$, the positive x -axis and the line $x = 2e$ is $2e \ln(2) + 1$. [5]

Total: 12

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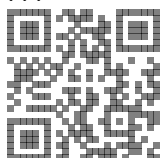
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6. The ellipse C has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

(a) Find an equation of the normal to C at the point $P(5 \cos(\theta), 3 \sin(\theta))$. [5]

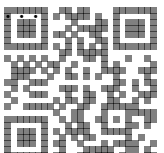
The normal to C at P meets the coordinate axes at Q and R .

Given that $ORSQ$ is a rectangle, where O is the origin,

(b) show that, as θ varies, the locus of S is an ellipse and find its equation in Cartesian form. [8]

Total: 13

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7.

$$I_n(x) = \int_0^x \cos^n(2t) dt, \quad n \geq 0.$$

(a) Show that [7]

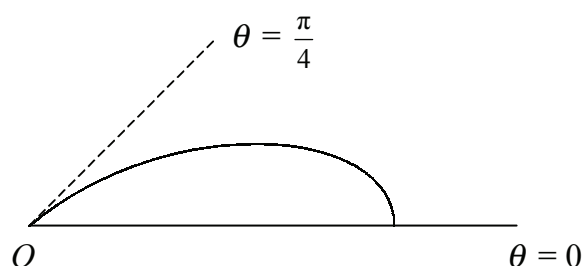
$$nI_n(x) = \frac{1}{2} \sin(2x) \cos^{n-1}(2x) + (n - 1)I_{n-2}(x), \quad n \neq 2.$$

(b) Find $I_0\left(\frac{\pi}{4}\right)$ in terms of π . [2]

Figure shows the curve with polar equation

$$r = a \cos^2(2\theta), \quad 0 \leq \theta \leq \frac{\pi}{4},$$

where a is a positive constant.



(c) Using your answers to parts (a) and (b), or otherwise, calculate the area bounded by the curve and the half-lines $\theta = 0$ and $\theta = \frac{\pi}{4}$. [7]

Total: 16

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