Solomon Practice Paper

Pure Mathematics 5B

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	7	
2	8	
3	9	
4	11	
5	12	
6	13	
7	15	
Total:	75	

How I can achieve better:

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1. Given that [7]

$$y \arccos(x) - \frac{x}{\pi} e^{2x} - 1 = 0,$$

find the value of at the point where $x = 0$, giving your answer in terms of π.

2. [8]

$$\mathrm{f}(x) = 5\cosh(x) + 3\sinh(x).$$

The minimum value of $\mathrm{f}(x)$ occurs at the point $(p\ln(q), r)$ where p, q and r are integers.

Find the values of p, q and r.

3. The line $y = mx + c$ is a tangent to the rectangular hyperbola with equation $xy = -9$.

   (a) Show that $c = \pm 6\sqrt{m}$. [4]

   (b) Hence, or otherwise, find the equations of the tangents from the point $(4, -2)$ to the rectangular hyperbola $xy = -9$. [5]

Total: 9

4. The curve C is defined by [11]

$$y^2 = x, \quad x \geq 0, \quad y \geq 0.$$

The region between C, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

Show that the area of the surface generated is

$$\frac{\pi}{6}\left(5\sqrt{5} - 1\right).$$

5. (a) Using the definition of $\cosh(x)$ in terms of exponential functions, express $\operatorname{sech}(x)$ in terms of e^{x} and e^{-x}. [1]

   (b) Sketch the graph of $y = \operatorname{sech}(x)$. [2]

   (c) Show that [4]

$$\int \operatorname{sech}(x)\,\mathrm{d}x = 2\arctan\left(\mathrm{e}^{x}\right) + c.$$

The curve C has equation $y = \operatorname{sech}(x)$. The region between C, the x-axis and the lines $x = -a$ and $x = a$, where a is a positive constant, is rotated through 2π about the x-axis.

   (d) Find the volume of revolution of the solid generated. [4]

   (e) Find the limit of the volume of revolution as $a \to \infty$. [1]

Total: 12

6.

$$I_n \int_0^{\sqrt{2}} \left(2 - x^2\right)^n \, \mathrm{d}x, \quad n \geq 0.$$

(a) Show that [9]

$$I_n = \frac{4n}{2n+1} I_{n-1}, \quad n \geq 1.$$

(b) Hence evaluate I_3, leaving your answer in surd form. [4]

Total: 13

7. The curve C has intrinsic equation

$$s = \ln\left(\tan\left(\frac{1}{2}\psi\right)\right), \quad 0 < \psi \leq \frac{\pi}{2}.$$

(a) Show that radius of curvature of C is given by $\rho\csc(\psi)$. [4]

Given that $y = \psi = \frac{\pi}{2}$ when $x = 0$,

(b) show that $y = \psi$, [4]

(c) use integration to show that a Cartesian equation of C is $x = \ln(\sin(y))$. [7]

Total: 15