Solomon Practice Paper

Pure Mathematics 5A

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	8	
3	9	
4	10	
5	10	
6	10	
7	11	
8	12	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. A curve has the equation [5]

$$y = x + 2x^2 + 5x^3.$$

Show that the radius of curvature of the curve at the origin is $\frac{1}{\sqrt{2}}$.

2. Show that [8]

$$\int_0^{\ln(2)} x \operatorname{sech}^2(x)\,\mathrm{d}x = \frac{3}{5}\ln(2) - \ln\left(\frac{5}{4}\right).$$

3. (a) Prove that [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\arcsin(2x) = \frac{2}{\sqrt{1-4x^2}}.$$

Given that

$$\mathrm{f}(x) = 2x\arcsin(2x) + \sqrt{1-4x^2},$$

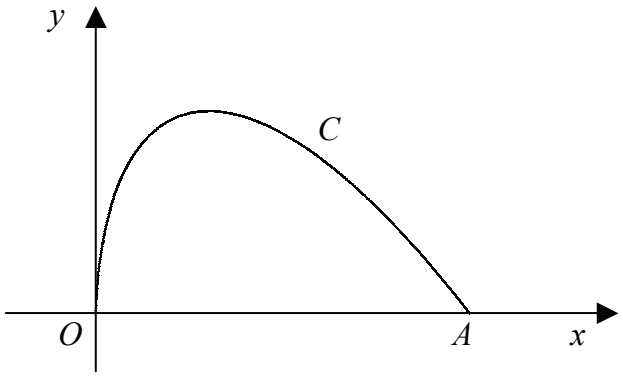
(b) Show that [6]

$$\mathrm{f}''(x)\left[\mathrm{f}(x) - x\mathrm{f}'(x)\right] = 4.$$

Total: 9

4. The parametric coordinates of the curve C shown above are

$$x = t^2, \quad \text{and} \quad y = t - \frac{1}{3}t^3, \quad 0 \leq t \leq a.$$



The curve C meets the x-axis at the point A where $t = a$.

(a) Find the value of a. [2]

The curve C is rotated through 2π about Ox.

(b) Find the surface area of the solid generated. [8]

Total: 10

5. (a) Using the definitions of $\cosh(x)$ and $\sinh(x)$ in terms of e^x and e^{-x}, prove that [3]

$$\cosh(2x) = 2\cosh^2(x) - 1.$$

(b) Solve the equation [7]

$$2\cosh(2x) = 13\cosh(x) - 12,$$

giving your answers in terms of natural logarithms.

Total: 10

6.

$$x^2 - 10x + 41 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b. [2]

(b) Show that [8]

$$\int_5^9 \frac{x}{\sqrt{x^2 - 10x + 41}}\,\mathrm{d}x = p\left(\sqrt{2} - 1\right) + q\ln(r),$$

stating your values of p, q and r.

Total: 10

7.

$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos(x)\,\mathrm{d}x \quad n \geq 0.$$

(a) Prove that [5]

$$I_n = \left(\frac{\pi}{2}\right) - n(n-1)I_{n-2}, \qquad n \geq 2.$$

(b) Hence find the value of I_4, giving your answer in terms of π. [6]

Total: 11

8. The rectangular hyperbola C has equation $xy = c^2$, where c is a positive constant.

   (a) Show that an equation of the tangent to C at the point $P\left(cp, \frac{c}{p}\right)$ is [4]

   $$x + yp^2 = 2cp.$$

   The tangent to C at P meets the x-axis at the point X.
   The point Q on C has coordinates $\left(cq, \frac{c}{q}\right), q \neq p$ such that QX is parallel to the y-axis.

   (b) Show that $q = 2p$. [3]

   M is the mid-point of PQ.

   (c) Find, in Cartesian form, an equation of the locus of M as p varies. [5]

Total: 12