Solomon Practice Paper

Pure Mathematics 4H

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	9	
4	9	
5	10	
6	15	
7	18	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. (a) Given that $f(r) = r!$, show that $f(r+1) - f(r) = r \times r!$. [2]

   (b) Hence find $\sum_{r=1}^{n}(r \times r!)$. [4]

Total: 6

2. (a) Given that [5]

$$y = \frac{2x}{x^2 + 9},$$

express x in terms of y.

(b) Hence prove that for all real values of x [3]

$$-\frac{1}{a} \leq \frac{2x}{x^2 + 9} \leq \frac{1}{a},$$

where a is a positive integer which you should find.

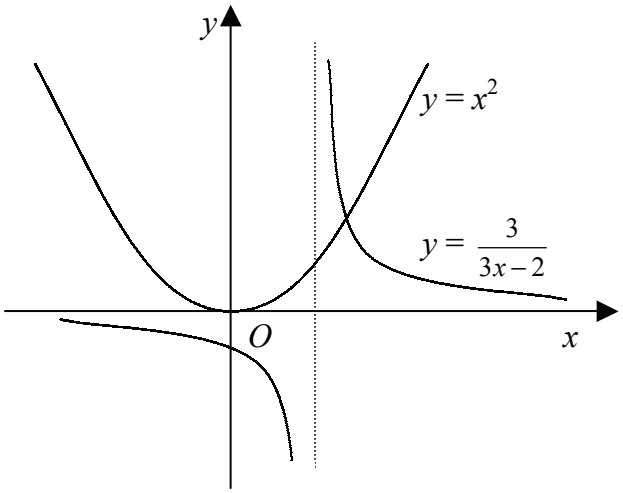
Total: 8

3. Find the general solution of the differential equation [9]

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 1 - y,$$

giving your answer in the form $y = \mathrm{f}(x)$.

4. Figure shows part of the curves $y = x^2$ and $y = \dfrac{3}{3x-2}$.



The curves meet at the point with x-coordinate α.

(a) Find the integer N such that $\dfrac{N}{10} < \alpha < \dfrac{N+1}{10}$. [4]

(b) Use interval bisection on the interval found in part (a) to find the value of α correct to 2 decimal places. [5]

Total: 9

5. Given that

$$\mathrm{f}(z) \equiv z^4 - 4z^3 + kz^2 - 4z + 13,$$

where k is a real constant, and that $z = \mathbf{i}$ is a solution of the equation $\mathrm{f}(z) = 0$,

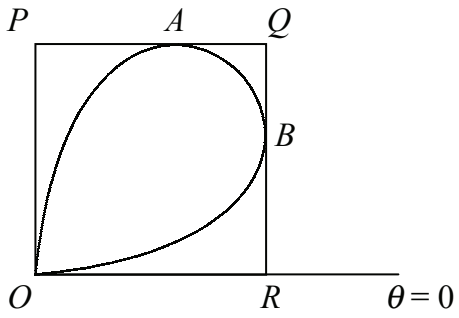
(a) show that $k = 14$, [3]

(b) find all solutions of the equation $\mathrm{f}(z) = 0$. [7]

Total: 10

6. The shape of a company logo is to be the region enclosed by the curve with polar equation

$$r^2 = a^2 \sin(2\theta), \quad 0 \leq \theta \leq \frac{\pi}{2}.$$



A sign in the shape of the logo is to be made by cutting the area enclosed by the curve from a square sheet of metal $OPQR$ where O is the pole and R lies on the initial line, $\theta = 0$, as shown in Figure.

PQ and QR are tangents to the curve, parallel and perpendicular to the initial line respectively, at the points A and B on the curve.

(a) Find the value of θ at the point A. [7]

(b) Show that the area of $OPQR$ is $\frac{3\sqrt{3}}{8}a^2$. [3]

(c) Find the area of the metal sheet which is not used. [5]

Total: 15

7. Given that $x = ke^{-t}$ satisfies the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 8e^{-t},$$

(a) find the value of k. [3]

(b) Hence find the solution of the differential equation for which $x = 1$ and $\frac{dx}{dt} = 3$ at $t = 0$. [8]

The maximum value of x occurs when $t = T$.

(c) Show that the maximum value of x is $\frac{40}{27}$ and find the value of T. [7]

Total: 18