Solomon Practice Paper

Pure Mathematics 4F

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	4	
2	7	
3	7	
4	7	
5	10	
6	10	
7	14	
8	16	
Total:	75	

How I can achieve better:

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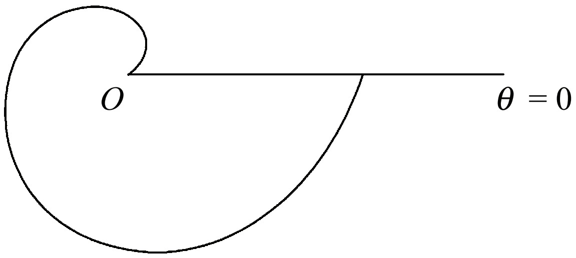
BY NC ND

Last updated: May 5, 2023

1. Figure shows the curve with polar equation [4]

$$r = a\theta, \quad 0 \leq \theta < 2\pi, \quad a > 0.$$



Find the area of the finite region bounded by the curve and the initial line $\theta = 0$.

2. Find the set of values of x for which [7]

$$\frac{(x-1)(x+2)}{x+4} > 4.$$

3.

$$f(x) = 3x^5 - 7x^2 + 3.$$

(a) Show that there is a root, α, of the equation $f(x) = 0$ in the interval $[0, 1]$. [2]

(b) Use linear interpolation once on the interval $[0, 1]$ to estimate the value of α. [2]

There is another root, β, of the equation $f(x) = 0$ close to -0.62.

(c) Use the Newton-Raphson method once to obtain a second approximation to β , giving your answer correct to 3 decimal places. [3]

Total: 7

4. The Cartesian equation of the curve C is

$$(x^2 + y^2)^2 = a^2(x^2 - y^2).$$

(a) Show that, in polar coordinates, the equation of curve C can be written as [4]

$$r^2 = a^2 \cos(2\theta).$$

(b) Sketch the curve C for $0 \leq \theta < 2\pi$. [3]

Total: 7

5. (a) Show that the substitution $y = \frac{1}{u}$ transforms the differential equation [3]

$$\frac{dy}{dx} + \frac{y}{x} - xy^2 = 0 \qquad (\star)$$

into the differential equation

$$\frac{du}{dx} - \frac{u}{x} + x = 0.$$

(b) Hence find the solution of differential equation $\star$ such that $y = 1$ when $x = 1$, giving your answer in the form $y = f(x)$. [7]

Total: 10

6. (a) Find $\displaystyle\sum_{r=n+1}^{2n} r^2$ in terms of n. [4]

   (b) Hence, or otherwise, show that [6]

$$4 \leq \frac{\displaystyle\sum_{r=n+1}^{2n} r^2}{\displaystyle\sum_{r=1}^{n} r^2} < 7$$

   for all positive integer values of n.

Total: 10

7. A particle moves along the x-axis such that at time t its x-coordinate satisfies the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 5\frac{\mathrm{d}x}{\mathrm{d}t} - 3x = 20\sin(t).$$

(a) Find the general solution of this differential equation. [10]

Initially the particle is at $x = 5$.

Given that the particle's x-coordinate remains finite as $t \to \infty$,

(b) find an expression for x in terms of t. [4]

Total: 14

8. The complex numbers z_1 and z_2 are given by

$$z_1 = \frac{1+\mathbf{i}}{1-\mathbf{i}}, \quad \text{and} \quad z_2 = \frac{\sqrt{2}}{1-\mathbf{i}}.$$

(a) Find z_1 in the form $a + \mathbf{i}b$ where a and b are real. [2]

(b) Write down the modulus and argument of z_1. [2]

(c) Find the modulus and argument of z_2. [4]

(d) Show the points representing z_1, z_2 and $z_1 + z_2$ on the same Argand diagram, and hence find the exact value of $\tan\left(\frac{3\pi}{8}\right)$. [8]

Total: 16