

Solomon Practice Paper

Pure Mathematics 4B

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

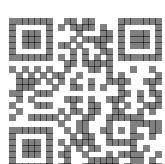
Question	Points	Score
1	6	
2	8	
3	9	
4	9	
5	10	
6	15	
7	18	
Total:	75	

How I can achieve better:

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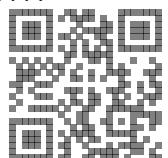
Last updated: May 5, 2023



1. Find the set of values of x for which

[6]

$$|2x^2 - 5x| < x.$$



2. (a) Sketch the curve C with the polar equation

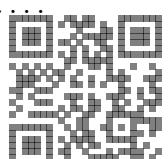
[3]

$$r^2 = a^2 \sin^2(2\theta), \quad 0 \leq \theta < 2\pi.$$

- (b) Find the exact area of the region enclosed by one loop of the curve C .

[5]

Total: 8



3. (a) Show that

[6]

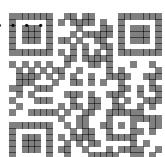
$$\sum_{r=1}^n (r^2 + 1)(r - 1) = \frac{1}{12}n(n - 1)(3n^2 + 5n + 8).$$

(b) Hence evaluate

[3]

$$\sum_{r=5}^{25} (r^2 + 1)(r - 1).$$

Total: 9



4. (a) Find the general solution of the differential equation

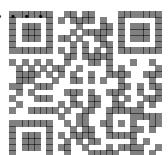
[6]

$$\frac{dy}{dx} - y \cot(x) = \sin(2x).$$

- (b) Given also that $y = 2$ when $x = \frac{\pi}{6}$, find the exact value of y when $x = \frac{2\pi}{3}$.

[3]

Total: 9



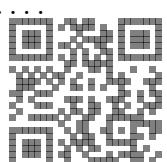
5.

$$f(x) \equiv x^3 - \ln(4 - x^2), \quad x \in \mathbb{R}, \quad -2 < x < 2.$$

- (a) Show that one root, α , of the equation $f(x) = 0$ lies in the interval $1.0 < \alpha < 1.1$. [2]

(b) Starting with $x = 1.0$, show that using the Newton-Raphson method twice gives an approximation to α that is correct to 6 decimal places. [8]

Total: 10



6. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 7 - \mathbf{i}, \quad z_2 = 1 + \mathbf{i}\sqrt{3}, \quad z_3 = a + \mathbf{i}b,$$

where a and b are rational constants.

Given that the modulus of $z_1 z_3$ is 50,

- (a) find the modulus of z_3 .

[3]

Given also that the argument of $\frac{z_2}{z_3}$ is $\frac{7\pi}{12}$,

- (b) find the argument of z_3 .

[3]

- (c) Find the values of a and b .

[2]

- (d) Show that $\frac{z_1}{z_3} = \frac{1}{5}(4 + 3\mathbf{i})$.

[3]

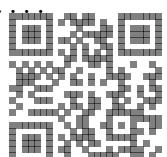
- (e) Represent z_1, z_3 and $\frac{z_1}{z_3}$ on the same Argand diagram.

[2]

- (f) By considering the modulus and argument of z_1 and z_3 , explain why

$$\frac{z_3}{z_1} = \left(\frac{z_1}{z_3} \right)^*$$

Total: 15



7. (a) Given that $x = e^t$, find $\frac{dy}{dx}$ in terms of $\frac{dy}{dt}$ and show that [5]

$$\frac{d^2y}{dx^2} = e^{-2t} \left(\frac{d^2y}{dt^2} - \frac{dy}{dt} \right).$$

- (b) Show that the substitution $x = e^t$ transforms the differential equation [3]

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = 6x^2$$

into the differential equation

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y = 6e^{2t}.$$

- (c) Given that when $x = 1, y = 3$ and $\frac{dy}{dx} = -5$, solve the differential equation [10]

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = 6x^2.$$

Total: 18

