Solomon Practice Paper

Pure Mathematics 2C

Time allowed: 90 **minutes**

Centre: www.CasperYC.club
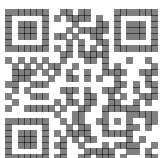
Name:

Teacher:

Question	Points	Score
1	6	
2	6	
3	6	
4	8	
5	10	
6	12	
7	13	
8	14	
Total:	75	

How I can achieve better:

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1. (a) Solve the equation [3]

$$\ln(2x + 1) = 3$$

giving your answer in terms of e.

(b) Given that [3]

$$2^x = 5^y,$$

show that $y = kx$ where k is a constant that you should find correct to an appropriate degree of accuracy.

Total: 6

2. (a) Use the identity [2]
$$\sin(A + B) \equiv \sin(A)\cos(B) + \cos(A)\sin(B)$$

to prove that
$$\sin(2A) \equiv 2\sin(A)\cos(A).$$

(b) Hence, or otherwise, use the fact that [4]

$$\sin(15°) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

to find the value of $\cos(15°)$ in exact form with a rational denominator.

Total: 6

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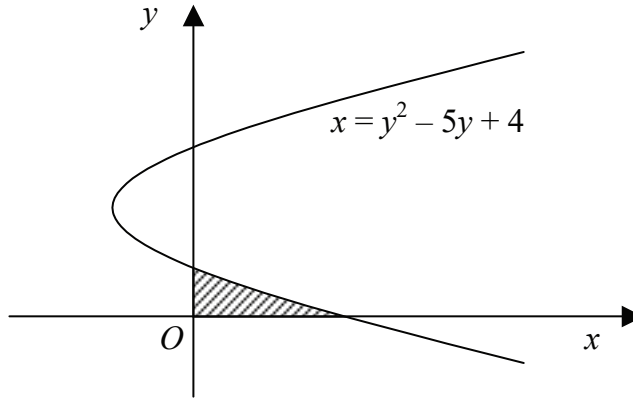...

3. Express   [6]

$$\frac{5x^2 - 11x + 9}{x^2 + 3x - 10} + \frac{3 - 2x}{x - 2}$$

as a single fraction in its simplest form.

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4. The coefficient of x^2 in the expansion of $(1 + 3x)^n$ is 252.

   Given that n is a positive integer,

   (a) find the value of n, [5]

   (b) show that the coefficient of x^3 is 1512. [3]

   Total: 8
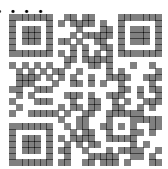
5. Figure shows the curve $x = y^2 - 5y + 4$.

(a) Express x^2 in descending powers of y. [3]

(b) Find $\displaystyle\int x^2 \, \mathrm{d}y$. [3]

(c) Show that the volume generated when the shaded region, bounded by the curve and the positive coordinate axes, is rotated through 2π radians about the y-axis is $\dfrac{47\pi}{10}$. [4]

Total: 10

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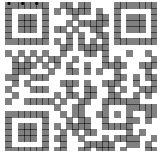
6. The functions f and g are defined by

$$f\colon x \mapsto x^2 - 2, \quad x \in \mathbb{R},$$
$$g\colon x \mapsto e^{\frac{3}{2}x} \qquad x \in \mathbb{R}.$$

(a) State the range of g. [1]

(b) Define fg as simply as possible. [3]

(c) Find, correct to 2 decimal places, the value of x for which $fg(x) = 5$. [3]

(d) Show that the only value of x for which $g(x) = fg(x)$ is $\dfrac{2}{3}\ln(2)$. [5]

Total: 12

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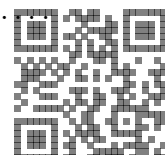...

7. (a) Prove that [6]

$$\cot^2(x) - \tan^2(x) \equiv 4\cot(2x)\csc(2x).$$

(b) Hence, find in terms of π the values of x in the interval $0 \le x \le \pi$ for which [7]

$$\cot^2(x) - \tan^2(x) = 8\cot(2x).$$

Total: 13

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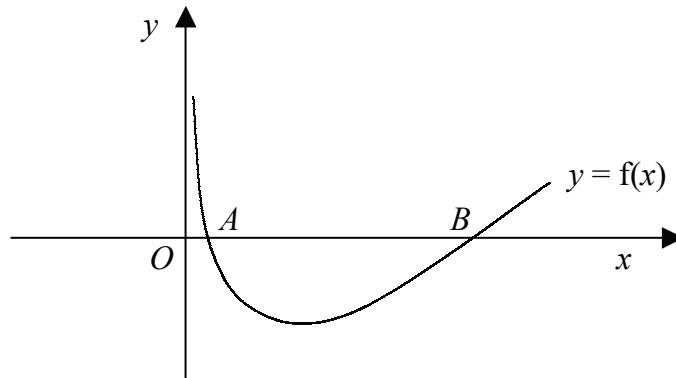………
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8. Figure shows part of the curve with equation $y = f(x)$, where

$$f(x) \equiv x - 3\ln(2x), \quad x \in \mathbb{R}, \quad x > 0.$$

The curve crosses the x-axis at the points A and B.

(a) Show that the x-coordinate of the point A lies in the interval $(0.6, 0.7)$.   [2]

(b) Find the value of N such that the x-coordinate of the point B lies in the interval $(N, N+1)$.   [4]

The line $y = x$ meets the curve at the point C.

(c) Find the coordinates of the point C.   [3]

(d) Show that the equation of the tangent to the curve at C is $y = 3 - 5x$.   [5]

Total: 14

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