Solomon Practice Paper

Pure Mathematics 1I

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	6	
4	7	
5	9	
6	10	
7	16	
8	16	
Total:	75	

How I can achieve better:

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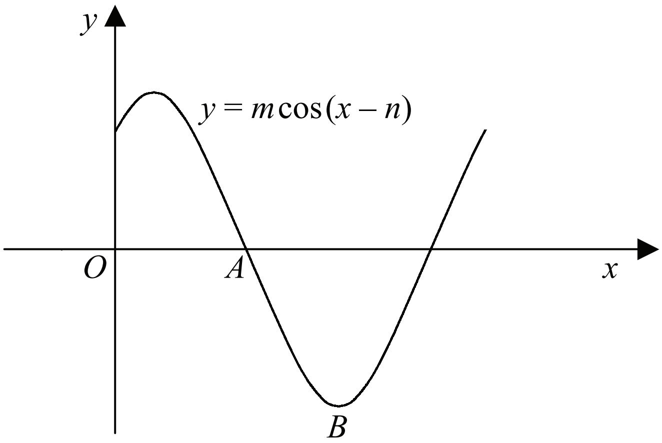
Last updated: May 5, 2023

1. Figure shows part of the curve $y = m\cos(x - n)$, where x is measured in degrees.



The constants m and n are integers and n is such that $0 < n < 90°$.

For $x > 0$, the curve first crosses the x-axis at the point $A(120, 0)$ and the first minimum is at the point $B(210, -4)$.

(a) Find the values of m and n. [3]

The curve above may also be written in the form $y = p\sin(x + q)$, where p and q are integers and $0 < q < 90°$.

(b) Write down the values of p and q. [2]

Total: 5

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2.

$$f(x) \equiv x^3 - 5x^2 + 3x + 2.$$

(a) Find $f'(x)$. [2]

(b) Hence, or otherwise, find the set of values of x for which $f(x)$ is decreasing. [4]

Total: 6

3. Given that $\sin(15°)$ is exactly [6]

$$\frac{\sqrt{3}-1}{2\sqrt{2}}$$

show that $\cos^2(15°)$ can be written as

$$\frac{m+n\sqrt{3}}{4}$$

where m and n are positive integers.

4.

$$f(x) \equiv x^2 - 2x - 6.$$

(a) By expressing $f(x)$ in the form $A(x+B)^2 + C$, prove that $f(x) \geq -7$. [4]

(b) Solve the equation $f(x) = 0$, giving your answers correct to 2 decimal places. [3]

Total: 7

5.

$$y^{\frac{1}{2}} = 2x^{\frac{1}{3}} + 1.$$

(a) Show that y can be written in the form [3]

$$y = Ax^{\frac{2}{3}} + Bx^{\frac{1}{3}} + C$$

where A, B and C are positive integers.

(b) Hence, evaluate [6]

$$\int_1^8 y \, \mathrm{d}x.$$

Total: 9

6. The first two terms of a geometric series are $(x + 2)$ and $(x^3 + 2x^2 - x - 2)$ respectively.

   (a) Find the common ratio of the series as a quadratic expression in terms of x. [3]

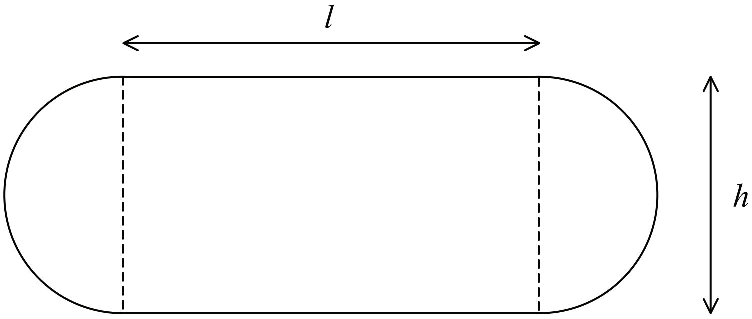
   (b) Express the second term of the series as a product of 3 linear factors. [3]

   Given that $x = \frac{1}{2}$,

   (c) show that the sum to infinity of the series is $\frac{10}{7}$. [4]

Total: 10

7. Figure shows the inside of a running track.



The track consists of two straight sections of length l metres, joined at either end by semicircles of diameter h metres.

(a) Find, in terms of h and l, expressions for [4]

i. the perimeter of the track,

ii. the area of the track.

Given that the track must have a perimeter of 400 metres,

(b) show that the area, A m^2, enclosed by the track is given by [5]

$$A = 200h - \frac{\pi h^2}{4}.$$

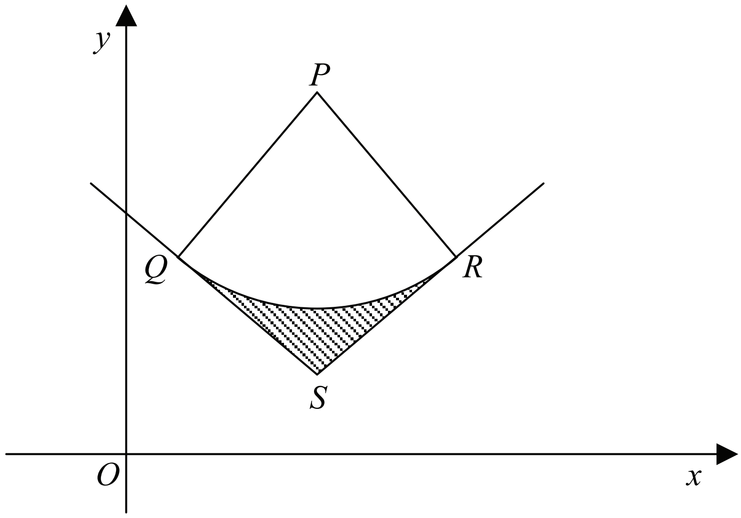
In order to stage the field events, A must be as large as possible. Given that h can vary,

(c) find the maximum value of A, giving your answer in terms of π, [5]

(d) justify that your value of A is a maximum. [2]

Total: 16

8. Figure shows the sector PQR of a circle, centre P.



The tangents to the circle at Q and R meet at the point S.

The shape $PQSR$ has $x = 4$ as a line of symmetry.

Given that P and Q are the points with coordinates $(4, 11)$ and $(1, 5)$ respectively,

(a) find the gradient of the line PQ, [2]

(b) find an equation of the tangent to the circle at Q, [3]

(c) show that the radius of the circle can be written in the form $a\sqrt{5}$ where a is a positive integer which you should find, [2]

(d) show that the angle subtended by the minor arc QR at P is 0.927 radians correct to 3 decimal places, [3]

(e) find the area of the shaded region enclosed by the arc QR and the lines QS and RS. [6]

Total: 16