

Edexcel (U.K.) Pre 2017

Questions By Topic

C4 Chap06 Integration

Compiled By: Dr Yu

Editors: Betül, Signal, Vivian

www.CasperYC.club

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DrYuFromShanghai@QQ.com

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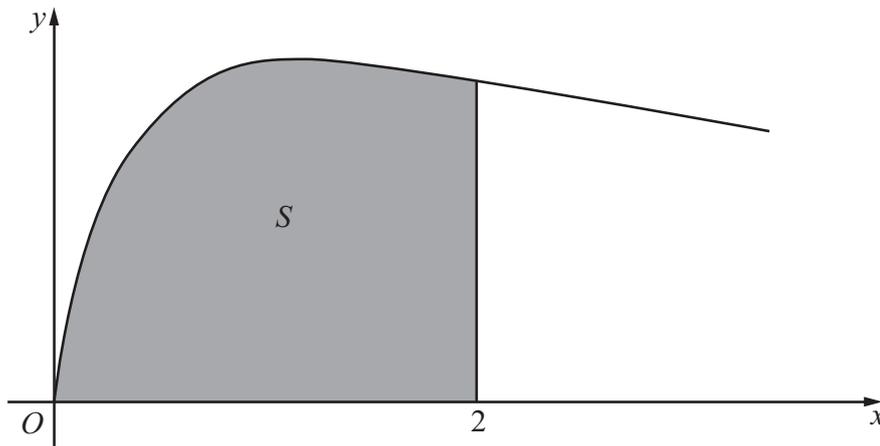


Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0$$

The finite region S , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 2$

The region S is rotated 360° about the x -axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

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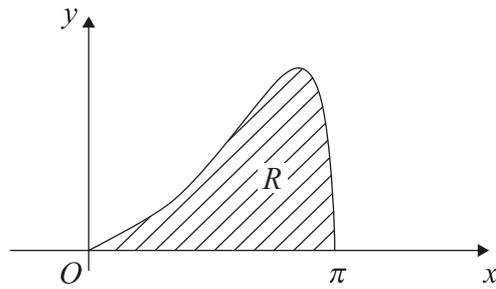


Figure 1

The curve shown in Figure 1 has equation $y = e^x \sqrt{\sin x}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x -axis is shown shaded in Figure 1.

- (a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

- (b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R . Give your answer to 4 decimal places.

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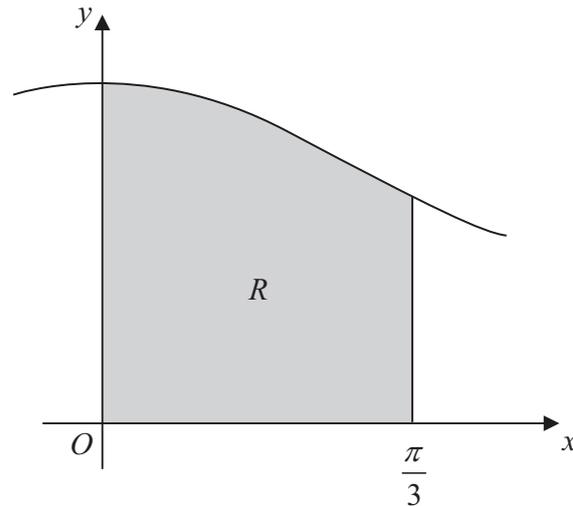


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{0.75 + \cos^2 x}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the y -axis, the x -axis and the line with equation $x = \frac{\pi}{3}$.

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R .

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a

further estimate of the area of R . Give your answer to 3 decimal places.

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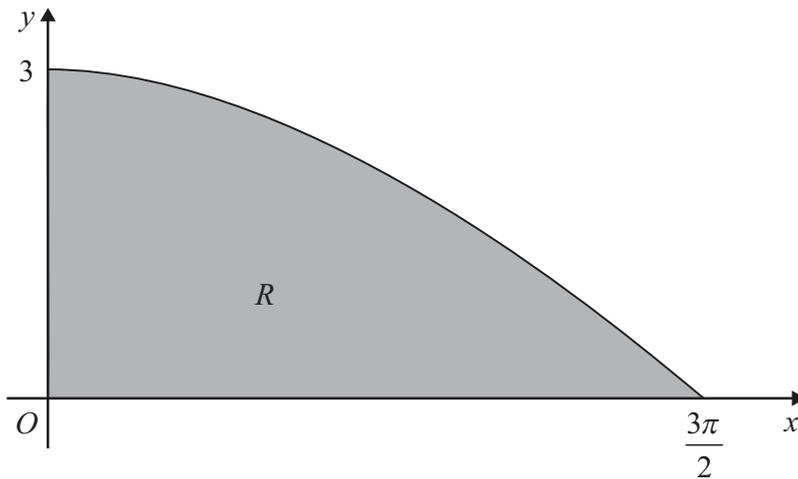
**Figure 1**

Figure 1 shows the finite region R bounded by the x -axis, the y -axis and the curve with

equation $y = 3 \cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places. (4)
- (c) Use integration to find the exact area of R . (3)

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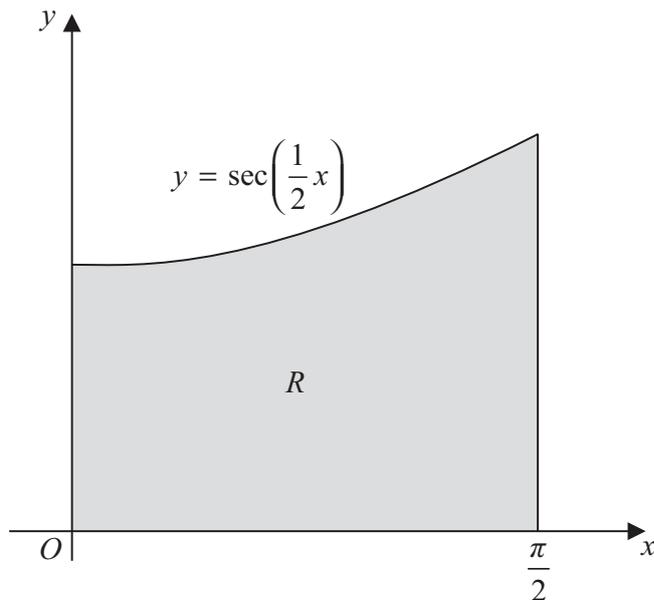


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

- (a) Complete the table above giving the missing value of y to 6 decimal places. (1)
- (b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R , giving your answer to 4 decimal places. (3)

Region R is rotated through 2π radians about the x -axis.

- (c) Use calculus to find the exact volume of the solid formed. (4)

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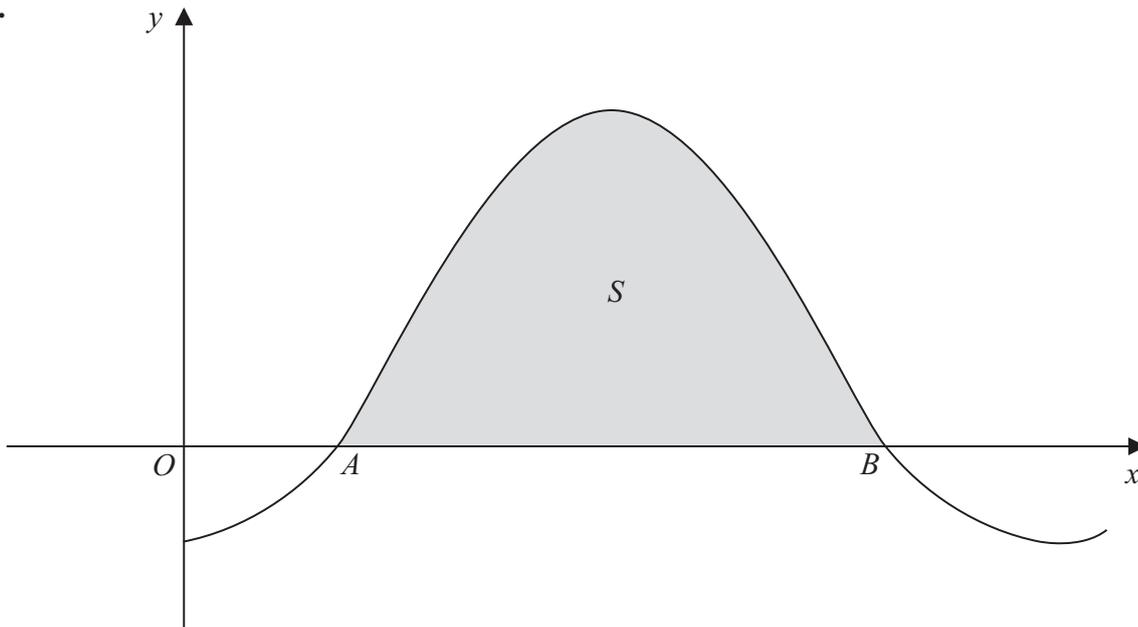
**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2 \cos x$, where x is measured in radians. The curve crosses the x -axis at the point A and at the point B .

- (a) Find, in terms of π , the x coordinate of the point A and the x coordinate of the point B . (3)

The finite region S enclosed by the curve and the x -axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x -axis.

- (b) Find, by integration, the exact value of the volume of the solid generated. (6)

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Figure 1

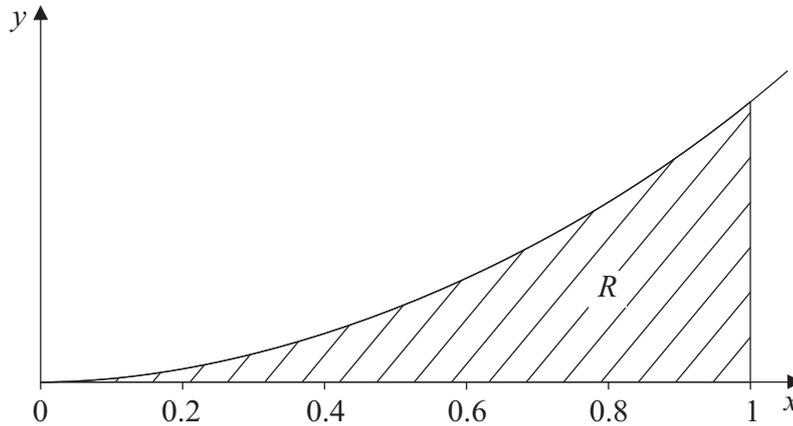


Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \quad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x -axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R .

(5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8 .

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

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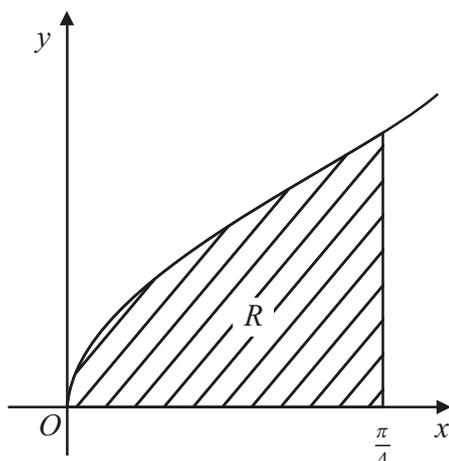


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R , which is bounded by the curve, the x -axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

- (a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

- (b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R , giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x -axis to generate a solid of revolution.

- (c) Use integration to find an exact value for the volume of the solid generated.

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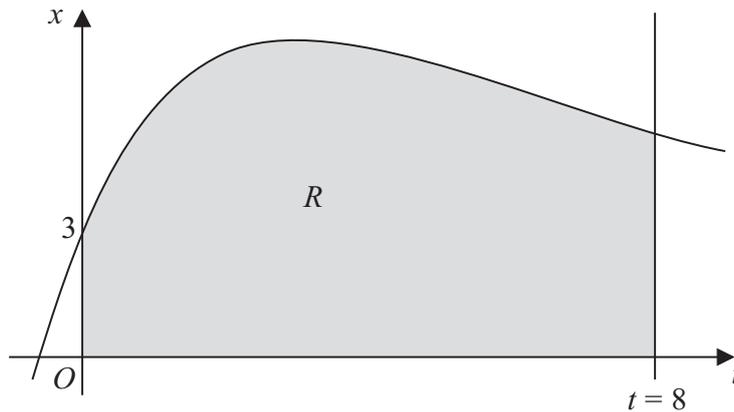


Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x -axis, the t -axis and the line $t = 8$.

- (a) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

- (b) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R , giving your answer to 2 decimal places.

(3)

- (c) Use calculus to find the exact value for the area of R .

(6)

- (d) Find the difference between the values obtained in part (b) and part (c), giving your answer to 2 decimal places.

(1)

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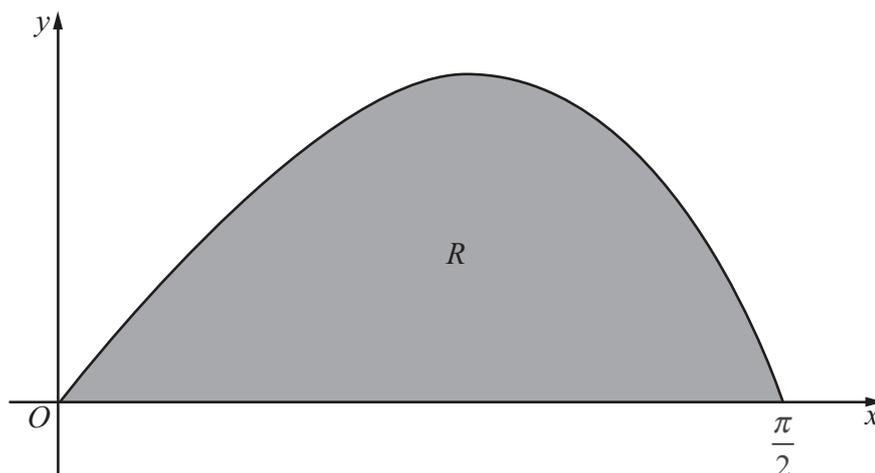


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2 \sin 2x}{1 + \cos x}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

The table below shows corresponding values of x and y for $y = \frac{2 \sin 2x}{1 + \cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 4 decimal places. (3)
- (c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2 \sin 2x}{1 + \cos x} dx = 4 \ln(1 + \cos x) - 4 \cos x + k$$

where k is a constant. (5)

- (d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. (3)

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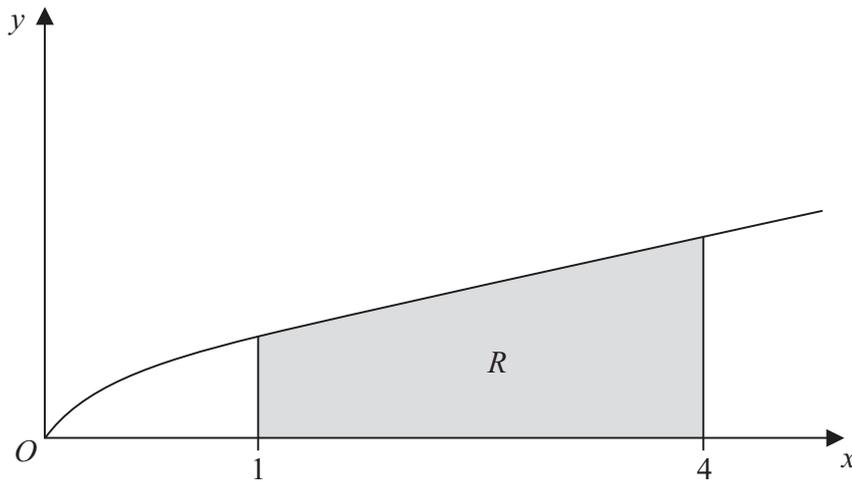


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, the line with equation $x = 1$ and the line with equation $x = 4$.

- (a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R , giving your answer to 3 decimal places.

(3)

- (c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R .

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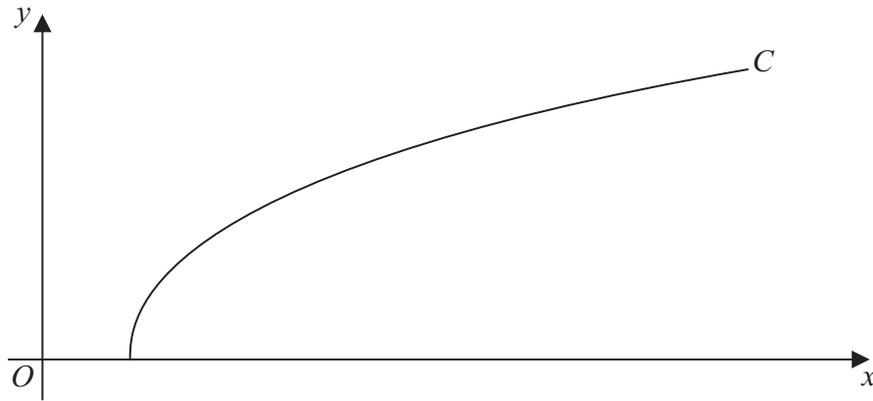
**Figure 2**

Figure 2 shows a sketch of the curve C with parametric equations

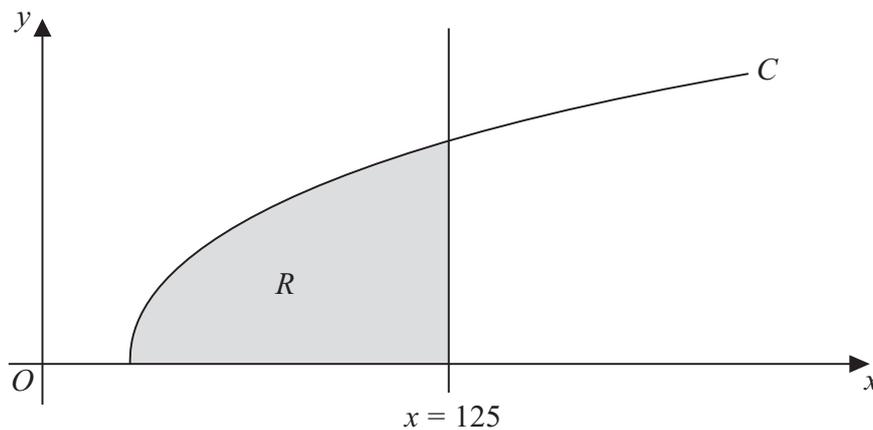
$$x = 27 \sec^3 t, \quad y = 3 \tan t, \quad 0 \leq t \leq \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \frac{\pi}{6}$ (4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \quad a \leq x \leq b$$

stating the values of a and b .

(3)**Figure 3**

The finite region R which is bounded by the curve C , the x -axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution. (5)

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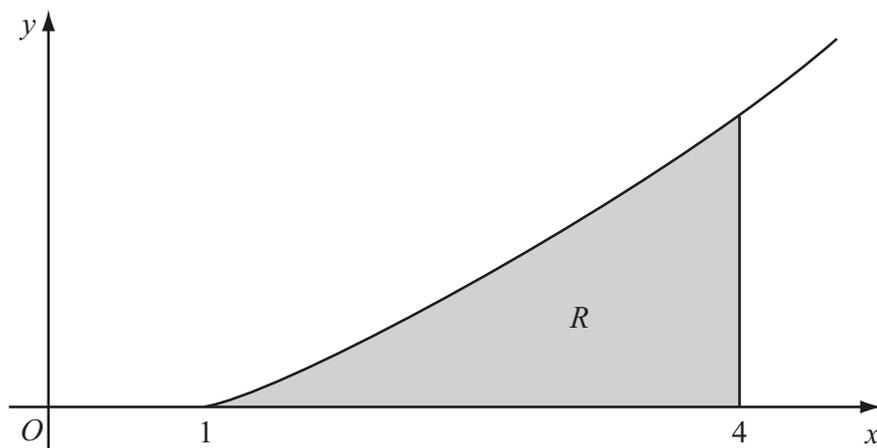


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

- (a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (4)
- (c) (i) Use integration by parts to find $\int x \ln x \, dx$.
- (ii) Hence find the exact area of R , giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers. (7)

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Figure 3

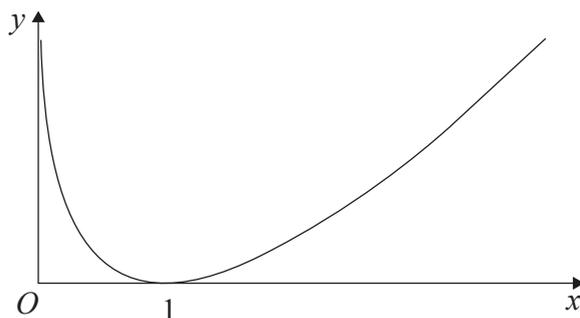


Figure 3 shows a sketch of the curve with equation $y = (x-1) \ln x$, $x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		$\ln 2$		$2 \ln 3$

(1)

Given that $I = \int_1^3 (x-1) \ln x \, dx$,

(b) use the trapezium rule

(i) with values of y at $x = 1, 2$ and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1, 1.5, 2, 2.5$ and 3 to find another approximate value for I to 4 significant figures.

(5)

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(d) Show, by integration, that the exact value of $\int_1^3 (x-1) \ln x \, dx$ is $\frac{3}{2} \ln 3$.

(6)

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6. The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 2, \quad t > 0$$

Find

- (a) an equation of the normal to C at the point where $t = 3$, (6)
- (b) a cartesian equation of C . (3)

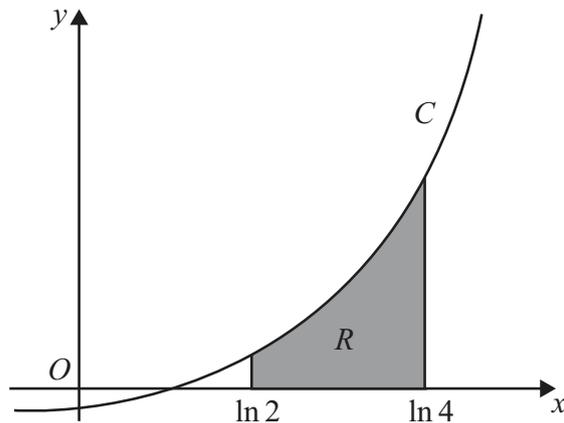


Figure 1

The finite area R , shown in Figure 1, is bounded by C , the x -axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x -axis.

- (c) Use calculus to find the exact volume of the solid generated. (6)

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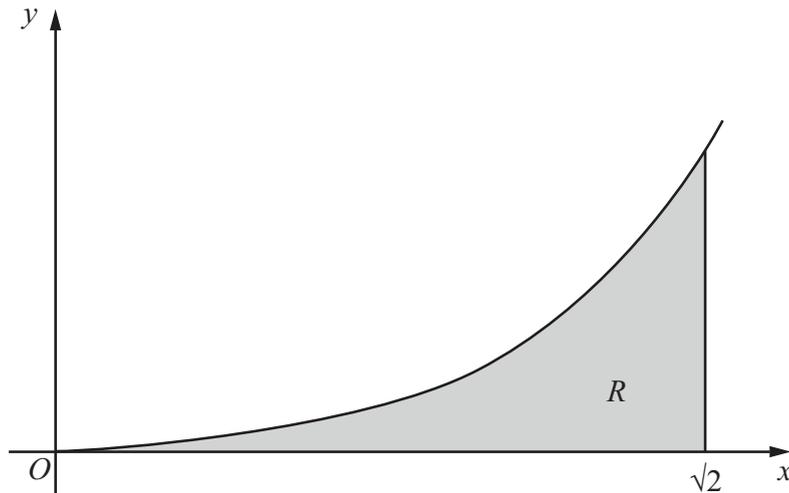


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$. The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

- (a) Complete the table above giving the missing values of y to 4 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)
- (c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \quad (4)$$

- (d) Hence, or otherwise, find the exact area of R . (6)

