Edexcel (U.K.) Pre 2017

Questions By Topic

C4 Chap05 Vectors

Compiled By: Dr Yu

Editors: Betül, Signal, Vivian

Leave blank

8. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(a) find the value of p.

(4)

Given also that B is a point on l such that $\angle BPA = 45°$,

(b) find the coordinates of the two possible positions of B.

(5)

Leave blank

5.

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(a) Show that l_1 and l_2 do not meet.

(4)

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(b) Find the cosine of the acute angle between AB and l_1.

(6)

Leave blank

4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(a) Find $\overrightarrow{AB}$.

(2)

(b) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant. Given that AC is perpendicular to l, find

(c) the value of p,

(4)

(d) the distance AC.

(2)

Leave
blank

6. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1.

(a) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(b) find the coordinates of P.

(5)

(c) Hence find the distance OP, giving your answer as a simplified surd.

(2)

Leave blank

8. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

Leave blank

6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between l_1 and l_2.

(3)

(d) Find the position vector of the point C.

(4)

Leave blank

4. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(a) Find the coordinates of A.

(2)

(b) Find the value of the constant p.

(3)

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point B lies on l_2 where $\mu = 1$

(d) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures.

(3)

Leave blank

4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(a) Write down the coordinates of A.

(1)

(b) Find the value of $\cos\theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X.

(1)

(d) Find the vector $\overrightarrow{AX}$.

(2)

(e) Hence, or otherwise, show that $|\overrightarrow{AX}| = 4\sqrt{26}$.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(f) find the length of AY, giving your answer to 3 significant figures.

(3)

非淡泊无以明志，非宁静无以致远。   郁博士的数学分享小站

Leave blank

7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(c) Hence, or otherwise, find the area of the triangle ABC.

(5)

书山有路勤为径，学海无涯苦作舟。

Leave blank

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

(b) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1.

(1)

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B.

(3)

Leave
blank

6. Relative to a fixed origin O, the point A has position vector $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and the point B has position vector $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$.

The line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where a, b and c are constants and λ is a parameter.

Given that the point A lies on the line l,

(a) find the value of a.

(3)

Given also that the vector $\overrightarrow{AB}$ is perpendicular to l,

(b) find the values of b and c,

(5)

(c) find the distance AB.

(2)

The image of the point B after reflection in the line l is the point B'.

(d) Find the position vector of the point B'.

(2)

书山有路勤为径，学海无涯苦作舟。

Leave blank

5. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(c) show that the points A, P and B are collinear and find the ratio $AP:PB$.

(4)

Leave
blank

4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

　　　　　　　　　　　书山有路勤为径，学海无涯苦作舟。

7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(a) Find the coordinates of B.

(4)

(b) Find the value of $\cos \theta$, giving your answer as a simplified fraction.

(4)

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
The point D is such that $ABCD$ is a parallelogram.

(c) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(3)

(d) Find the position vector of the point D.

(2)

Leave
blank

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2.

(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1.

(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.

(4)

Leave
blank

7. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$,
the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$,
and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(a) Find a vector equation for the line l.

(3)

(b) Find $\left|\overrightarrow{CB}\right|$.

(2)

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

(3)

(d) Find the shortest distance from the point C to the line l.

(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures.

(3)

Leave blank

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection.

(5)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place.

(3)

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1,

(c) find the exact coordinates of P.

(6)

Leave
blank

7. The point A has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point B has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin O.

(a) Find the position vector of the point C, with position vector $\mathbf{c}$, given by

$$\mathbf{c} = \mathbf{a} + \mathbf{b}.$$

(1)

(b) Show that $OACB$ is a rectangle, and find its exact area.

(6)

The diagonals of the rectangle, AB and OC, meet at the point D.

(c) Write down the position vector of the point D.

(1)

(d) Find the size of the angle ADC.

(6)

Leave blank

7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,
the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,
and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle BAD is $109°$, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.

(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)

Leave
blank

8. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Hence find a vector equation for the line l_1

(1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle PBA is θ,

(c) show that $\cos\theta = \dfrac{1}{3}$

(3)

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2

(2)

The points C and D both lie on the line l_2

Given that $AB = PC = DP$ and the x coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D.

(3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd.

(4)