Edexcel (U.K.) Pre 2017

Questions By Topic

C3 Chap03 Exponentials and Logarithms

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3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20\,\mathrm{e}^{1.5t}, \quad t \geqslant 0$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

Leave blank

1. Find the exact solutions to the equations

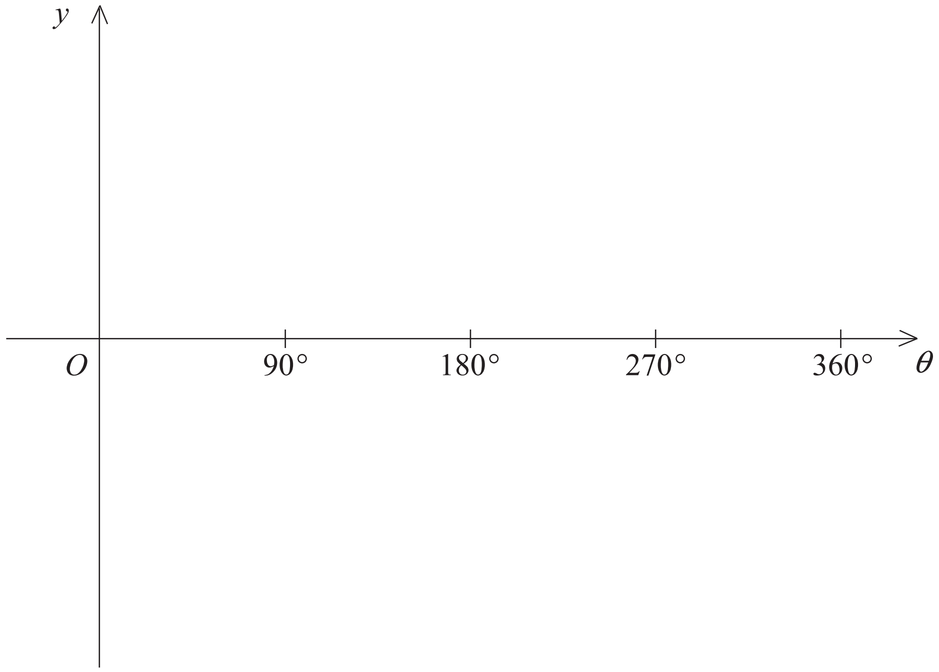
(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^x + 3e^{-x} = 4$.

(4)

Question 7 continued


y
O
90°
180°
270°
360°
θ

4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$.

(3)

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

5. The radioactive decay of a substance is given by

$$R = 1000\mathrm{e}^{-ct}, \qquad t \geqslant 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when $t = 22\ 920$.

(2)

(d) In the space provided on page 13, sketch the graph of R against t .

(2)

8. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$

(1)

(b) Calculate the exact value of t when $V = 9500$

(4)

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)

4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\,e^{-0.05t} + 25, \quad t \geqslant 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)

7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800a\,\mathrm{e}^{0.2t}}{1 + a\,\mathrm{e}^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(a) show that $a = 0.12$,

(3)

(b) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(c) Show that $p = \dfrac{336}{0.12 + \mathrm{e}^{-0.2t}}$.

(1)

(d) Hence show that the population cannot exceed 2800.

(2)

6. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \qquad -1 < x < 2$

(5)

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\dfrac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

Leave blank

5. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\,e^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$.

(4)

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$.

(6)

8.

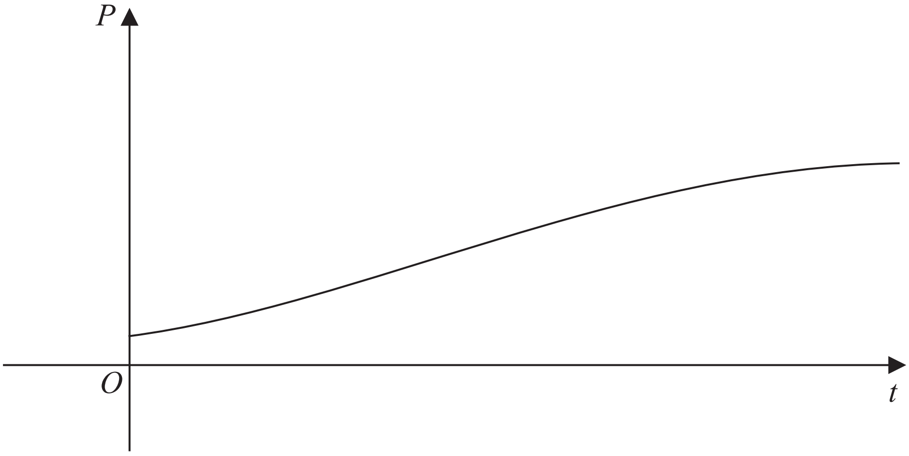


Figure 3

The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1+7e^{-kt}}, \qquad t \geqslant 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study,

(2)

(b) find a value for the expected upper limit of the population.

(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places.

(5)

Using this value for k,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.

(2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study.

(3)

Leave blank

4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y=\sqrt{(4x+1)}, \quad x>-\tfrac{1}{4}, \ y>0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax+by+c=0$, where a, b and c are integers.

(6)

Leave blank

5.

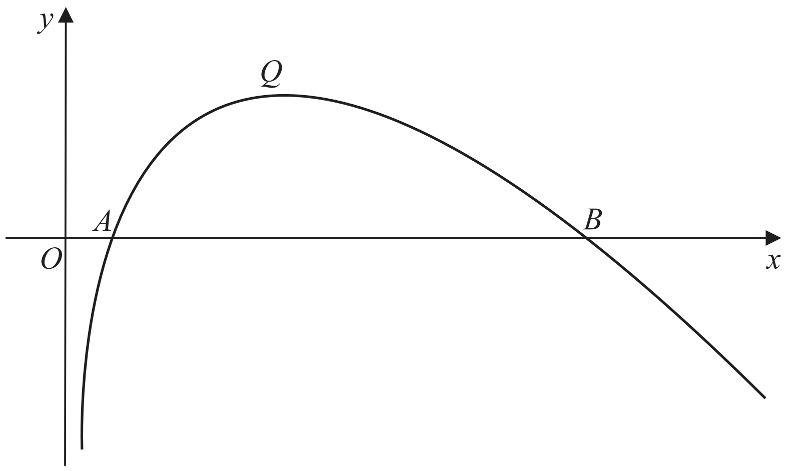


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \quad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(d) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

Leave blank

9. (i) Find the exact solutions to the equations

(a) $\ln(3x-7)=5$

(3)

(b) $3^x e^{7x+2}=15$

(5)

(ii) The functions f and g are defined by

$$\begin{aligned} f(x) &= e^{2x}+3, & x &\in \mathbb{R} \\ g(x) &= \ln(x-1), & x &\in \mathbb{R},\ x>1 \end{aligned}$$

(a) Find f^{-1} and state its domain.

(4)

(b) Find fg and state its range.

(3)