Question Number	Scheme	Marks
1 (a)	You would assign an average rank between the tied ranks	B1
		(1)
(b)	Rank for total tournaments 1 3 4 6 8 9 2 5 10 7	M1
	$\sum d^2 = 0+1+1+4+9+9+25+9+1+9\ [=68]$	M1
	$r_s = 1 - \frac{6\times '68'}{10(10^2-1)}$	dM1
	$= 0.5878\ldots$ awrt 0.588	A1
		(4)
(c)	$H_0: \rho = 0$, $H_1: \rho > 0$	B1
	Critical Value = 0.5636 or CR … 0.5636	B1
	Reject H_0 or significant or lies in the critical region	dM1
	There is sufficient evidence of a positive correlation between rank and total tournaments won	A1
		(4)
(d)	2.5% and $r_s = 0.6485$ or CR … 0.6485	B1
		(1)
	Notes	**Total 10**

(a)	**B1**	for an appropriate explanation of how to deal with tied ranks. Ignore any comments regarding PMCC Do not allow add 0.5 to both ranks
(b)	**M1**	attempt to rank total tournaments (at least four correct) Condone reversed ranks
	M1	finding the difference between players rank and each of their total tournaments ranks and evaluating $\sum d^2$ May be implied by 68
	dM1	dependent on 1st M1. Using $1 - \frac{6\sum d^2}{10(99)}$ with their $\sum d^2$ (you will need to check their $\sum d^2$ if no value shown)
	A1	awrt 0.588 Allow $\frac{97}{165}$
(c)	**B1**	both hypotheses correct. Must be in terms of ρ. Must be attached to H_0 and H_1 If r_s is negative in part (b) then allow $H_1: \rho < 0$
	B1	critical value of 0.5636 If r_s is negative in part (b) then allow –0.5636
	dM1	dependent on 2nd B1. A correct statement ft their part (b) and their CV– no context needed but do not allow contradicting non contextual comments. This may be implied by a correct contextual conclusion.
	A1	correct conclusion which is rejecting H_0, which must mention **rank** and **total tournaments**. No hypotheses is A0.
		NB If they have used $H_1: \rho < 0$ then the maximum they can score is B1B1dM1A0
(d)	**B1**	for 2.5% and a correct critical value of 0.6485

Question Number	Scheme		Marks
2 (a)	$\bar{x} = \left[\frac{7690}{100}\right] = 76.9$		B1
	$s_x^2 = \frac{669.24}{99} = 6.76$		M1 A1
			(3)
(b)	$H_0 : \mu_x = \mu_y \qquad H_1 : \mu_x \neq \mu_y$		B1
	$Z = \frac{"76.9" - 75.9}{\sqrt{\frac{"6.76"}{100} + \frac{2.2^2}{80}}} = 2.793...$	awrt ± 2.79	M1 M1 A1
	2 tailed critical value $z = \pm 2.5758$		B1
	Reject H_0/Significant/In the critical region		M1
	There is sufficient evidence to suggest that the mean water temperature after 4 hours for brand *A* is different to brand B		A1ft
			(7)
(c)	(It is reasonable) since both samples are (reasonably) large		B1
			(1)
	Notes		**Total 11**
(a)	**B1**	for 76.9	
	M1	for use of $\frac{1}{n-1}\sum(x-\bar{x})^2$ oe	
	A1	for 6.76	
(b)	**B1**	for both hypotheses correct. Must be attached to H_0 and H_1 Allow equivalent hypotheses. Must be in terms of μ Allow any letter for the subscripts	
	M1	for a correct method to find the standard error. Follow through their values from (a)	
	M1	an attempt at $\pm \frac{a-b}{\sqrt{\frac{c}{100} + \frac{d^2}{80}}}$ with at least 3 of *a*, *b*, *c* or *d* correct.	
	A1	awrt ± 2.79	
	B1	$z =$ awrt ± 2.5758 seen (Allow $z =$ awrt ± 2.3263 if a one tailed test is used)	
	M1	a correct statement consistent with their CV and *Z* value – need not be contextual but do not allow contradicting non contextual comments. This may be implied by a correct contextual conclusion.	
	A1ft	This mark is dependent on the 2nd M mark being awarded. A correct contextual statement ft their CV and their *Z* value	
(c)	**B1**	a correct explanation, which makes reference to both samples. e.g. Do not allow the sample is large enough	

Question Number	Scheme	Marks
3 (a)	$\left[\dfrac{26.624+28.976}{2}\right]=27.8$	B1
		(1)
(b)	$28.976-26.624=2\times1.96\times\dfrac{\sigma}{\sqrt{25}}$ or $26.624='27.8'-1.96\times\dfrac{\sigma}{\sqrt{25}}$ or $28.976='27.8'+1.96\times\dfrac{\sigma}{\sqrt{25}}$	M1 B1
	$\sigma=3$ *	A1* cso
		(3)
(c)	$2\times z\times\dfrac{3}{\sqrt{25}}=2.1$ So $z=1.75$	M1 A1
	$\text{P}(Z>'1.75')=\text{P}(Z<-'1.75')=1-'0.9599'='0.0401'$	M1 A1ft
	Confidence level $=100\times(1-2\times'0.0401')=91.98\%$	M1 A1
		(6)
(d)	$2\times1.96\times\dfrac{3}{\sqrt{n}}<1.5$	M1
	$\sqrt{n}>\dfrac{6\times1.96}{1.5}$	dM1
	$\sqrt{n}>$ awrt 7.84 So $n=62$	A1 A1
		(4)
	Notes	**Total 14**

(a)	**B1**	for 27.8
(b)	**M1**	for $28.976-26.624=2\times z$ value $\times\dfrac{\sigma}{\sqrt{25}}$ or $26.624='27.8'-z$ value $\times\dfrac{\sigma}{\sqrt{25}}$ or $28.976='27.8'-z$ value $\times\dfrac{\sigma}{\sqrt{25}}$ where $1.5<z<2.4$
	B1	awrt 1.96
	A1* cso	answer is given so no incorrect working must be seen
(c)	**M1**	for $2\times z\times\dfrac{3}{\sqrt{25}}=2.1$
	A1	for $z=1.75$
	M1	for $1-p$, where p is a probability
	A1ft	for 0.0401 or ft their z value (Allow 0.04)
	M1	for $100\times(1-2\times0.0401)$ ft their $\text{P}(Z<-1.75)$
	A1	awrt 92.0 (allow 92)
(d)	**M1**	for $2\times z$ value $\times\dfrac{3}{\sqrt{n}}<1.5$ oe z value must either be correct or consistent with part (b) Allow $\leqslant$ or = Condone $>$ or $\geqslant$
	dM1	Dependent on previous M mark. Correct rearrangement to get $\sqrt{n}>\ldots$ or $n>\ldots$ oe Allow $\geqslant$ or = Condone $<$ or $\leqslant$
	A1	awrt 7.84 may be implied by awrt 61.5
	A1	for $n=62$

Question Number	Scheme	Marks
4 (a)	[Continuous] uniform on the interval [0, 7]	B1
		(1)
(b)	mean = 3.5	B1
	standard deviation = $\sqrt{\frac{(7-0)^2}{12}}$	M1
	$= \frac{7}{\sqrt{12}} = 2.0207...$ awrt 2.02	A1
		(3)
(c)	By the CLT $\bar{T} \sim \text{N}\left(3.5, \frac{49}{552}\right)$	M1
	$\text{P}(3.4 < \bar{T} < 3.6) = \text{P}\left(\frac{3.4 - "3.5"}{"\sqrt{\frac{49}{552}}"} < Z < \frac{3.6 - "3.5"}{"\sqrt{\frac{49}{552}}"}\right) = [\text{P}(-0.34 < Z < 0.34)]$	M1 A1
	= 0.6331 – (1 – 0.6331) (Calculator gives 0.6314…)	M1
	= 0.2662 (Calculator gives 0.2628…) awrt 0.263 to 0.266	A1
		(5)
(d)	Large/ independent/ random sample allows use of CLT	B1
		(1)
	Notes	**Total 10**

(a)	**B1**	For the correct distribution stated (need uniform and correct interval) Allow U[0, 7] A fully correct pdf implies B1 e.g. $\text{f}(x) = \begin{cases} \frac{1}{7} & 0 \leqslant x \leqslant 7 \\ 0 & \text{otherwise} \end{cases}$
(b)	**B1**	For 3.5
	M1	For a correct method for finding the standard deviation
	A1	awrt 2.02 (Allow $\frac{7}{\sqrt{12}}$ or $\frac{7\sqrt{3}}{6}$ oe)
(c)	**M1**	For writing or using $\text{N}\left(3.5, \frac{49}{552}\right)$ oe Allow $\text{N}\left(3.5, \frac{2.02^2}{46}\right)$ or ft from part (b) e.g. if Po(7) given in part (a) allow $\text{N}\left(7, \frac{7}{46}\right)$
	M1	For standardising using either 3.4 or 3.6 and their mean and standard deviation
	A1	For a fully correct expression for either 3.4 or 3.6. May be implied by ± awrt 0.34
	M1	For $p - (1-p)$ or $2(p - 0.5)$ oe
	A1	awrt 0.263 to 0.266
(d)	**B1**	Any suitable assumption

Question Number	Scheme	Marks
5 (a)	It is not a statistic as it involves unknown [population parameters]	B1
		(1)
(b)	An estimator for μ is unbiased if its expected value is equal to μ	B1
		(1)
(c)	$\mathrm{E}(U_1) = 3\mathrm{E}(X_1) - 2\mathrm{E}(X_2)$ or $\mathrm{E}(U_2) = \frac{1}{4}(\mathrm{E}(X_1) + 3\mathrm{E}(X_2))$	M1
	$\mathrm{E}(U_1) = 3\mu - 2\mu = \mu$ (therefore unbiased)	A1cso
	$\mathrm{E}(U_2) = \frac{1}{4}(\mu + 3\mu) = \mu$ (therefore unbiased)	A1cso
		(3)
(d)	$\mathrm{Var}(U_1) = 9\mathrm{Var}(X_1) + 4\mathrm{Var}(X_2)$ or $\mathrm{Var}(U_2) = \frac{1}{16}\mathrm{Var}(X_1) + \frac{9}{16}\mathrm{Var}(X_2)$	M1
	$[\mathrm{Var}(U_1) =] 13\sigma^2$	A1
	$[\mathrm{Var}(U_2) =] \frac{5}{8}\sigma^2$	A1
	As $\mathrm{Var}(U_1) > \mathrm{Var}(U_2)$ U_2 is the most efficient estimator for μ	A1
		(4)
	Notes	**Total 9**

(a)	**B1**	for a correct explanation, must include unknown
(b)	**B1**	for a correct explanation that refers to expected *X*. Allow $\mu - \mathrm{E}(X) = 0$, but bias = 0 is B0
(c)	**M1**	for use of $a\mathrm{E}(X_1) + b\mathrm{E}(X_2)$ May be implied by $3\mu - 2\mu$ or $\frac{1}{4}(\mu + 3\mu)$
	A1cso	for a correct solution for $\mathrm{E}(U_1)$ with no incorrect working Condone missing notation. Condone missing subscripts
	A1cso	for a correct solution for $\mathrm{E}(U_2)$ with no incorrect working seen Condone missing notation. Condone missing subscripts
(d)	**M1**	for use of $a^2\mathrm{Var}(X_1) + b^2\mathrm{Var}(X_2)$
	A1	Allow $9\sigma^2 + 4\sigma^2$
	A1	Allow $\frac{1}{16}\sigma^2 + \frac{9}{16}\sigma^2$ or $\frac{5}{8}\sigma^2$ oe
	A1	for U_2 with a correct reason
		NB It is possible to score M1 A0 A0 A1 if $\mathrm{Var}(U_1)$ and $\mathrm{Var}(U_2)$ are correct

Question Number	Scheme	Marks
6 (a)	$M \sim \mathrm{N}(80,100)$ $\quad W \sim \mathrm{N}(69,25)$ $X = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + W_1 + W_2 + W_3$	
	$X \sim \mathrm{N}(687, 675)$	M1 A1
	$\mathrm{P}(X > 700) = \mathrm{P}\left(Z > \frac{700-687}{\sqrt{675}}\right) = \mathrm{P}(Z > 0.500...)$	M1
	$(=1-0.6915) = 0.3085$ (Calculator gives 0.3084)	A1
		(4)
(b)	Let Y = Number of men in the lift	
	$Y \sim \mathrm{N}(80x, 100x)$	M1
	$\mathrm{P}(Y > 700) = \mathrm{P}\left(Z > \frac{700-80x}{10\sqrt{x}}\right) < 0.025$	M1
	$\frac{700-80x}{10\sqrt{x}} > 1.96$	B1
	$80x + 19.6\sqrt{x} - 700[<0]$ | $6400x^2 - 112384.16x + 490000[>0]$	M1
	Solving leading to $\sqrt{x} < 2.838...$ | Solving leading to $x < 8.05...$	M1
	So $c = 8$ (people)	A1
		(6)
	Notes	**Total 10**

(a)	**B1**	for setting up normal distribution with mean 687
	B1	for a correct variance (675) or for standard deviation ($15\sqrt{3}$)
	M1	for standardising with 700, 687 and their standard deviation
	A1	for answer between 0.308 – 0.309
(b)	**M1**	for setting up normal distribution with mean $80x$ and variance $100x$ (may be implied by use of sd = $10\sqrt{x}$) Allow any letter
	M1	for standardising with 700, their mean and their standard deviation (if not stated then these must be correct)
	B1	for an equation or inequality set = to 1.96 (Allow – 1.96)
	M1	for a correct 3TQ ft their mean and standard deviation
	M1	for an attempt to solve their 3TQ with either $\sqrt{x} < ...$ or $x < ...$ Allow = instead of < Condone > or $\geqslant$ If the answer is incorrect then we must see use of the quadratic formula/completing the square (Allow one error)
	A1	cao

Question Number	Scheme	Marks
7 (a)	H_0 : The observed distribution can be modelled by a discrete uniform distribution H_1 : The observed distribution cannot be modelled by a discrete uniform distribution	B1 (1)
(bi)	(see table below)	B1 M1

Observed	Expected	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
$x+6$	x	$\frac{36}{x}$	$\frac{(x+6)^2}{x}$
$x-8$	x	$\frac{64}{x}$	$\frac{(x-8)^2}{x}$
$x+8$	x	$\frac{64}{x}$	$\frac{(x+8)^2}{x}$
$x-5$	x	$\frac{25}{x}$	$\frac{(x-5)^2}{x}$
$x+4$	x	$\frac{16}{x}$	$\frac{(x+4)^2}{x}$
$x-5$	x	$\frac{25}{x}$	$\frac{(x-5)^2}{x}$
Total = $6x$	Total = $6x$	Total = $\frac{230}{x}$	Total = $\frac{6x^2+230}{x}$

Question Number	Scheme	Marks
	$X^2 = \sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E} - 6x$; $\frac{230}{x}$ or $\frac{6x^2+230}{x} - 6x$	M1 ; A1
	$v = 6-1=5$; $\chi^2_5(0.05) = 11.070 \Rightarrow$ CR: $X^2 \geqslant 11.070$	B1 ; B1
	Do not reject H_0 if $\frac{230}{x} \leqslant$ '11.070' or $\frac{6x^2+230}{x} - 6x \leqslant$ '11.070'	M1
	$x \geqslant 20.7768...$ So $x = 21$	A1 (8)
(bii)	Hence the die was rolled "21" × 6 = 126 times	M1 A1 (2)

	Notes		**Total 11**
(a)	**B1**	for both hypotheses correct Allow H_0 : the die is not biased H_1 : the die is biased	
(bi)	**B1**	for expected frequency = x	
	M1	for one correct $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ ft their expected frequency	
	M1	for an attempt at X^2 ft their values (At least 4 of these need to be seen and added)	
	A1	for either $\frac{230}{x}$ or $\frac{6x^2+230}{x} - 6x$	
	B1	for $v = 6-1=5$ May be implied by a correct critical value	
	B1	for a correct critical value ft their DOF (**NB** common error is $v = 4$ so $\chi^2_4(0.05) = 9.488$)	
	M1	for either $\frac{230}{x} \leqslant$ their CV or $\frac{6x^2+230}{x} - 6x \leqslant$ their CV Allow $<$ rather than $\leqslant$	
	A1	for $x = 21$ provided the previous M mark has been awarded	
(bii)	**M1**	for their 21 × 6 Allow 6 × x or the answer to 6 × their value for x	
	A1	cao	