S1_2021_06_MS

Question Number	Scheme	Marks	
1. (a)	First Counter Second Counter A Red H 3 Blue 3 Vellow 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 13 14 15 16 17 18 19 19 10 11 12 12 12 12 13 14 15 16 17 18 19	B1 B1	
(b)	$P(Y) = \frac{7}{12} \times \frac{2}{11} + \frac{3}{12} \times \frac{2}{11} + \frac{2}{12} = \left\{ \frac{42}{132} \text{ or } \frac{7}{22} \right\} \underline{\text{or}}$ $P(\text{Yellow and two counters}) = \frac{7}{12} \times \frac{2}{11} + \frac{3}{12} \times \frac{2}{11} = \left\{ \frac{20}{132} \text{ or } \frac{5}{33} \right\}$	(2) M1	
	$\frac{P([Y \cap R] \cup [Y \cap B])}{P(Y)} = \frac{\frac{20}{132}}{\frac{42}{132}}$	M1	
	$=\frac{20}{42} \underline{\text{or}} \frac{10}{21} \text{oe}$	A1 (3) [5 marks]	
	Notes		
(a)	$1^{\text{st}} \text{ B1} \text{for the remaining probs on first set of branches and at least one on 2^{\text{nd}} \text{ set}}$ $2^{\text{nd}} \text{ B1} \text{for a fully correct tree diagram with all the correct probabilities}}$		
(b)	1 st M1 for a correct ft expression for P(<i>Y</i>) or P(Yellow and two counters)ft their tree diagram eg $1 - \frac{7}{12} \times \frac{6+3}{11} - \frac{3}{12} \times \frac{7+2}{11}$ NB: The method is implied by the numbers in curly brackets but we do not need to see them to award the mark.		
	2 nd M1 for a correct ratio formula (symbols or words) <u>and</u> at least one correct fully correct ft ratio. Do not follow through probabilities > 1 or < 0 A1 for $\frac{10}{21}$ or exact equivalent. (Allow $0.\dot{4}7619\dot{0}$) NB if an exact correct fraction is not given and an awrt 0.476 is given	-	
	get M1M1A0 if from correct working Generally if the answer is correct then award full marks (unless from incorrect working) or notes indicate otherwise	obvious	

-	stion nber		Scheme	Mar	rks
2.	(a)	B and C		B1	(1)
	(b)	A and C	independent gives:		(1)
			$65 = 0.13$ or $0.65 \times (r+0.13) = 0.13$ or $0.65 \times (0.48 - s) = 0.13$	M1	
		P(C = 0.2 or $r + 0.13 = 0.2$ or $0.48 - s = 0.2$	A1	
		$\mathbf{D}(\mathbf{A}) \perp \mathbf{r}$	$r \{= 0.2 - 0.13\} = 0.07$ or $s \{= 0.48 - 0.2\} = 0.28$ r + s = 1 or $0.65 + 0.07$ $r + s = 1$ or $0.65 + 0.28$ $r + r = 1$	A1 M1	
		$\Gamma(A) + I$	s = 1 - 0.72 = 0.07 + s = 1 of 0.03 + 0.28 + 7 = 1 s = 1 - 0.72 = 0.28 and r = 1 - 0.93 = 0.07	A1	
					(5)
	(c)	$\mathbf{P}[(B\cup 0$	C] = "0.2" + q or 0.13 + "0.07" + q	B1ft	
			$\mathbf{P}(A \cap C') = p + q \{= 0.52\}$	B1	
		$\{\mathbf{P}[(A \in$	$(C') \cap (B \cup C) = q \Longrightarrow \qquad "(p+q)" \times "(0.2+q)" = q \text{ or}$		
			$"\times"(0.13 + "0.07" + q)" = q$ or $"(p+q)"\times"(1-s-p)" = 0.52-p$	M1	
		```	$p + q = 0.52$ ] $0.52 \times "(0.2 + q)" = q \text{ or } 0.52(0.72 - p) = 0.52 - p$	M1	
				IVI I	
			$q = \frac{13}{60}$	A1	
			—	A 1	
			$p = \frac{91}{300}$	A1	
			Notes	[12 ma	(6) rkcl
	(a)	B1	<i>B</i> and <i>C</i> seen. If they include <i>A</i> then B0	[12 IIIa	<u>II K5j</u>
	<b>(b)</b>	1 st M1	for a correct equation for $P(C)$ using independence.		
		1 st A1	for $P(C) = 0.2$ correct linear equation for <i>r</i> or <i>s</i>		
		2 nd A1	for either $r = 0.07$ or $s = 0.28$		
		2 nd M1	for using $\sum p = 1$ Allow letter r and s or their values for r and s prov	vided they	are
			probabilities.		
		3 rd A1	for both $s = 0.28$ and $r = 0.07$		
		1 St D 1 ft	NB: The quotations around the 0.07 ("0.07") imply that we ft their v		
	(c)	1 st B1ft	for an expression (in q) for $P(B \cup C)$ ft their value of r or their "0.2"		
		2 nd B1	eg 0.13 + "their $r$ " + $q$ Implied by 1 st or 2 nd M1 below. for a correct expression for P( $A \cap C'$ ) in terms of $p$ and $q$ or 0.52		
			Implied by $1^{\text{st}}$ or $2^{\text{nd}}$ M1below		
		1 st M1	for a correct use of independence (ft their probabilities), values or le	tters.	
		1	Implied by 2 nd M1		
		2 nd M1	using $n \perp a = 0.52$ to goin a linear equation in one variable		
		2 nd M1	using $p + q = 0.52$ to gain a linear equation in one variable for a correct fraction for $q$		
		2 nd M1 1 st A1 2 nd A1	for a correct fraction for $q$ for a correct fraction for $p$	c c	
		1 st A1	for a correct fraction for q	3 0.07 C	

Question Number	Scheme	Marks
<b>3</b> (a)	Width = $2.5$ (cm)	B1
	1.5 cm ² for freq of 5 so $6 \times 1.5 = 9$ cm ² for freq of 30 or fd $= \frac{5}{3}$ w $\times h = 9$	M1
	So $h = 9 \div 2.5$ or $6 \div \frac{5}{3} = 3.6$ (cm)	A1
(b)	$Q_2 = [12] + \frac{16}{25} \times 3$ allow use of $(n + 1)$ giving $[12] + \frac{16.5}{25} \times 3$	(3) M1
	$\frac{25}{25} = 13.92 = \text{ awrt } \underline{13.9}$	A1
(c)(i)	$\sum fx = 5 \times 6.5 + 13 \times 9 + 16 \times 11 + 25 \times 13.5 + 30 \times 17.5 + 11 \times 24 = 1452$	(2) M1
(ii)	$\overline{x} = 14.52 = \text{awrt } \underline{14.5}$	A1 (2) M1
(11)	$\sum fx^2 = 6.5^2 \times 5 + 9^2 \times 13 + 11^2 \times 16 + 13.5^2 \times 25 + 17.5^2 \times 30 + 24^2 \times 11 = 23280$	1011
	$\sigma_x = \sqrt{\frac{"23280"}{100} - ("14.52")^2}  \underline{\text{or}}  \sqrt{21.9696}$	M1
	$\sigma_x = 4.687 = $ awrt <b><u>4.69</u></b>	A1 (3)
( <b>d</b> )	$\frac{1}{2} \times 13 + 16 + 25 + 30 + \frac{1}{4} \times 11$	M1
	So proportion is 80.25 % or 0.8025 awrt <b>0.803</b>	A1 (2)
(e)	Profit = $2.2 \times "0.8025" + 0.8 \times \frac{0.75 \times 11}{100} - 1.2 \times "\left(1 - \left[0.8025 + \frac{0.75 \times 11}{100}\right]\right)"$	M1
	= 1.6935 awrt <u>1.7 (p)</u>	A1 (2)
	Notes	[14 marks]
(a)	B1 for width = 2.5 (cm) M1 for sight of 9 cm ² or $w \times h = 9$ or fd = $\frac{5}{3}$ (o.e.)	
	A1 for height = 3.6 (cm)	
(b)		
	M1 for $\frac{16}{25} \times 3$ or $\frac{9}{25} \times 3$ or $\frac{m-12}{15-m} = \frac{16}{9}$	
	For any correct equation leading to $Q_2$ or correct fraction as part of $Q_2$	
(c)(i)	A1 for awrt 13.9 (use of $(n + 1)$ giving $13.98 = awrt 14.0$ ) M1 for attempt at $\Sigma fx$ with at least 3 correct terms or $900 < \Sigma fx < 1800$	
	<b>for info</b> $\Sigma f x = 32.5 + 117 + 176 + 337.5 + 525 + 264$	
	A1 for awrt 14.5 (correct answer only 2/2)	
(ii)	1 st M1 for attempt at $\Sigma fx^2$ with at least 3 correct terms or 20 000 < $\Sigma fx^2$ < 26 ( for info $\Sigma fx^2$ = 211.25 + 1053 + 1936 + 4556.25 + 9187.5 + 6336	000
	$2^{\text{nd}}$ M1 for a correct expression including $\sqrt{(\text{ft their }\Sigma fx^2 \text{ if clear it is }\Sigma fx^2)}$ Do not allow	
	$\begin{array}{c} \Sigma fx^{2} & (\Sigma fx)^{2} \text{ for } \Sigma fx^{2} \\ A1 & \text{ for awrt 4.69 (allow } s = 4.7107 \text{ awrt 4.71 }) (correct answer only 3/$	3)
( <b>d</b> )	M1 for attempt at a correct expression (allow 1 error or omission) $eg100 - ($	
	A1 for awrt 80.3% or 0.803	/ .
(e)	M1 for a correct expression ft their 0.8025 o.e. eg $[2.2 \times (100 - 11.5 - 8.25) + 0.8 \times 8.25 - 1.2 \times 11.5] \div 100$	
	Condone $[2.2 \times "80" + 0.8 \times (8) - 1.2 \times (12)] \div 100$	
	A1 for awrt 1.7 Allow $\pm 0.017$ (this must have units)	

Question Number	Scheme	Marks
<b>4.</b> (a)	$P(W < 120) = P\left(Z < \frac{120 - 165}{35}\right)$	M1
	= P(Z < -1.2857) = 1 - 0.9015  or  1 - 0.9007285 $= 0.09927 = awrt  0.0985 - 0.0994$	M1 A1
(b)	e.g. $P(W > x) = \frac{1}{3}$ gives $\frac{x - 165}{35} = \pm 0.43$ (calculator 0.430727)	(3) M1B1
	Limits 149.9245 to 180.0754 awrt <u>150</u> to <u>180</u>	A1, A1 (4)
(c)	$P(W < 200   W > "180")  \underline{\text{or}}  \frac{P("180" < W < 200)}{P(W > "180")  \text{or}  \frac{1}{3}}$	M1
	$=\frac{0.8413(44739)-\frac{2}{3}}{\frac{1}{3}}$	A1 (num)
	= 0.52403 <u>(0.523~0.5264)</u>	A1 (3)
( <b>d</b> )	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}; \times 3!$	M1;M1
	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}; \times 3! = \frac{2}{9}$	A1
		(3) [ <b>13 marks</b> ]
	Notes	[ []
(a)	$1^{st}$ M1for standardising with 120 (allow 210), 165 and 35. Accept $\pm$ $2^{nd}$ M1for attempting $1 - p$ [where $0.85 ]A1for awrt 0.0985~0.0994 (Correct ans only 3/3)$	
(b)	M1 for standardising with x (o.e.) 165 and 35 and setting equal to a z value, (Accept $\frac{165 - x}{35} = \pm z$ where $0.4 <  z  < 0.5$ )	0.4 <  z  < 0.5
	B1 for use of $z = 0.43$ or better We must see 0.43 or better. 1 st A1 for lower limit of awrt 150 2 nd A1 for upper limit of awrt 180	
SC	11	
(c)	M1 for a correct probability statement (either form) ft their 180 or a correct ratio 1 st A1 for a correct numerator (awrt 0.175) 2 nd A1 for an answer in the range awrt 0.523~0.5264 (use of 180 gives 0.5263869)	
( <b>d</b> )	1 st M1 for $\left(\frac{1}{3}\right)^3$ (or equivalent)	
	$2^{nd}$ M1 for $p \times 3!$ (or equivalent) where $0$	
	A1 for $\frac{2}{9}$ or any exact equivalent	

Question Number	Scheme	Marks
<b>5.</b> (a)	{E(X) = } $-2a - b + 0 \times c + b + 4a$ or $2a$ { $2a = 0.5$ so } <u>$a = 0.25$</u>	M1 A1
(b)	$\{ E(X^{2}) = \} (-2)^{2} \times a + (-1)^{2} \times b + 0 + 1^{2} \times b + 4^{2} \times a \text{ or } 20a + 2b \text{ (o.e.)} \\ \{ Var(X) = \}^{"}20a + 2b^{"} - 0.5^{2} \\ 20a + 2b - 0.25 = 5.01 \text{ (o.e.) e.g. } "4.75" + 2b = 5.01 \\ \{ 2b = 0.26 \text{ so} \} \underline{b} = 0.13 \\ \{ Use \text{ of sum of probs} = 1 \text{ to calculate a } 2^{nd} \text{ value} \} \underline{c} = 0.24 \\ \end{bmatrix}$	(2) M1 M1 A1 A1 A1ft
(c)(i) (ii)	${E(Y) = 5 - 8 \times 0.5} = 1$ ${Var(Y) =} (-8)^2 \times 5.01$	(5) B1 M1
( <b>d</b> )	$= 320.64 \text{ awrt } \underline{321}$ $4X^{2} > 5 - 8X$ $(2X - 1)(2X + 5) > 0 \implies X > 0.5$ So need X = 1 or 4 <u>or</u> probability of a + b $= \underline{0.38}$	A1 (3) M1 M1A1 M1 A1
		(5) [15 marks]
(a)	NotesM1for any correct expression for $E(X)$ in terms of $a$ (or $a, b, c$ )A1for $a = 0.25$	
(b)	1st M1for attempt at an expression for $E(X^2)$ with at least 3 correct non-zero terms2nd M1for a correct expression for $Var(X)$ eg"18a - c + 1" - 0.5² Allow with their value of a substituted1st A1for a correct equation for b (or possibly c) eg"18a - c + 1" - 0.5² = 5.01 Allow with their value of a substituted2nd A1for either b = 0.13 or c = 0.243rd A1ftfor using c = 1 - 2×"0.25" - 2×"0.13" or b = (1 - 2×"0.25" - "0.24") ÷ 2 to gai the correct ft answer for their 2 nd value	
(c)	B1 for $\{E(Y) =\} 1$ M1 for correct use of $Var(aX + b) = a^2 Var(X)$ A1 for awrt 321	
( <b>d</b> )	$1^{st}$ M1for correct quadratic inequality (may be inside prob statement) or $2^{nd}$ M1for an attempt to solve or identifying correct X values $1^{st}$ A1for $X > 0.5$ [ may also have $X < -2.5$ ] $3^{rd}$ M1for realising need $X = 1$ and 4 only or answer of their $(a + b)$ $2^{nd}$ A1for 0.38 (or exact equivalent) only (correct ans only 5/5)	table of values

Question Number	Scheme	Marks
6. (a)	$\{\mathbf{S}_{yy} =\} 42.63 - \frac{23.7^2}{16} = [7.524375]$	B1
		(1)
(b)	Use of $\overline{y} = 3.684 - 0.3242\overline{x}$ ; so $\sum x = 16 \times \left(\frac{3.684 - \frac{23.7}{16}}{0.3242}\right) = 108.71067$	M1; A1
	$\{\mathbf{S}_{xx} =\}756.81 - \frac{("108.71")^2}{16}; = 18.18435 \text{ awrt } \mathbf{\underline{18.2}}$	M1; A1 (4)
(c)	$b = \frac{S_{xy}}{S_{xx}} \Longrightarrow S_{xy} = "18.1843" \times (-0.3242) [= -5.8953]; r = \frac{"-5.89536"}{\sqrt{"18.184" \times 7.524375}}$	M1; M1
	$= -0.50399 = -0.49 \sim -0.51$	A1 (2)
(d)	Sub $x = 2$ in the regression line gives $y = 3.0356$	(3) B1 (1)
(e)	St.dev = $\sqrt{\frac{S_{xx}}{n}} = \sqrt{\frac{"18.184"}{16}} = 1.066$	M1
	So limits are: $\frac{"108.71"}{16} \pm 3 \times "1.066" = 3.5965~9.9929 = awrt 3.6~10$	
( <b>f</b> )	The probability of $\underline{x = 2}$ being in the range is very small; so Behrouz's estimate is <u>unreliable</u>	(3) B1ft; dB1ft (2)
(g)	Should use regression of $x$ on $y$ to estimate unemployment or equivalent So Andi's suggestion is not suitable <u>or</u> not to be recommended	$ \begin{array}{c} (2)\\ B1\\ dB1 \end{array} $
		(2) [16 marks]
	Notes	
(a)		or $23.7^2$
(b)	<b>—</b>	
	1 st A1 for $\sum x$ = awrt 109 2 nd M1 for a correct expression for S _{xx} ft their $\sum x$	
	$2^{nd}$ M1 for a correct expression for $S_{xx}$ ft their $\Sigma x$ $2^{nd}$ A1 for awrt 18.2	
(c)		
	$2^{nd}$ M1 for a correct expression for <i>r</i> ft their S _{xy} and S _{xx}	
( <b>d</b> )	A1for an answer in the range $-0.49 \sim -0.51$ B1for sight of $y = 3.03$ or better. Allow 3.04	
(e)	1 st M1 for a correct attempt at st. dev. ft their $S_{xx}$ or $\sqrt{\frac{756.81}{16} - \left(\frac{"108.71"}{16}\right)}$	$\int^2$ ft their $\Sigma x$
	2 nd M1 for one correct calcft their values	
( <b>f</b> )	A1 for a range awrt 3.6~10 1st D1ft for a correct reason ft their range in part (e) eg $x = 2$ is <u>outside</u> the range	ge Allow
	extrapolation	50. 1 110 11
(g)	$2^{nd}$ dB1ft dep on $1^{st}$ B1 for stating a correct conclusion for their range $1^{st}$ B1 for a suitable reason based on reg line, eg regression line (y on x) can to estimate wages. Allow x instead of unemployment and y instead of	•
	$2^{nd}$ dB1 dep on $1^{st}$ B1 for suggesting not suitable (or equivalent)	