Pearson Edexcel IAL (Further) Mathematics

# Pure Mathematics 4

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# Past Paper Collection

Last updated: January 21, 2025



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Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel**
**International Advanced Level**

Centre Number | Candidate Number

# Tuesday 13 October 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA14/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

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- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

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*Turn over* ▶

Leave blank

**1.** Given that $n$ is an integer, use algebra, to prove by contradiction, that if $n^3$ is even then $n$ is even.

**(4)**

Leave blank

**Question 1 continued**

**Q1**

**(Total 4 marks)**

Leave blank

**2.** (a) Use the binomial expansion to expand

$$(4 - 5x)^{-\frac{1}{2}} \qquad |x| < \frac{4}{5}$$

in ascending powers of $x$, up to and including the term in $x^2$ giving each coefficient as a fully simplified fraction.

**(4)**

$$f(x) = \frac{2 + kx}{\sqrt{4 - 5x}} \qquad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of f($x$), in ascending powers of $x$, is

$$1 + \frac{3}{10}x + mx^2 + \ldots \qquad \text{where } m \text{ is a constant}$$

(b) find the value of $k$,

**(2)**

(c) find the value of $m$.

**(2)**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

Q2

**(Total 8 marks)**

Leave blank

**3.**

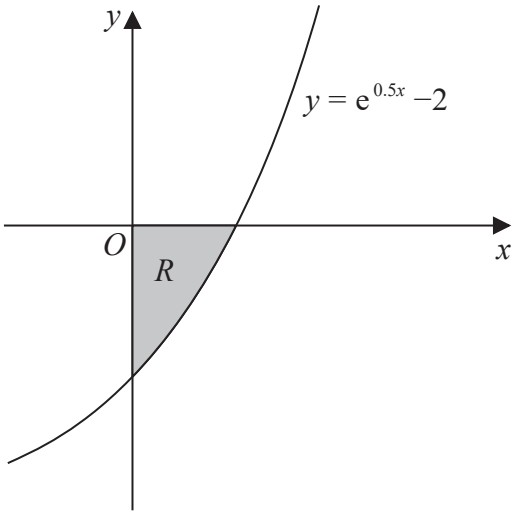


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = e^{0.5x} - 2$

The region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the $y$-axis.

The region $R$ is rotated 360° about the $x$-axis to form a solid of revolution.

Show that the volume of this solid can be written in the form $a \ln 2 + b$, where $a$ and $b$ are constants to be found.

**(6)**

Leave blank

**Question 3 continued**

**Q3**

**(Total 6 marks)**

Leave blank

**4.**

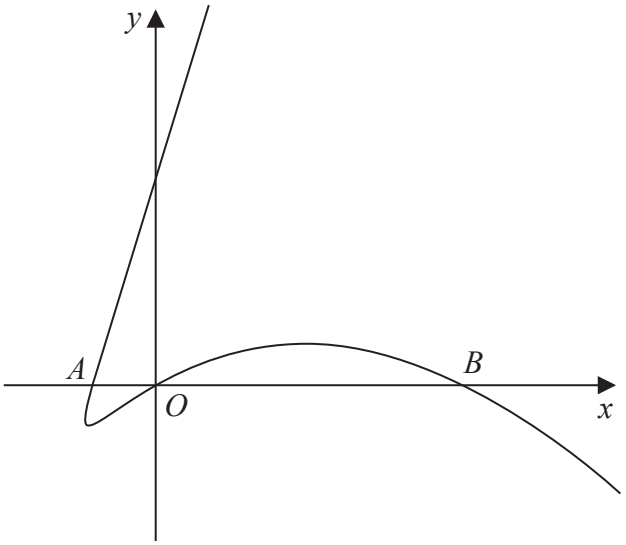


**Figure 2**

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the $x$-axis at the origin and at the points $A$ and $B$, as shown in Figure 2.

(a) Find the coordinates of $A$ and show that $B$ has coordinates (20, 0).

**(3)**

(b) Show that the equation of the tangent to the curve at $B$ is

$$7y + 4x - 80 = 0$$

**(5)**

The tangent to the curve at $B$ cuts the curve again at the point $P$.

(c) Find, using algebra, the $x$ coordinate of $P$.

**(4)**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

**(Total 12 marks)**

**Q4**

Leave blank

**5.**

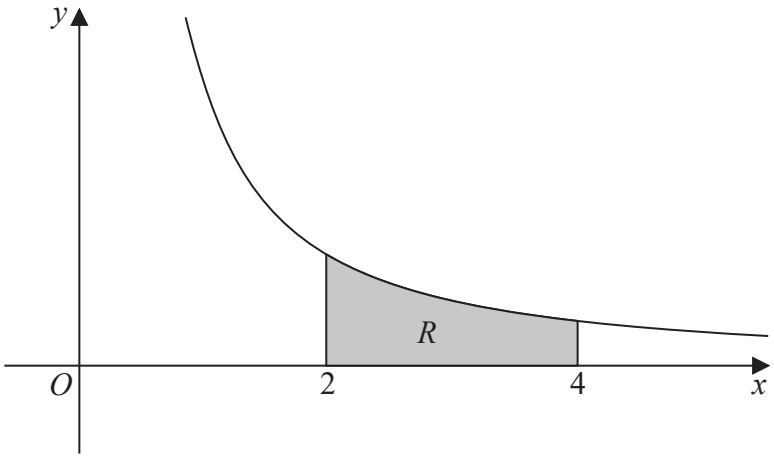


**Figure 3**

(a) Find $\int \frac{\ln x}{x^2} \mathrm{d}x$

**(3)**

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \qquad x > 0.5$$

The finite region $R$, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the $x$-axis and the line with equation $x = 4$

(b) Use the answer to part (a) to find the exact area of $R$, writing your answer in simplest form.

**(4)**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Q5

**(Total 7 marks)**

Leave blank

**6.** A curve $C$ has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(5)**

(b) Hence show that the $x$ coordinates of the stationary points of $C$ are solutions of the equation

$$\tan x + x \ln x = 0$$

**(2)**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

**Q6**

**(Total 7 marks)**

Leave blank

**7.** (i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(6)**

(ii) Find

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,\mathrm{d}x$$

**(6)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**(Total 12 marks)**

**Q7**

Leave blank

**8.** Relative to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \text{where } \mu \text{ is a scalar parameter}$$

Given that $l_1$ and $l_2$ meet at the point $X$,

(a) find the position vector of $X$.

**(5)**

The point $P(10, -7, 0)$ lies on $l_1$

The point $Q$ lies on $l_2$

Given that $\overrightarrow{PQ}$ is perpendicular to $l_2$

(b) calculate the coordinates of $Q$.

**(5)**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

**Q8**

**(Total 10 marks)**

Leave blank

**9.** Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, $A\,\text{cm}^2$, covered by the bacteria, $t$ days after the bacteria start to grow, is modelled by the differential equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r}\right)^2$$

where $p$, $q$ and $r$ are integers to be found.

**(7)**

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

**(2)**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

**Q9**

**(Total 9 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

**END**

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**Pearson Edexcel**
**International Advanced Level**

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# Thursday 7 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA14/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

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## Information

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- There are 10 questions in this question paper. The total mark for this paper is 75.
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## Advice

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*Turn over* ▶

Leave blank

**1.** (a) Find the first 4 terms, in ascending powers of $x$, of the binomial expansion of

$$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \qquad |x| < \frac{1}{20}$$

giving each coefficient in its simplest form.

**(5)**

By substituting $x = \frac{1}{100}$ into the answer for (a),

(b) find an approximation for $\sqrt{5}$

Give your answer in the form $\frac{a}{b}$ where $a$ and $b$ are integers to be found.

**(2)**

Leave blank

**Question 1 continued**

**Q1**

**(Total 7 marks)**

Leave blank

**2.**

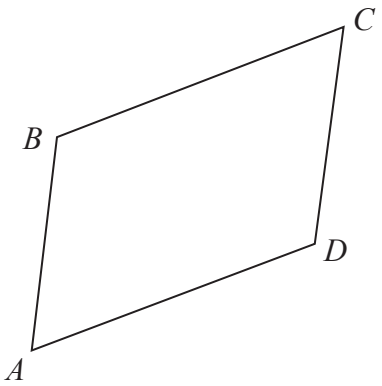


**Figure 1**

Figure 1 shows a sketch of parallelogram $ABCD$.

Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle $ABC$, giving your answer in degrees, to 2 decimal places.

**(3)**

(b) Find the area of parallelogram $ABCD$, giving your answer to one decimal place.

**(2)**

Leave blank

**Question 2 continued**

**Q2**

**(Total 5 marks)**

Leave blank

**3.** Prove by contradiction that there is no greatest odd integer.

**(2)**

Leave blank

**Question 3 continued**

**Q3**

**(Total 2 marks)**

Leave blank

**4.** The curve $C$ is defined by the parametric equations

$$x = \frac{1}{t} + 2 \qquad y = \frac{1 - 2t}{3 + t} \qquad t > 0$$

(a) Show that the equation of $C$ can be written in the form $y = \mathrm{g}(x)$ where g is the function

$$\mathrm{g}(x) = \frac{ax + b}{cx + d} \qquad x > k$$

where $a$, $b$, $c$, $d$ and $k$ are integers to be found.

**(5)**

(b) Hence, or otherwise, state the range of g.

**(2)**

Leave blank

**Question 4 continued**

**Q4**

**(Total 7 marks)**

Leave blank

**5.** **In this question you should show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_{1}^{13} \frac{4}{3 + \sqrt{2x - 1}} \, dx$$

giving your answer in the form $p + q \ln 2$, where $p$ and $q$ are integers to be found.

**(8)**

Leave blank

**Question 5 continued**

**Q5**

**(Total 8 marks)**

Leave blank

**6.** A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(5)**

The curve crosses the $y$-axis at the origin and at the point $P$.

(b) Find the equation of the normal to the curve at $P$, writing your answer in the form $y = mx + c$ where $m$ and $c$ are constants to be found.

**(4)**

Leave blank

**Question 6 continued**

**Q6**

**(Total 9 marks)**

Leave blank

**7.**

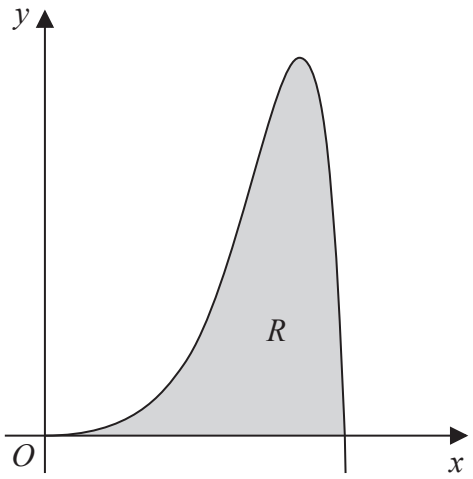


**Figure 2**

(a) Find $\int e^{2x} \sin x \, dx$

**(5)**

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \qquad x \geqslant 0$$

The finite region $R$ is bounded by the curve and the $x$-axis and is shown shaded in Figure 2.

(b) Show that the exact area of $R$ is $\dfrac{e^{2\pi} + 1}{5}$

(*Solutions relying on calculator technology are not acceptable.*)

**(2)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**Q7**

**(Total 7 marks)**

Leave blank

**8.** With respect to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ b \end{pmatrix}$$

where $\lambda$ and $\mu$ are scalar parameters and $b$ is a constant.

Prove that for all values of $b \neq 7$, the lines $l_1$ and $l_2$ are skew.

**(6)**

Leave blank

**Question 8 continued**

**Q8**

**(Total 6 marks)**

Leave blank

**9.**

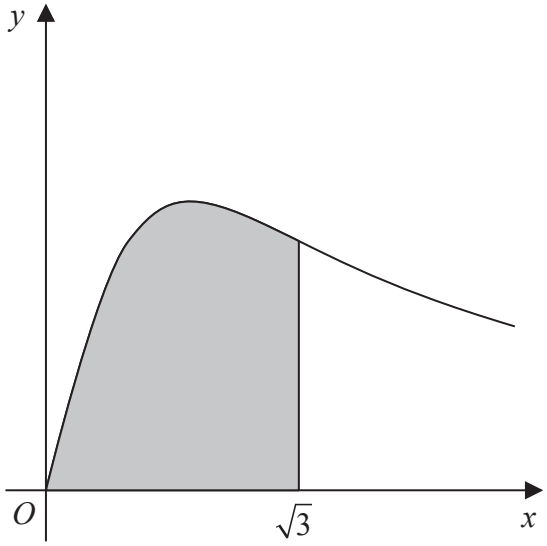


**Figure 3**

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan\theta \qquad y = 2\sin 2\theta \qquad \theta \geqslant 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the $x$-axis and the line with equation $x = \sqrt{3}$

The region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta)\, \mathrm{d}\theta$$

where $p$ and $k$ are constants to be found.

**(7)**

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

**(3)**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

**Q9**

**(Total 10 marks)**

Leave blank

**10.** (a) Write $\dfrac{1}{(H-5)(H+3)}$ in partial fraction form.

**(3)**

The depth of water in a storage tank is being monitored.

The depth of water in the tank, $H$ metres, is modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{(H-5)(H+3)}{40}$$

where $t$ is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10 + 3\mathrm{e}^{-0.2t}}{2 - \mathrm{e}^{-0.2t}}$$

**(7)**

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(3)**

According to the model, the depth of water in the tank will eventually fall to $k$ metres.

(d) State the value of the constant $k$.

**(1)**

Leave blank

**Question 10 continued**

Leave blank

**Question 10 continued**

Leave blank

**Question 10 continued**

Leave blank

**Question 10 continued**

**Q10**

**(Total 14 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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**Pearson Edexcel**
**International Advanced Level**

Centre Number | Candidate Number

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

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- Try to answer every question.
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- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

*Turn over* ▶

Leave blank

**1.** Given that $k$ is a constant and the binomial expansion of

$$\sqrt{1+kx} \qquad |kx| < 1$$

in ascending powers of $x$ up to the term in $x^3$ is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of $k$,

(ii) find the value of the constant $A$ and the constant $B$.

**(5)**

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

**(2)**

Leave blank

**Question 1 continued**

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**Question 1 continued**

Leave blank

**Question 1 continued**

**Q1**

**(Total 7 marks)**

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**2.**

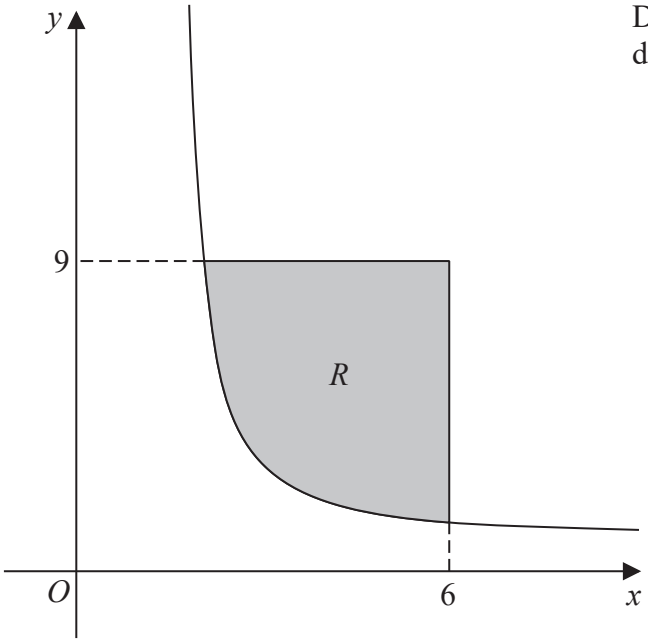


Diagram **NOT** drawn to scale

**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x-3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

**(7)**

Leave blank

**Question 2 continued**

**Q2**

**(Total 7 marks)**

Leave blank

**3.**

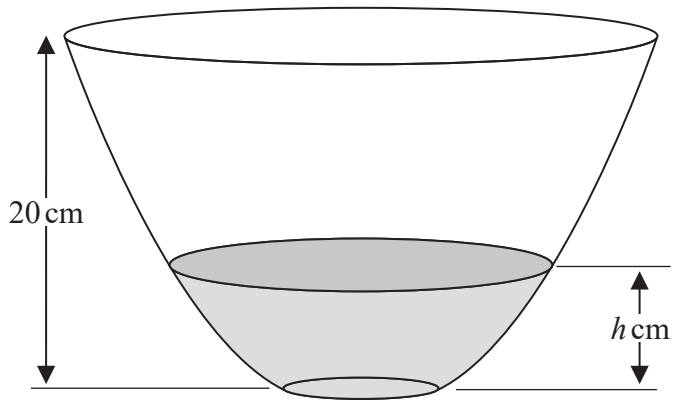


**Figure 2**

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is $h$ cm, the volume of water in the bowl, $V$ cm$^3$, is modelled by the equation

$$V = \frac{1}{3}h^2(h + 4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm$^3$ s$^{-1}$, find, according to the model,

(a) the time taken to fill the bowl,

**(2)**

(b) the rate of change of the depth of the water, in cm s$^{-1}$, when $h = 5$

**(5)**

Leave blank

**Question 3 continued**

Leave blank

**Question 3 continued**

Leave blank

**Question 3 continued**

**Q3**

**(Total 7 marks)**

Leave blank

**4.** Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}} \mathrm{d}x$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(8)**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

**Q4**

**(Total 8 marks)**

Leave blank

**5.** A curve has equation

$$y^2 = ye^{-2x} - 3x$$

(a) Show that

$$\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$$

**(4)**

The curve crosses the $y$-axis at the origin and at the point $P$.

The tangent to the curve at the origin and the tangent to the curve at $P$ meet at the point $R$.

(b) Find the coordinates of $R$.

**(5)**

Leave blank

**Question 5 continued**

**Q5**

**(Total 9 marks)**

Leave blank

**6.**

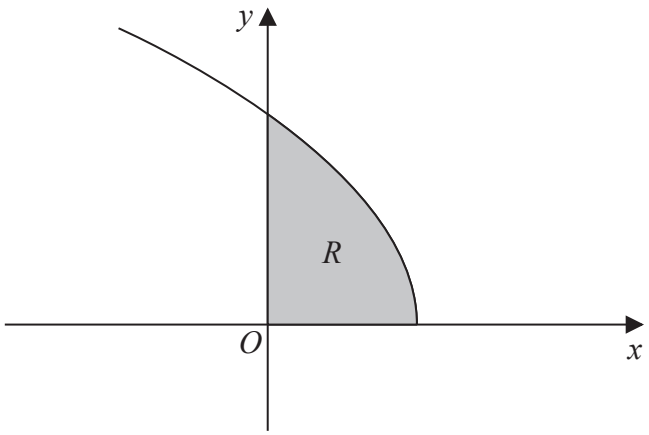


**Figure 3**

Figure 3 shows a sketch of the curve $C$ with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region $R$, shown shaded in Figure 3, is bounded by the curve, the $x$-axis and the $y$-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t \cos t \, \mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of $R$.

**(6)**

(b) Show that all points on $C$ satisfy $y = \sqrt{ax + b}$, where $a$ and $b$ are constants to be found.

**(3)**

The curve $C$ has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and $a$ and $b$ are the constants found in part (b).

(c) State the range of f.

**(1)**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

**(Total 10 marks)**

**Q6**

Leave blank

**7.** Relative to a fixed origin $O$, the line $l$ has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to $l$,

(a) find $\overrightarrow{OA}$

**(2)**

The point $X$ lies on $l$.

Given that $X$ is the point on $l$ that is closest to the origin,

(b) find the coordinates of $X$.

**(5)**

The points $O$, $X$ and $A$ form the triangle $OXA$.

(c) Find the exact area of triangle $OXA$.

**(3)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**(Total 10 marks)**

**Q7**

Leave blank

**8.** (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = g(x)$.

**(7)**

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

**(2)**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

**Q8**

**(Total 9 marks)**

Leave blank

**9.** (i) Relative to a fixed origin $O$, the points $A$, $B$ and $C$ have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points $A$, $B$ and $C$ lie in a straight line, with $B$ lying between $A$ and $C$.

Given $AB : AC = 1 : 3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

**(3)**

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if $n^2$ is a multiple of 3 then $n$ is a multiple of 3

**(5)**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

**Q9**

**(Total 8 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

**END**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
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- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

Leave blank

**1.** The curve $C$ has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on $C$.

Find the equation of the normal to $C$ at the point $P$, writing your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers to be found.

**(7)**

Leave blank

**Question 1 continued**

Q1

**(Total 7 marks)**

Leave blank

**2.** Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

**(6)**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

Leave blank

**Question 2 continued**

**Q2**

**(Total 6 marks)**

Leave blank

**3.** $$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants $A$, $B$, $C$ and $D$.

**(5)**

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

**(2)**

(c) Hence, explain why $g'(x) > 3$ for all values of $x$ in the domain of g.

**(1)**

Leave blank

**Question 3 continued**

Leave blank

**Question 3 continued**

Leave blank

**Question 3 continued**

**Q3**

**(Total 8 marks)**

Leave blank

**4.** $f(x) = \sqrt{1 - 4x^2}$ $\quad |x| < \frac{1}{2}$

(a) Find, in ascending powers of $x$, the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

**(4)**

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

**(2)**

Leave blank

**Question 4 continued**

**Q4**

**(Total 6 marks)**

Leave blank

**5.**

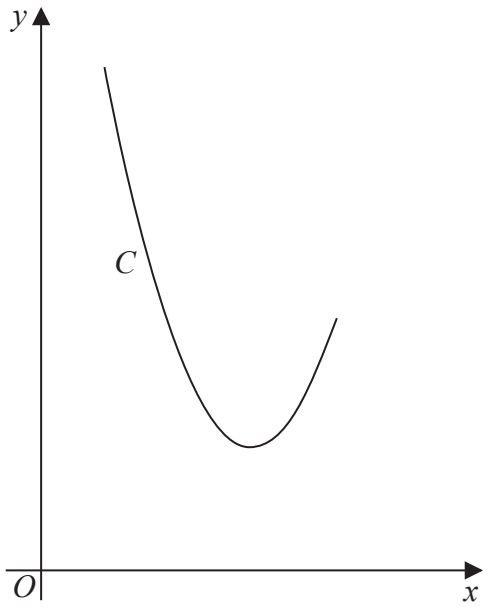


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of $C$ at $x = 3$

**(4)**

The curve $C$ has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where $a$, $b$ and $c$ are constants to be found.

**(3)**

(c) Find the range of f.

**(2)**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

**Q5**

**(Total 9 marks)**

Leave blank

**6.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

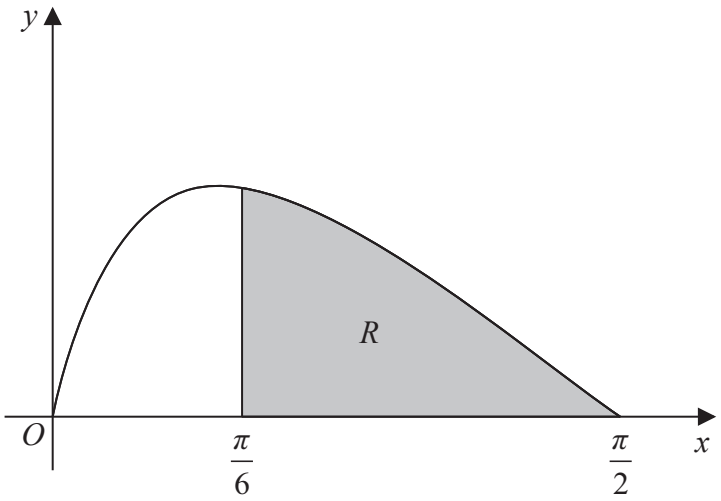


**Figure 2**

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of $R$ can be written in the form $a + \ln b$, where $a$ and $b$ are rational constants to be found.

**(7)**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

**Q6**

**(Total 7 marks)**

Leave blank

**7.** With respect to a fixed origin $O$,

- the line $l$ has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where $\lambda$ is a scalar constant
- the point $A$ has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that $X$ is the point on $l$ nearest to $A$,

(a) find

(i) the coordinates of $X$

(ii) the shortest distance from $A$ to $l$.
Give your answer in the form $\sqrt{d}$, where $d$ is an integer.

**(7)**

The point $B$ is the image of $A$ after reflection in $l$.

(b) Find the position vector of $B$.

**(2)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**Q7**

**(Total 9 marks)**

Leave blank

**8.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

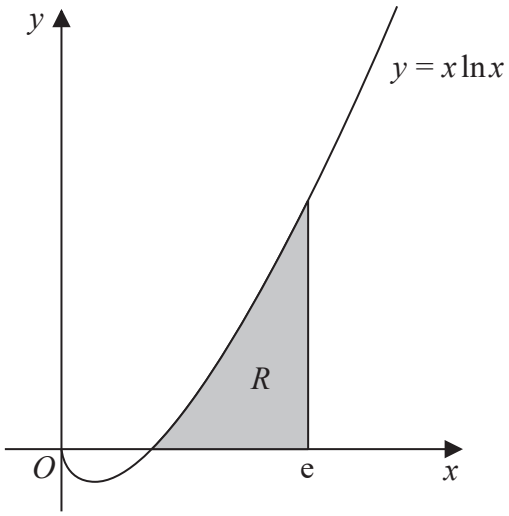


**Figure 3**

(a) Find $\int x^2 \ln x \, dx$

**(3)**

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \qquad x > 0$$

The region $R$, shown shaded in Figure 3, lies entirely above the $x$-axis and is bounded by the curve, the $x$-axis and the line with equation $x = e$.

This region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

**(4)**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

**Q8**

**(Total 7 marks)**

Leave blank

**9.**

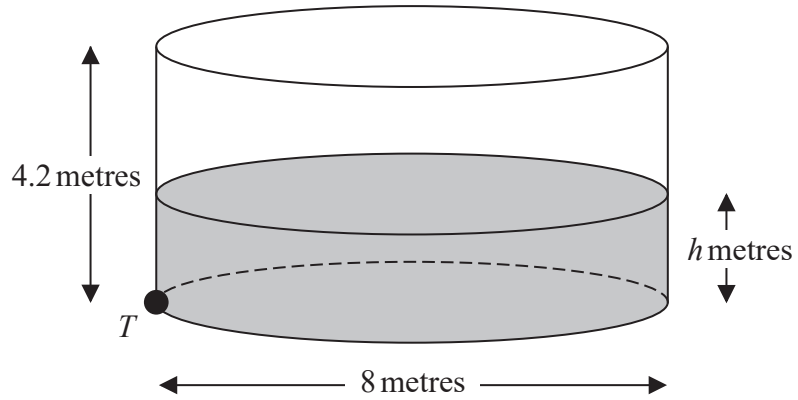


**Figure 4**

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\text{m}^3$ per minute.

There is a tap at point $T$ at the bottom of the tank.

At time $t$ minutes after the tap has been opened,

- the depth of the water is $h$ metres
- the water is leaving the tank at a rate of $(0.15\pi h)\text{m}^3$ per minute

(a) Show that

$$\frac{\text{d}h}{\text{d}t} = \frac{12 - 3h}{320}$$

**(4)**

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

**(6)**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

**Q9**

**(Total 10 marks)**

Leave blank

**10.** (a) A student's attempt to answer the question

"Prove by contradiction that if $n^3$ is even, then $n$ is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number $n$ such that $n^3$ is even, but $n$ is odd.

If $n$ is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$

So $n^3 = (2p + 1)^3$

$= 8p^3 + 12p^2 + 6p + 1$

$=$

This contradicts our initial assumption, so if $n^3$ is even, then $n$ is even.

Complete this proof by filling in line 5.

**(1)**

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

**(5)**

Leave blank

**Question 10 continued**

Leave blank

**Question 10 continued**

**Q10**

**(Total 6 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

Please check the examination details below before entering your candidate information

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|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

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## Information

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- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

Leave blank

**1.** The curve $C$ has equation

$$xy^2 = x^2y + 6 \qquad x \neq 0 \quad y \neq 0$$

Find an equation for the tangent to $C$ at the point $P(2, 3)$, giving your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**(6)**

Leave blank

**Question 1 continued**

Q1

**(Total 6 marks)**

Leave blank

**2.** (a) Find, in ascending powers of $x$, the first three non-zero terms of the binomial series expansion of

$$\sqrt[3]{1+4x^3} \qquad |x| < \frac{1}{\sqrt[3]{4}}$$

giving each coefficient as a simplified fraction.

**(4)**

(b) Use the expansion from part (a) with $x = \frac{1}{3}$ to find a rational approximation to $\sqrt[3]{31}$

**(3)**

Leave blank

**Question 2 continued**

**(Total 7 marks)**

**Q2**

Leave blank

**3.** The curve $C$ has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that $C$ has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where $p$ and $q$ are constants to be found.

**(6)**

(b) Hence, find a Cartesian equation for $C$ in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where $a$, $b$, $c$ and $d$ are constants.

**(3)**

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**Question 3 continued**

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**Question 3 continued**

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**Question 3 continued**

**Q3**

**(Total 9 marks)**

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**4.**

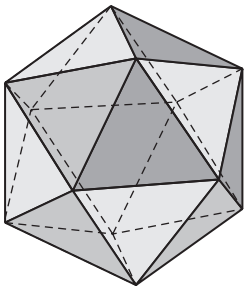

**Figure 1**

A regular icosahedron of side length $x$ cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length $x$ cm.

(a) Show that the surface area, $A$ cm$^2$, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

**(2)**

Given that the volume, $V$ cm$^3$, of the icosahedron is given by

$$V = \frac{5}{12}\left(3 + \sqrt{5}\right)x^3$$

(b) show that $\dfrac{\mathrm{d}V}{\mathrm{d}A} = \dfrac{\left(3 + \sqrt{5}\right)x}{8\sqrt{3}}$

**(3)**

The surface area of the icosahedron is increasing at a constant rate of $0.025\,\text{cm}^2\,\text{s}^{-1}$

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

**(3)**

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**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

**Q4**

**(Total 8 marks)**

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**5.**

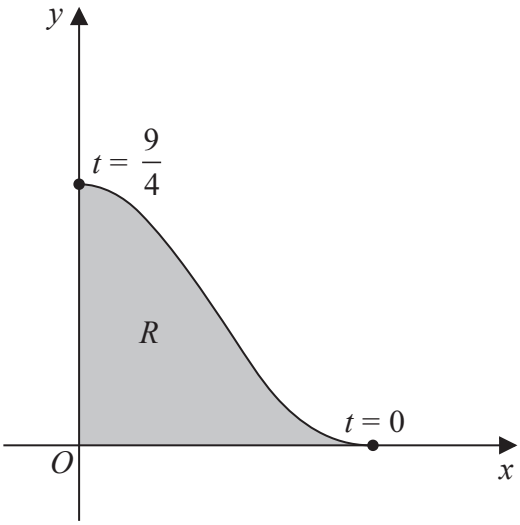


**Figure 2**

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the $x$-axis when $t = 0$ and meets the $y$-axis when $t = \frac{9}{4}$

The region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the $y$-axis.

(a) Show that the area of $R$ is given by

$$K\int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}}\,\mathrm{d}t$$

where $K$ is a constant to be found.

**(4)**

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of $R$.

(*Solutions relying on calculator technology are not acceptable.*)

**(6)**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

**(Total 10 marks)**

**Q5**

Leave blank

**6.** Three **consecutive** terms in a sequence of real numbers are given by

$$k,\ 1 + 2k \text{ and } 3 + 3k$$

where $k$ is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

**(5)**

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**Question 6 continued**

**Q6**

**(Total 5 marks)**

Leave blank

**7.**

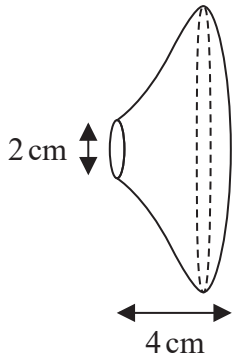


**Figure 3**

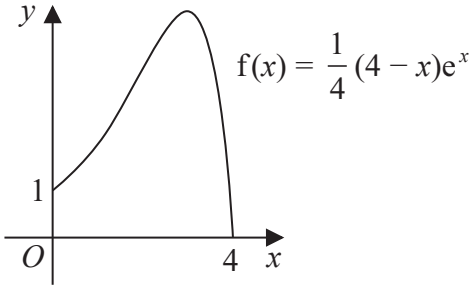


**Figure 4**

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the $x$-axis, where the units are centimetres.

The equation of the curve is given by

$$\mathrm{f}(x) = \frac{1}{4}(4 - x)\mathrm{e}^{x} \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, $V\,\text{cm}^3$, of the doorknob is given by

$$V = K\int_0^4 \left(x^2 - 8x + 16\right)\mathrm{e}^{2x}\,\mathrm{d}x$$

where $K$ is a constant to be found.

**(3)**

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(\mathrm{e}^q + r)\,\text{cm}^3$ where $p$, $q$ and $r$ are simplified rational numbers to be found.

**(5)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**(Total 8 marks)**

**Q7**

Leave blank

**8.** With respect to a fixed origin $O$ the points $A$ and $B$ have position vectors

$$\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix}$$

respectively.

The line $l_1$ passes through the points $A$ and $B$.

(a) Write down an equation for $l_1$

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where $\lambda$ is a scalar parameter.

**(2)**

The line $l_2$ has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

where $\mu$ is a scalar parameter.

(b) Show that $l_1$ and $l_2$ do **not** meet.

**(4)**

The point $C$ is on $l_2$ where $\mu = -1$

(c) Find the acute angle between $AC$ and $l_2$

Give your answer in degrees to one decimal place.

**(5)**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

**(Total 11 marks)**

**Q8**

Leave blank

**9.** (a) Find the derivative with respect to $y$ of

$$\frac{1}{(1+2\ln y)^2}$$

**(2)**

(b) Hence find a general solution to the differential equation

$$3\text{cosec}(2x)\frac{\mathrm{d}y}{\mathrm{d}x} = y(1+2\ln y)^3 \qquad y>0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

**(4)**

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = \mathrm{e}^{A\sec x - \frac{1}{2}}$$

where $A$ is an irrational number to be found.

**(5)**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

Leave blank

**Question 9 continued**

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**Question 9 continued**

**Q9**

**(Total 11 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

**END**

Please check the examination details below before entering your candidate information

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|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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## Instructions

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## Information

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- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
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## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** The binomial expansion of

$$(3 + kx)^{-2} \qquad |kx| < 3$$

where $k$ is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \ldots$$

where $A$, $B$, $C$ and $D$ are constants.

(a) Find the value of $A$

**(1)**

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

**(3)**

(c) Hence (i) find the value of $k$

(ii) find the value of $D$

**(3)**

**Question 1 continued**

**(Total for Question 1 is 7 marks)**

**2.** (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

**(3)**

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where $n$ is an integer to be found.

**(6)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 9 marks)**

**3.**

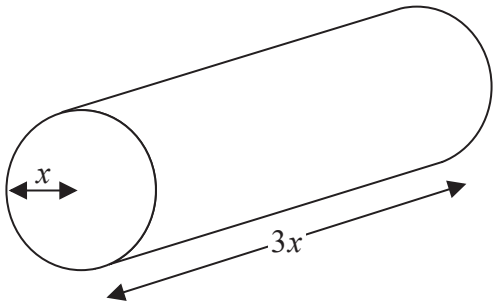



**Figure 1**

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At $t$ seconds after the tablet is dropped into the water, the radius of the tablet is $x$ mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5\,\text{mm}^2\,\text{s}^{-1}$

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 7$

**(4)**

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$

**(4)**

**Question 3 continued**

**(Total for Question 3 is 8 marks)**

**4.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where $k$ is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

**(4)**

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of $k$

**(4)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 8 marks)**

**5.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(a) Use the substitution $x = 2\sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x = \int_0^p \left(\frac{3}{2}\sec u\tan u + \frac{1}{2}\sec^2 u\right)\mathrm{d}u$$

where $p$ is a constant to be found.

**(4)**

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

**(4)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 8 marks)**

**6.** Relative to a fixed origin $O$,

- the point $A$ has position vector $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
- the point $B$ has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point $C$ has position vector $3\mathbf{i} + p\mathbf{j} - \mathbf{k}$

where $p$ is a constant.

The line $l$ passes through $A$ and $B$.

(a) Find a vector equation for the line $l$

**(3)**

Given that $\overrightarrow{AC}$ is perpendicular to $l$

(b) find the value of $p$

**(3)**

(c) Hence find the area of triangle $ABC$, giving your answer as a surd in simplest form.

**(3)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 9 marks)**

**7.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The curve $C$ has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leqslant t \leqslant 5$$

(a) Show that $\dfrac{dy}{dx} = 3$ where $t = \pi$

**(4)**

The point $P$ lies on $C$ where $t = \pi$

(b) Find the equation of the tangent to the curve at $P$ in the form $y = mx + c$ where $m$ and $c$ are constants to be found.

**(3)**

Given that the tangent to the curve at $P$ cuts $C$ at the point $Q$

(c) show that the value of $t$ at point $Q$ satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$

**(2)**

(d) Hence find the exact value of the $y$ coordinate of $Q$

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

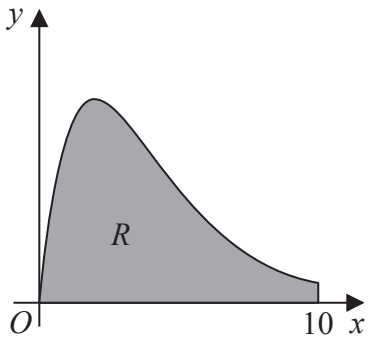



**Figure 2**

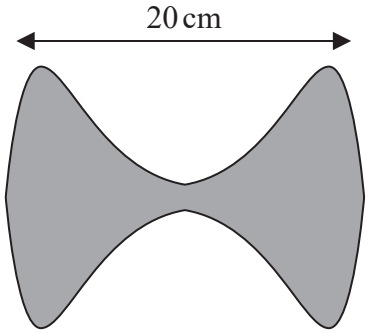



**Figure 3**

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the line with equation $x = 10$

The region $R$ is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(a) Show that the volume, $V$, of this solid is given by

$$V = k\int_0^{10} x^2 e^{-x}\,dx$$

where $k$ is a constant to be found.

**(2)**

(b) Find $\int x^2 e^{-x}\,dx$

**(3)**

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per $cm^3$ to 3 significant figures.

**(5)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9.** Use proof by contradiction to show that, when $n$ is an integer,

$$n^2 - 2$$

is **never** divisible by 4

**(4)**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 4 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** A curve $C$ has parametric equations

$$x = \frac{t}{t-3} \qquad y = \frac{1}{t} + 2 \qquad t \in \mathbb{R} \quad t > 3$$

Show that all points on $C$ lie on the curve with Cartesian equation

$$y = \frac{ax - 1}{bx}$$

where $a$ and $b$ are constants to be found.

**(3)**

**Question 1 continued**

**(Total for Question 1 is 3 marks)**

**2.** (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

**(3)**

(b) Hence show that

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,\mathrm{d}x = \ln k$$

where $k$ is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(4)**

**Question 2 continued**

**(Total for Question 2 is 7 marks)**

**3.**

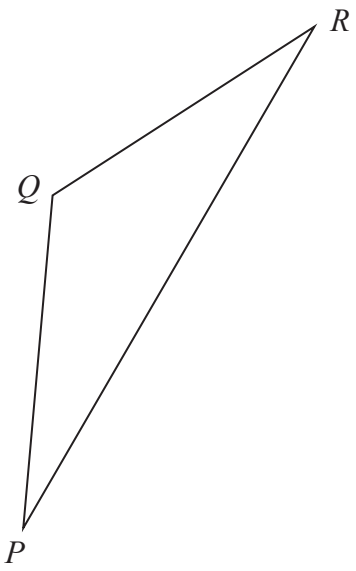



**Figure 1**

Figure 1 shows a sketch of triangle $PQR$.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$

**(2)**

(b) Find the size of angle $PQR$, in degrees, to three significant figures.

**(3)**

**Question 3 continued**

**(Total for Question 3 is 5 marks)**

**4.**

$$g(x) = \frac{1}{\sqrt{4 - x^2}}$$

(a) Find, in ascending powers of $x$, the first four non-zero terms of the binomial expansion of $g(x)$. Give each coefficient in simplest form.

**(5)**

(b) State the range of values of $x$ for which this expansion is valid.

**(1)**

(c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear.

**(2)**

**Question 4 continued**

**(Total for Question 4 is 8 marks)**

**5.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

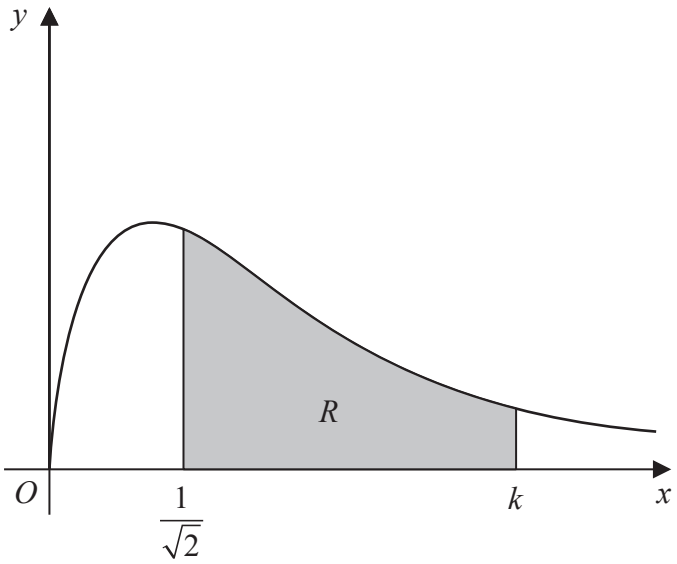



**Figure 2**

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region $R$, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the $x$-axis and the line with equation $x = k$.

This region is rotated through 360° about the $x$-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant $k$.

**(6)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 6 marks)**

**6.**

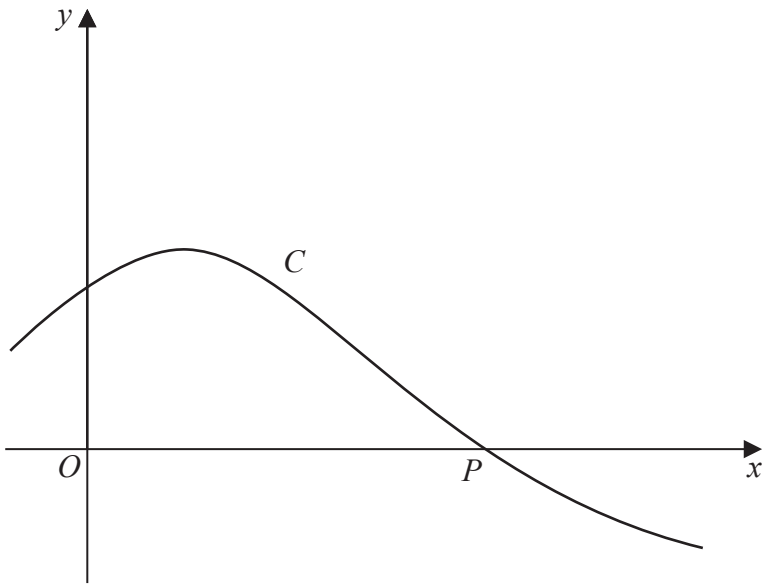



**Figure 3**

Figure 3 shows a sketch of the curve $C$ with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the $x$-axis at point $P$, as shown in Figure 3.

(a) Find the equation of the tangent to $C$ at $P$, writing your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(5)**

The curve $C$ has equation $y = \mathrm{f}(x)$, where f is a function with domain $\left[k, 1 + 3\sqrt{3}\right]$

(b) Find the exact value of the constant $k$.

**(1)**

(c) Find the range of f.

**(2)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 8 marks)**

**7.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = a + b\ln 2$$

where $a$ and $b$ are constants to be found.

**(7)**

(ii) Show, by integration, that

$$\int 3e^x \cos 2x\,dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where $p$ and $q$ are constants to be found and $c$ is an arbitrary constant.

**(5)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8.** A student was asked to prove by contradiction that

"there are no positive integers $x$ and $y$ such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers $x$ and $y$ exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{aligned} 3x - y &= 1 \\ x + y &= 25 \end{aligned}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5$$ Not integers

Show the calculations and statements that are needed to complete the proof.

**(4)**

**Question 8 continued**

**(Total for Question 8 is 4 marks)**

**9.** With respect to a fixed origin $O$, the equations of lines $l_1$ and $l_2$ are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where $\lambda$ and $\mu$ are scalar parameters.

Prove that lines $l_1$ and $l_2$ are skew.

**(5)**

**Question 9 continued**

**(Total for Question 9 is 5 marks)**

**10.** A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- $t$ minutes after the ball of ice is placed in the bucket, its radius is $r$ cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking $r$ and $t$,

**(5)**

(b) find the time taken for the ball of ice to melt completely.

**(2)**

(c) On Diagram 1 on page 27, sketch a graph of $r$ against $t$.

**(1)**

**Question 10 continued**

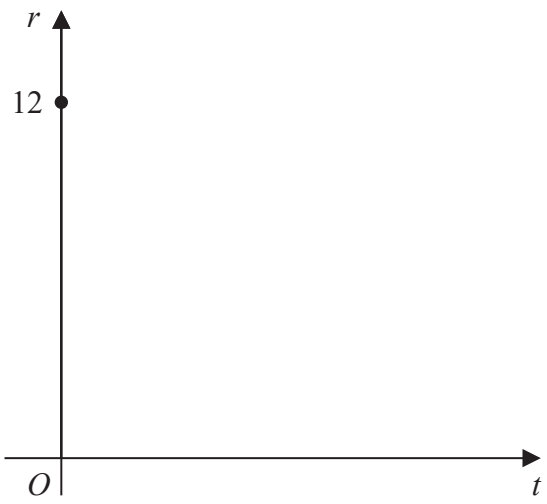



**Diagram 1**

**Question 10 continued**

**Question 10 continued**

**(Total for Question 10 is 8 marks)**

**11.**

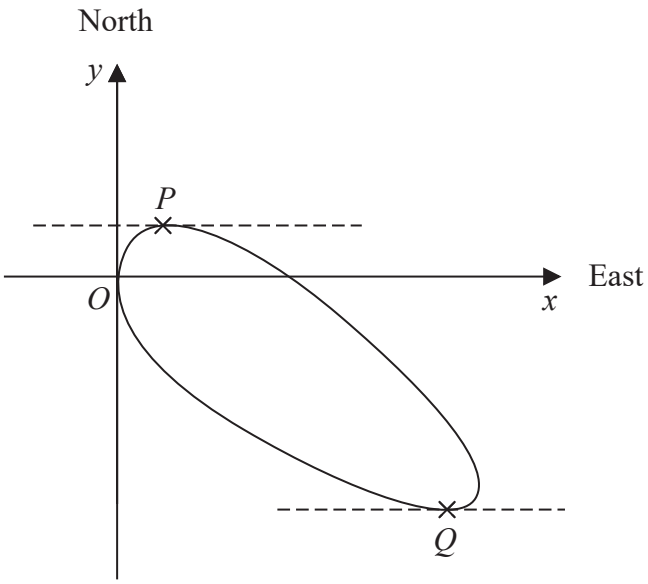



**Figure 4**

Figure 4 shows a sketch of the closed curve with equation

$$(x+y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$$

**(5)**

The curve is used to model the shape of a cycle track with both $x$ and $y$ measured in km.

The points $P$ and $Q$ represent points that are furthest north and furthest south of the origin $O$, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point $Q$ is south of $O$. Give your answer to the nearest 100 m.

**(4)**

**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 9 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** $$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

(a) Write $f(x)$ in partial fraction form.

**(3)**

(b) (i) Hence find, in ascending powers of $x$ up to and including the terms in $x^2$, the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.

**(5)**

(ii) Find the range of values of $x$ for which this expansion is valid.

**(1)**

**Question 1 continued**

**Question 1 continued**

**Question 1 continued**

**(Total for Question 1 is 9 marks)**

**2.** A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

**(4)**

(b) Hence or otherwise, find the $x$ coordinate of the point of intersection of this line and the line with equation $y = x + 12$

**(2)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3.**

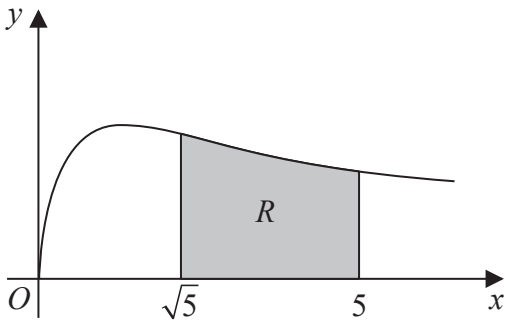



**Figure 1**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region $R$ is rotated through 360° about the $x$-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where $a$ is an irrational number and $b$ is a prime number.

**(5)**

**Question 3 continued**

**(Total for Question 3 is 5 marks)**

**4.** (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

may be expressed in the form

$$\int_a^b ku^2 e^u\, du$$

where $a$, $b$ and $k$ are constants to be found.

**(4)**

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

giving your answer in simplest form.

**(5)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 9 marks)**

**5.**

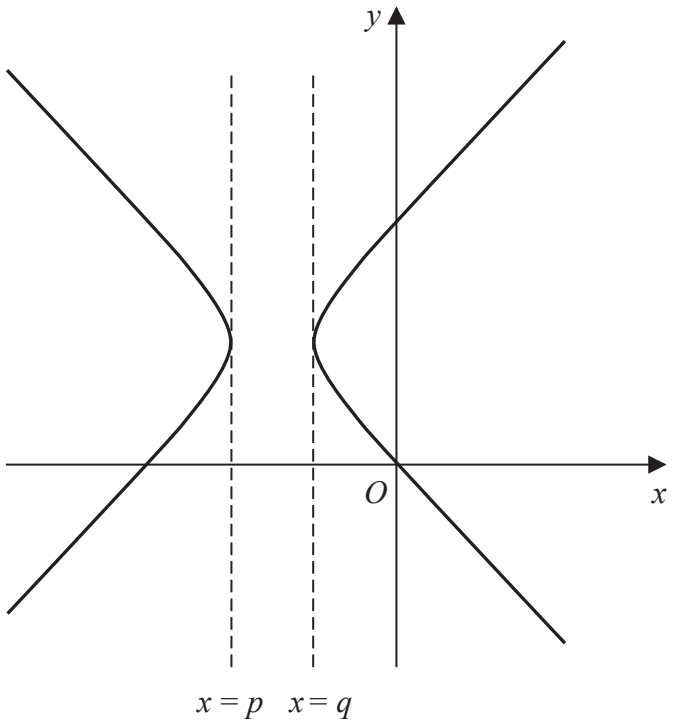



**Figure 2**

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(4)**

The curve is not defined for values of $x$ in the interval $(p, q)$, as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of $p$ and the value of $q$.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(3)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 7 marks)**

**6.** Relative to a fixed origin $O$.

- the point $A$ has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line $l$ passes through $A$ and $B$.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line $l$

**(3)**

The point $C$ has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point $P$ lies on $l$

Given that $\overrightarrow{CP}$ is perpendicular to $l$

(b) find the position vector of the point $P$

**(5)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 8 marks)**

**7.** The volume $V\,\text{cm}^3$ of a spherical balloon with radius $r\,\text{cm}$ is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ giving your answer in simplest form.

**(1)**

At time $t$ seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

**(5)**

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

**(3)**

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8.**

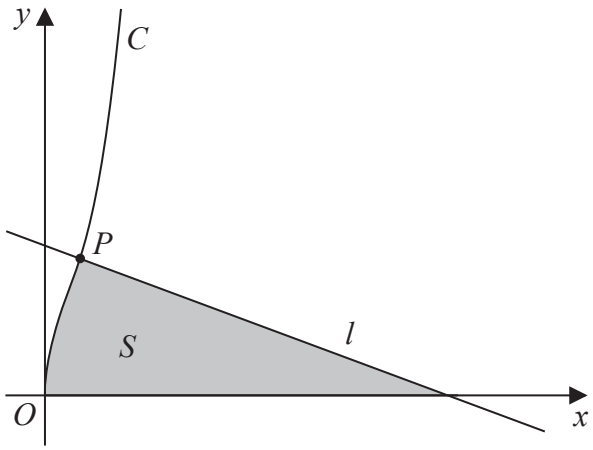



**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A curve $C$ has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point $P$ with parameter $t = \frac{\pi}{4}$ lies on $C$.

The line $l$ is the normal to $C$ at $P$, as shown in Figure 3.

(a) Show, using calculus, that an equation for $l$ is

$$8y + 2x = 17$$

**(5)**

The region $S$, shown shaded in Figure 3, is bounded by $C$, $l$ and the $x$-axis.

(b) Find, using calculus, the exact area of $S$.

**(6)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 11 marks)**

**9.** A student was asked to prove, for $p \in \mathbb{N}$, that

"if $p^3$ is a multiple of 3, then $p$ must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

Assumption:

There exists a number $p$, $p \in \mathbb{N}$, such that $p^3$ is a multiple of 3, and $p$ is NOT a multiple of 3

Let $p = 3k + 1$, $k \in \mathbb{N}$.

Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$

$= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

**(3)**

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

**(5)**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 8 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Friday 9 June 2023

Afternoon (Time: 1 hour 30 minutes)

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** (a) Find the first 4 terms of the binomial expansion, in ascending powers of $x$, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

**(5)**

Given that

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{n}\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$$

(b) write down the value of $n$.

**(1)**

(c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of $x$, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

**(3)**

**Question 1 continued**

**Question 1 continued**

**Question 1 continued**

**(Total for Question 1 is 9 marks)**

**2.**

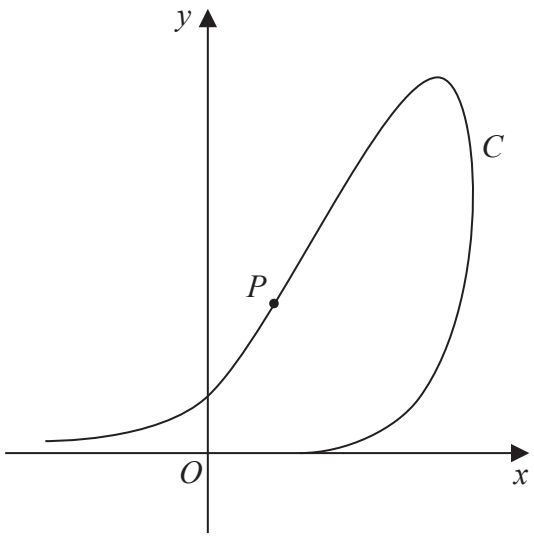



**Figure 1**

Figure 1 shows a sketch of part of the curve $C$ with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point $P$ lies on $C$ and has $x$ coordinate 2

(a) Find the $y$ coordinate of $P$.

**(2)**

(b) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$.

**(5)**

The tangent to $C$ at $P$ crosses the $x$-axis at the point $Q$.

(c) Find the $x$ coordinate of $Q$, giving your answer in the form $\dfrac{a\ln 2 + b}{c\ln 2 + d}$ where $a$, $b$, $c$ and $d$ are integers to be found.

**(3)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 10 marks)**

**3.** $$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

**(3)**

(b) Hence find $\int f(x)\,dx$

**(3)**

(c) Use the answer to part (b) to find the value of $k$ for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

**(5)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 11 marks)**

**4.** Relative to a fixed origin $O$,

- the point $A$ has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point $B$ has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point $P$ has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line $l$ passes through $A$ and $B$.

(a) Find a vector equation for $l$.

**(2)**

The point $C$ lies on $l$ so that $PC$ is perpendicular to $l$.

(b) Find the coordinates of $C$.

**(4)**

The point $P'$ is the reflection of $P$ in the line $l$.

(c) Find the coordinates of $P'$

**(2)**

(d) Hence find $\left|\overrightarrow{PP'}\right|$, giving your answer as a simplified surd.

**(2)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 10 marks)**

**5.** (i) Find

$$\int x^2 e^x dx$$

**(4)**

(ii) Use the substitution $u = \sqrt{1-3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} dx = -2(1-3x)^{\frac{1}{2}}(Ax + B) + k$$

where $A$ and $B$ are integers to be found and $k$ is an arbitrary constant.

**(6)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 10 marks)**

**6.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The temperature, $\theta$ °C, of a car engine, $t$ minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where $k$ is a constant.

Given that the temperature of the car engine
- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where $a$, $b$, $c$ and $d$ are integers to be found.

**(7)**

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

**(2)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 9 marks)**

**7.** Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if $k$ is an integer and $k^2$ is a multiple of 7 then $k$ is a multiple of 7)

**(4)**

**Question 7 continued**

**(Total for Question 7 is 4 marks)**

**8.**

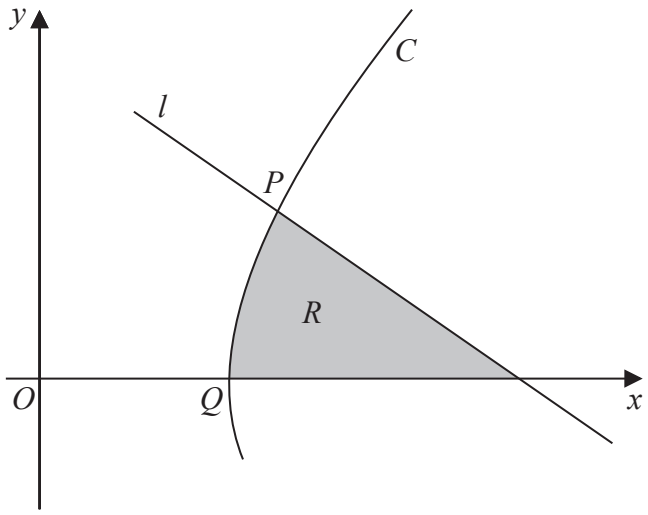



**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve $C$ intersects the $x$-axis at the point $Q$.

(a) Find the $x$ coordinate of $Q$.

**(1)**

The line $l$ is the normal to $C$ at the point $P$ as shown in Figure 2.

Given that $t = 2$ at $P$

(b) write down the coordinates of $P$

**(1)**

(c) Using calculus, show that an equation of $l$ is

$$3x + 5y = 15$$

**(3)**

The region, $R$, shown shaded in Figure 2 is bounded by the curve $C$, the line $l$ and the $x$-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region $R$ is rotated through $2\pi$ radians about the $x$-axis.

**(7)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 12 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Monday 23 October 2023

Afternoon (Time: 1 hour 30 minutes)

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** (a) Find the first four terms, in ascending powers of $x$, of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

**(4)**

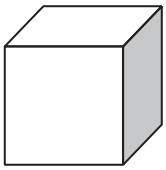
(b) Find the range of values of $x$ for which this expansion is valid.

**(1)**

**Question 1 continued**

**(Total for Question 1 is 5 marks)**

**2.**



$x$ cm

**Figure 1**

Figure 1 shows a cube which is increasing in size.

At time $t$ seconds,

- the length of each edge of the cube is $x$ cm
- the surface area of the cube is $S$ cm$^2$
- the volume of the cube is $V$ cm$^3$

Given that the surface area of the cube is increasing at a constant rate of $4\,\text{cm}^2\,\text{s}^{-1}$

(a) show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{k}{x}$ where $k$ is a constant to be found,

**(4)**

(b) show that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = V^p$ where $p$ is a constant to be found.

**(3)**

**Question 2 continued**

**(Total for Question 2 is 7 marks)**

**3.** **In this question you must show all stages of your working.**

**Solutions based on calculator technology are not acceptable.**

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x}\, dx$$

giving your answer in simplest form.

**(5)**

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2}\, dx = a + \ln b$$

where $a$ and $b$ are constants to be found.

**(7)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 12 marks)**

**4.** (a) Prove by contradiction that for all positive numbers $k$

$$k + \frac{9}{k} \geqslant 6$$

**(4)**

(b) Show that the result in part (a) is not true for all real numbers.

**(1)**

**Question 4 continued**

**(Total for Question 4 is 5 marks)**

**5.**

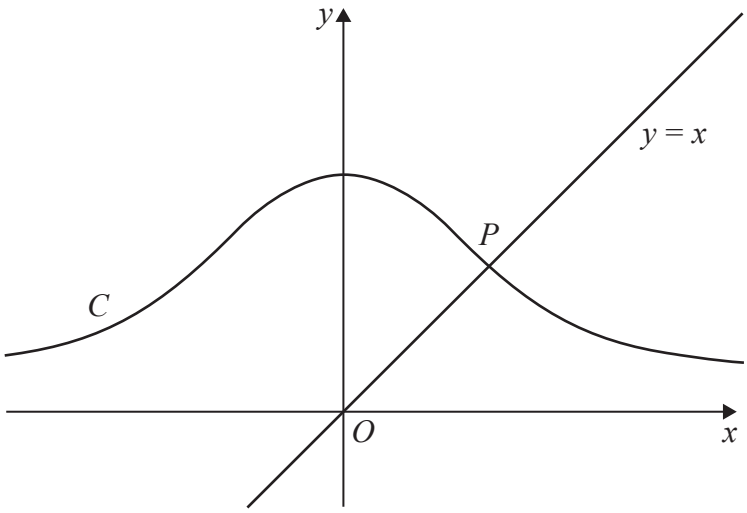



**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation

$$y^3 - x^2 + 4x^2y = k$$

where $k$ is a positive constant greater than 1

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(5)**

The point $P$ lies on $C$.

Given that the normal to $C$ at $P$ has equation $y = x$, as shown in Figure 2,

(b) find the value of $k$.

**(5)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 10 marks)**

**6.** The line $l_1$ has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where $\lambda$ is a scalar parameter.

The line $l_2$ has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where $\mu$ is a scalar parameter.

Given that $l_1$ and $l_2$ meet at the point $P$

(a) state the coordinates of $P$

**(1)**

Given that the angle between lines $l_1$ and $l_2$ is $\theta$

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

**(3)**

The point $Q$ lies on $l_1$ where $\lambda = 6$

Given that point $R$ lies on $l_2$ such that triangle $QPR$ is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle $QPR$

**(3)**

(d) find the coordinates of the possible positions of point $R$

**(3)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 10 marks)**

**7.** The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, $x$, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where $k$ is a constant and $t$ is the number of years after monitoring began.

(a) Show that $k = 1.8$

**(2)**

(b) Hence calculate the long-term population of goats predicted by this model.

**(1)**

In a **second** model, the number of goats, $x$, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

**(3)**

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

**(5)**

(e) Find the long-term population of goats predicted by this **second** model.

**(1)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8.**

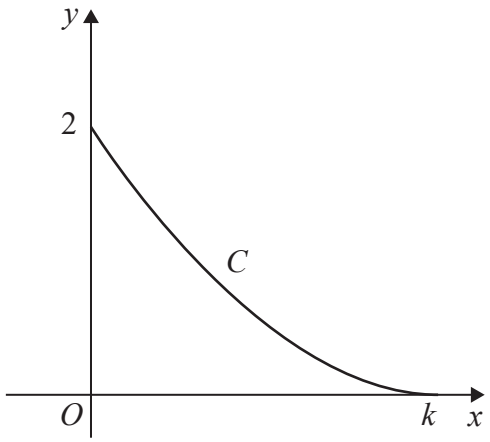



**Figure 3**

Figure 3 shows a sketch of the curve $C$ with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the $y$-axis at 2 and the $x$-axis at $k$, where $k$ is a constant.

(a) State the value of $k$.

**(1)**

(b) Use parametric differentiation to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \operatorname{cosec} t$$

where $\lambda$ is a constant to be found.

**(4)**

The point $P$ with parameter $t = \frac{\pi}{4}$ lies on $C$.

The tangent to $C$ at the point $P$ cuts the $y$-axis at the point $N$.

(c) Find the exact $y$ coordinate of $N$, giving your answer in simplest form.

**(3)**

The region bounded by the curve, the $x$-axis and the $y$-axis is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta(1 - \cos 4t)\,\mathrm{d}t$$

where $\alpha$ and $\beta$ are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

**(6)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 14 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Thursday 18 January 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

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- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** Find, in ascending powers of $x$ up to and including the term in $x^3$, the binomial expansion of

$$(1 - 4x)^{-3} \qquad |x| < \frac{1}{4}$$

fully simplifying each term.

**(4)**

**Question 1 continued**

**(Total for Question 1 is 4 marks)**

**2.** Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants $A$, $B$ and $C$.

**(4)**

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,\mathrm{d}x$$

giving your answer in the form $p\ln q + r$ where $p$, $q$ and $r$ are rational numbers.

**(6)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 10 marks)**

**3.**

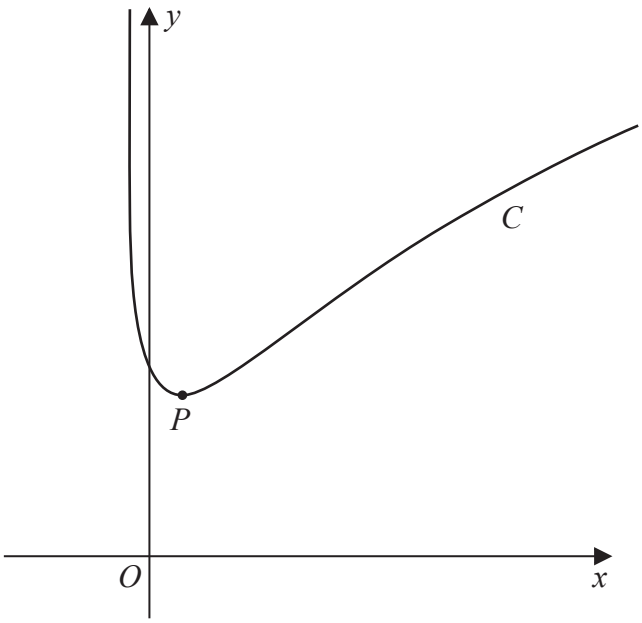


**Figure 1**

The curve $C$, shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \qquad y > 0$$

where $k$ is a constant.

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$

**(5)**

The point $P(p, 2)$, where $p$ is a constant, lies on $C$.

Given that $P$ is the minimum turning point on $C$,

(b) find

(i) the value of $p$

(ii) the value of $k$

**(4)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4.**

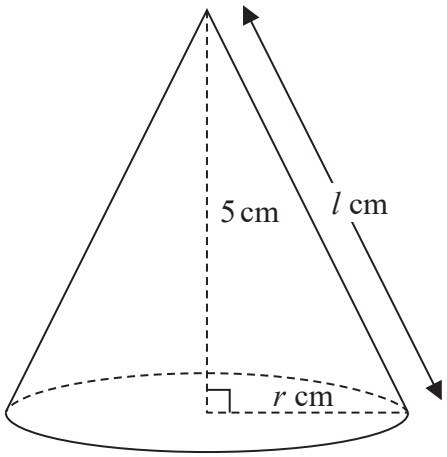


**Figure 2**

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius $r$ cm
- slant height $l$ cm

(a) Find an expression for $l$ in terms of $r$

**(1)**

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in $cm^2$ per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi rl$]

**(4)**

**Question 4 continued**

**(Total for Question 4 is 5 marks)**

**5.** (a) Find $\int x^2 \cos 2x \, dx$

**(4)**

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left( \frac{t \cos t}{y} \right)^2$$

giving your answer in the form $y^n = f(t)$ where $n$ is an integer.

**(5)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 9 marks)**

**6.** Relative to a fixed origin $O$, the lines $l_1$ and $l_2$ are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where $\lambda$ and $\mu$ are scalar parameters and $p$ is a constant.

Given that $l_1$ and $l_2$ intersect,

(a) find the value of $p$,

**(4)**

(b) find the position vector of the point of intersection.

**(2)**

(c) Find the acute angle between $l_1$ and $l_2$

Give your answer in degrees to one decimal place.

**(3)**

The point $A$ lies on $l_1$ with parameter $\lambda = 2$

The point $B$ lies on $l_2$ with $\overrightarrow{AB}$ perpendicular to $l_2$

(d) Find the coordinates of $B$

**(5)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 14 marks)**

**7.** (a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x\,dx = \frac{1}{8}(e^{2\pi} - 1)$$

**(5)**

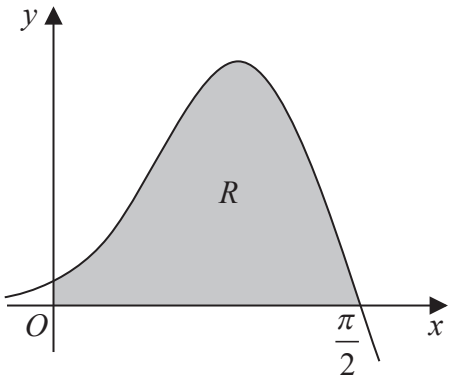


**Figure 3**

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x}\cos x$$

The region $R$, shown shaded in Figure 3, is bounded by the positive $x$-axis, the positive $y$-axis and the curve.

The region $R$ is rotated through $2\pi$ radians about the $x$-axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 8 marks)**

**8.** Use proof by contradiction to prove that the curve with equation

$$y = 2x + x^3 + \cos x$$

has no stationary points.

**(4)**

**Question 8 continued**

**(Total for Question 8 is 4 marks)**

**9.**

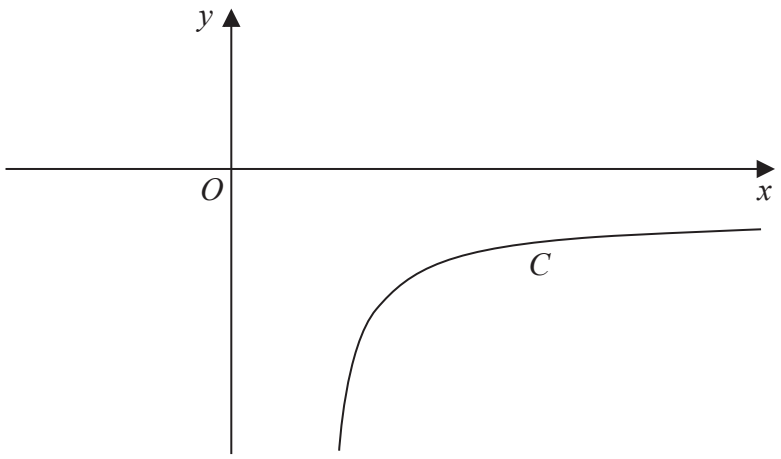


**Figure 4**

Figure 4 shows a sketch of the curve $C$ with parametric equations

$$x = \sec t \qquad y = \sqrt{3}\tan\left(t + \frac{\pi}{3}\right) \qquad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of $t$

**(3)**

(b) Find an equation for the tangent to $C$ at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where $m$ and $c$ are constants.

**(4)**

(c) Show that all points on $C$ satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where $A$ and $B$ are constants to be found.

**(5)**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 12 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Thursday 6 June 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
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- Answer the questions in the spaces provided
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

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- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**1.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

Find

$$\int_0^{\frac{\pi}{6}} x\cos 3x \,\mathrm{d}x$$

giving your answer in simplest form.

**(5)**

**Question 1 continued**

**(Total for Question 1 is 5 marks)**

**2.** With respect to a fixed origin, $O$, the point $A$ has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point $B$.

**(2)**

The point $C$ has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where $a$ is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of $a$.

**(4)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3.** The curve $C$ is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to $C$ at the point (2, 5), giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(7)**

**Question 3 continued**

**(Total for Question 3 is 7 marks)**

**4.**

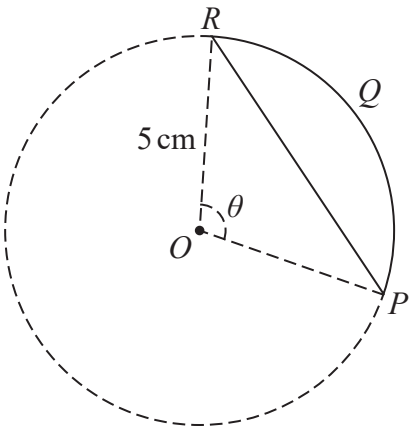


**Figure 1**

Figure 1 shows a sketch of a segment $PQRP$ of a circle with centre $O$ and radius 5 cm.

Given that

- angle $POR$ is $\theta$ radians
- $\theta$ is increasing, from 0 to $\pi$, at a constant rate of 0.1 radians per second
- the area of the segment $PQRP$ is $A\,\text{cm}^2$

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K(1 - \cos\theta)$$

where $K$ is a constant to be found.

**(2)**

(b) Find, in $\text{cm}^2\,\text{s}^{-1}$, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

**(4)**

**Question 4 continued**

**(Total for Question 4 is 6 marks)**

**5.**

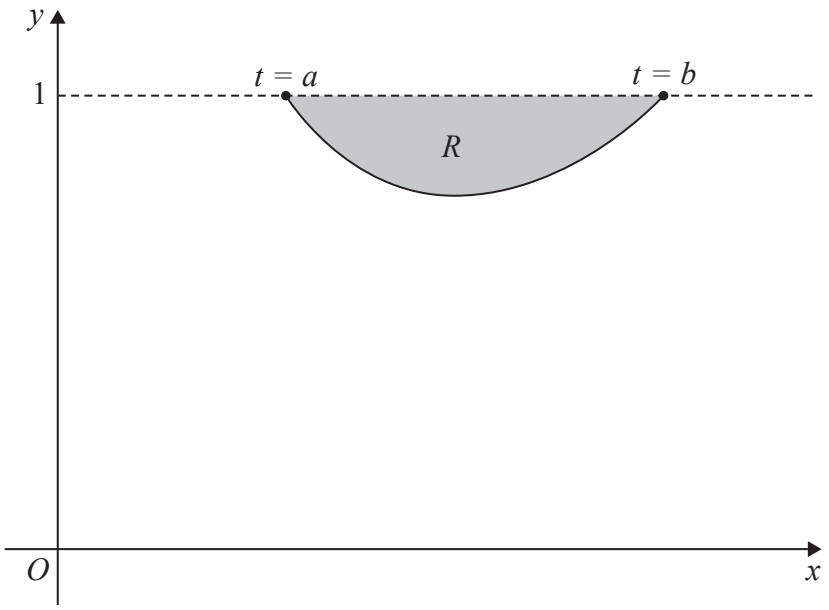


**Figure 2**

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where $a$ and $b$ are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of $a$ and the value of $b$

**(2)**

The region $R$, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region $R$ is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where $M$ and $k$ are constants to be found.

**(5)**

(c) (i) Write $\dfrac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of $R$, giving your answer in simplest form.

**(6)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 13 marks)**

**6.** With respect to a fixed origin $O$, the line $l_1$ is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $A$ lies on $l_1$

Given that $|\overrightarrow{OA}| = 5\sqrt{10}$

(a) show that at $A$ the parameter $\lambda$ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

**(3)**

Hence

(b) (i) show that one possible position vector for $A$ is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for $A$.

**(3)**

The line $l_2$ is parallel to $l_1$ and passes through $O$.

Given that

- $\overrightarrow{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point $B$ lies on $l_2$ where $|\overrightarrow{OB}| = 4\sqrt{10}$

(c) find the area of triangle $OAB$, giving your answer to one decimal place.

**(4)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 10 marks)**

**7.** The current, $x$ amps, at time $t$ seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where $k$ is a constant.

Initially there is zero current in the circuit.

(a) Solve the differential equation to find an equation, in terms of $k$, for the current in the circuit at time $t$ seconds.
Give your answer in the form $x = f(t)$.

**(6)**

Given that in the long term the current in the circuit approaches 7 amps,

(b) find the value of $k$.

**(2)**

(c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 11 marks)**

**8.** $f(x) = (8 - 3x)^{\frac{4}{3}} \qquad 0 < x < \frac{8}{3}$

(a) Show that the binomial expansion of $f(x)$ in ascending powers of $x$ up to and including the term in $x^3$ is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \ldots$$

where $A$ and $B$ are constants to be found.

**(4)**

(b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \qquad 0 < x < \frac{8}{3}$$

where $A$ and $B$ are the constants found in part (a).

(*Solutions relying on calculator technology are not acceptable.*)

**(4)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 8 marks)**

**9.**

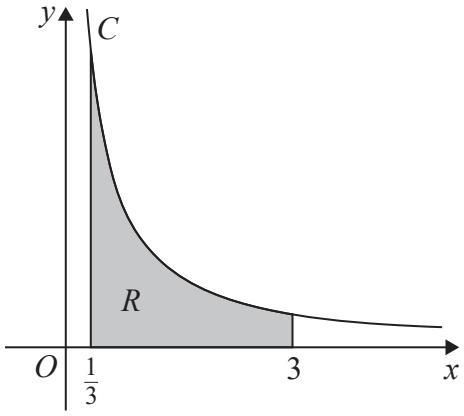


**Figure 3**

The curve $C$, shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\left(\arctan\sqrt{x}\right)}$$

The region $R$, shown shaded in Figure 3, is bounded by $C$, the line with equation $x = 3$, the $x$-axis and the line with equation $x = \frac{1}{3}$

The region $R$ is rotated through 360° about the $x$-axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume $V$ of the solid formed is given by

$$k\int_a^b \frac{1}{u^2}\,\mathrm{d}u$$

where $k$, $a$ and $b$ are constants to be found.

**(6)**

(b) Hence, using algebraic integration, find the value of $V$ in simplest form.

**(3)**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 9 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Monday 28 October 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

**1.** (a) Find the first 4 terms of the binomial expansion, in ascending powers of $x$, of

$$(8 - 3x)^{-\frac{1}{3}} \qquad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

**(4)**

(b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

**(2)**

**Question 1 continued**

**(Total for Question 1 is 6 marks)**

**2.** **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

The curve $C_1$ has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve $C_2$ has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that $C_1$ and $C_2$ do **not** intersect.

**(4)**

**Question 2 continued**

**(Total for Question 2 is 4 marks)**

**3.**

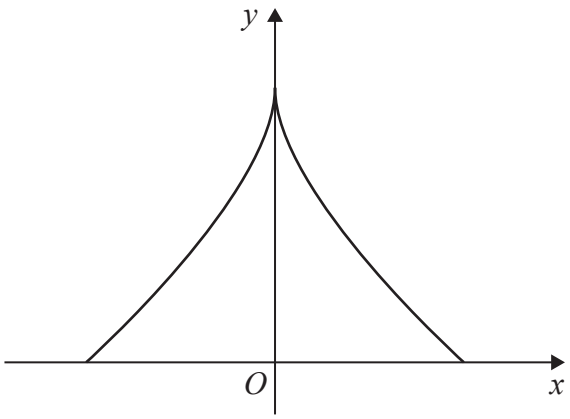


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with parametric equations

$$x = 3\sin^3\theta \qquad y = 1 + \cos 2\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k\,\mathrm{cosec}\,\theta \qquad \theta \neq 0$$

where $k$ is a constant to be found.

**(3)**

The point $P$ lies on $C$ where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to $C$ at $P$, giving your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**(3)**

(c) Show that $C$ has Cartesian equation

$$8x^2 = 9(2 - y)^3 \qquad -q \leqslant x \leqslant q$$

where $q$ is a constant to be found.

**(3)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4.**

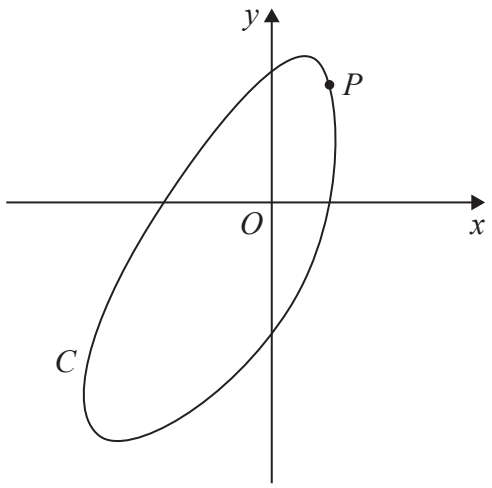


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation

$$3x^2 + 2y^2 - 4xy + 8^x - 11 = 0$$

The point $P$ has coordinates $(1, 2)$.

(a) Verify that $P$ lies on $C$.

**(1)**

(b) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(5)**

The normal to $C$ at $P$ crosses the $x$-axis at a point $Q$.

(c) Find the $x$ coordinate of $Q$, giving your answer in the form $a + b\ln 2$ where $a$ and $b$ are integers.

**(3)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 9 marks)**

**5.**

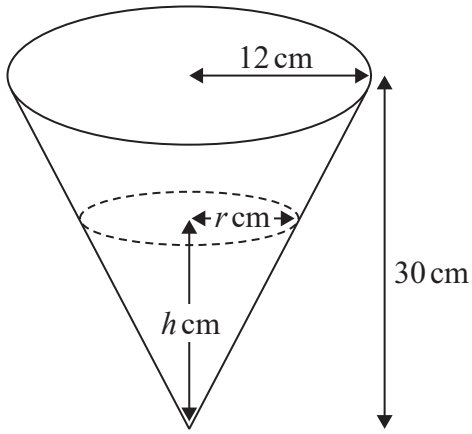


**Figure 3**

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is $h$ cm, the surface of the water has radius $r$ cm and the volume of water is $V$ cm$^3$

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume $V$ of a right circular cone with vertical height $h$ and base radius $r$ is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

**(2)**

Given that water flows into the container at a constant rate of $2\pi$ cm$^3$ s$^{-1}$

(b) find, in cm s$^{-1}$, the rate at which $h$ is changing, exactly 1.5 **minutes** after water starts flowing into the container.

**(4)**

**Question 5 continued**

**(Total for Question 5 is 6 marks)**

**6.** Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} \mathrm{d}x = 2\left(x^3 + 1\right)^k \left(x^3 - A\right) + c$$

where $k$ and $A$ are constants to be found and $c$ is an arbitrary constant.

**(5)**

**Question 6 continued**

**(Total for Question 6 is 5 marks)**

**7.**

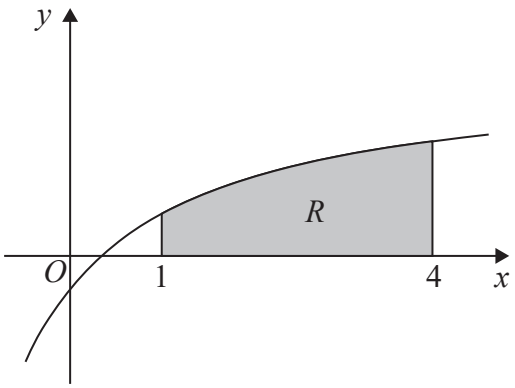


**Figure 4**

Figure 4 shows a sketch of part of the curve with equation

$$y = \frac{3x - 1}{x + 2} \qquad x > -2$$

(a) Show that

$$\frac{3x - 1}{x + 2} \equiv A + \frac{B}{x + 2}$$

where $A$ and $B$ are constants to be found.

**(2)**

The finite region $R$, shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 4$, the $x$-axis and the line with equation $x = 1$

This region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Use the answer to part (a) and algebraic integration to find the exact volume of the solid generated, giving your answer in the form

$$\pi(p + q \ln 2)$$

where $p$ and $q$ are rational constants.

**(6)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 8 marks)**

**8.** Relative to a fixed origin $O$

- the point $A$ has coordinates $(-10, 5, -4)$
- the point $B$ has coordinates $(-6, 4, -1)$

The straight line $l_1$ passes through $A$ and $B$.

(a) Find a vector equation for $l_1$

**(2)**

The line $l_2$ has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where $p$ and $q$ are constants and $\mu$ is a scalar parameter.

Given that $l_1$ and $l_2$ intersect at $B$,

(b) find the value of $p$ and the value of $q$.

**(3)**

The acute angle between $l_1$ and $l_2$ is $\theta$

(c) Find the exact value of $\cos\theta$

**(3)**

Given that the point $C$ lies on $l_2$ such that $AC$ is perpendicular to $l_2$

(d) find the exact length of $AC$, giving your answer as a surd.

**(2)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9.** (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

**(2)**

The height above ground, $h$ metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where $t$ seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where $k$ is a constant to be found.

**(6)**

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(2)**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 10 marks)**

**10.**

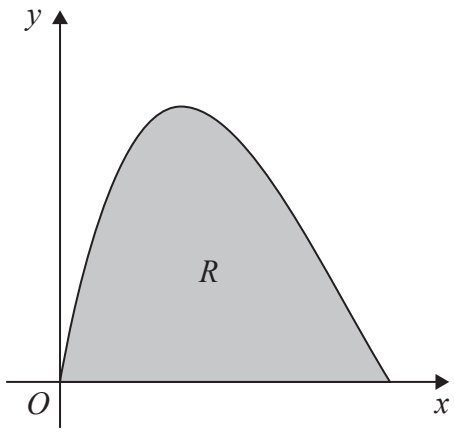


**Figure 5**

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region $R$, shown shaded in Figure 5, is bounded by the curve and the $x$-axis.

(a) Show that the area of $R$ is

$$k\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t\,\mathrm{d}t$$

where $k$ is a constant to be found.

**(3)**

(b) Hence, using algebraic integration, find the exact area of $R$, giving your answer in the form

$$p\pi + q$$

where $p$ and $q$ are constants.

**(5)**

**Question 10 continued**

**Question 10 continued**

**(Total for Question 10 is 8 marks)**

**TOTAL FOR PAPER IS 75 MARKS**