Question	Scheme	Marks
1(a)	$A = \frac{1}{9}$	B1
		(1)
(b)	$"3^{-2}"\left(1 + (-2)\left(\frac{kx}{3}\right) + \frac{(-2)(-3)}{2}\left(\frac{kx}{3}\right)^2 + \ldots\right)$ or $x: \quad 3^{-2}(-2)\left(\frac{k}{3}\right)\left(= -\frac{2k}{27}\right)$ and $x^2: \quad 3^{-2}\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2 = \frac{k^2}{27}$	B1
	$\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2 = 3\times(-2)\left(\frac{k}{3}\right) \Rightarrow \ldots k^2 = \ldots k$	M1
	$k^2 + 6k = 0$ *	A1*
		(3)
(c)(i)	$k = -6$	B1
(ii)	$3^{-2}\frac{(-2)(-3)(-4)}{3!}\left(\frac{"-6"}{3}\right)^3 = \frac{32}{9}$	M1A1
		(3)
		(7 marks)

Notes

Mark parts (a) and (b) as a whole.

(a)

B1: $A = \frac{1}{9}$

(b)

B1: Correct unsimplified coefficients for x and x^2 either in an expansion or separate for $(3 + kx)^{-2}$ or for $\left(1 + \frac{k}{3}\right)^{-2}$ (accept the 3^{-2} missing or incorrect). May be implied. Accept $B = -\frac{2k}{3}$ and $C = \frac{k^2}{3}$ if they forget the multiple outside. B0 if brackets on $\left(\frac{k}{3}\right)^2$ missing unless implied by recovery.

M1: Sets their coefficient of x^2 equal to 3 times their coefficient of x to produce a two term quadratic equation in terms of k.

A1*: Achieves given answer from a correct equation, but condone if B and C both missed the 3^{-2}. May be scored if A was incorrect.

(c)(i)

B1: $k = -6$ only. The $k = 0$ solution must be rejected.

(ii)

M1: Substitutes their non-zero value for k into a correct expression for the coefficient of x^3. Must include the 3^{-2}

A1: $\frac{32}{9}$ oe

Question	Scheme	Marks
2(a)	$\left(\frac{1}{(1+3x)(1-x)}=\frac{A}{1+3x}+\frac{B}{1-x}\Rightarrow\right)1=A(1-x)+B(1+3x)$	B1
	when $x=1\ \Rightarrow 1=4B\Rightarrow B=...$ or when $x=-\frac{1}{3}\ \Rightarrow 1=\frac{4}{3}A\Rightarrow A=...$	M1
	$\frac{3}{4(1+3x)}+\frac{1}{4(1-x)}$	A1
		(3)
(b)	$\int\cot y\ \mathrm{d}y=\int...\ \mathrm{d}x\Rightarrow$ "$\ln\sin y$" $=\int...\ \mathrm{d}x$	M1
	$...=\int\left(\text{"}\frac{3}{4(1+3x)}\text{"+"}\frac{1}{4(1-x)}\text{"}\right)\mathrm{d}x=...\ln(1+3x)\pm...\ln(1-x)\ \ (+c)$	M1
	$\ln\sin y=\frac{1}{4}\ln(1+3x)-\frac{1}{4}\ln(1-x)\ \ (+c)$ oe	A1ft
	$\ln\sin\left(\frac{\pi}{2}\right)=\frac{1}{4}\ln\left(1+3\times\frac{1}{2}\right)-\frac{1}{4}\ln\left(1-\frac{1}{2}\right)+c\Rightarrow c=...\left(=-\frac{1}{4}\ln 5\right)$	dM1
	$k\ln\sin y=m\ln(...)\Rightarrow\sin^k y=...^m$ or $k\ln\sin y=...\Rightarrow\sin^k y=\exp(...)$	M1
	$\sin^4 y=\frac{1+3x}{5(1-x)}$	A1
		(6)
		(9 marks)

Notes

(a)

B1: For a correct suitable identity without fractions, such as $1=A(1-x)+B(1+3x)$, seen or implied.

M1: Attempts to find one of the constants by either substitution or equating coefficients. May be implied by a correct value for A or B via cover up rule.

A1: $\frac{3}{4(1+3x)}+\frac{1}{4(1-x)}$ oe allow values for A and B to be stated following a correct partial fraction form, or if correct partial fractions see in (b).

(b)

M1: Attempts to separate variables to form $\cot y\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{g}(x)$ (oe for cot y) and integrate $\cot y$. Accept any changed function for the attempt but must be attempting to integrate cot y (oe).

M1: Attempts to integrate their partial fractions from (a) so award for $\frac{...}{(1+3x)}\to...\ln(1+3x)$ or $...\ln(4+12x)$ and $\frac{...}{(1-x)}\to...\ln(1-x)$ or $...\ln(4-4x)$ oe

A1ft: Correct expression (any equivalent) (both sides). Follow through on their constants for the partial fractions. Condone the absence of the constant of integration.

dM1: Depends on second M, and must have attempted to integrate both sides. Uses the initial conditions in an equation with a constant of integration. May integrate between limits to achieve this. (Accept if a value for c cannot be reached from their equation.)

M1: Attempts to rearrange their equation by correctly using log work to reach the required form $\sin^n y=\mathrm{f}(x)$. Must have had $k$ln sin y = … (k may be 1). Not dependent - may be gained before finding the constant if ln A is used, and allow if the constant is missing.

A1: $\sin^4 y=\frac{1+3x}{5(1-x)}$ (oe in correct form)

Question	Scheme	Marks
3(a)	$\frac{dA}{dt} = -0.5$	B1
	$A = \pi x^2 \Rightarrow \frac{dA}{dx} = 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \frac{"-0.5"}{"2\pi x"} \quad \left(= \frac{-1}{4\pi x}\right)$	M1
	$\frac{dx}{dt} = -0.011368...$	A1cso
		(4)
(b)	$V = \pi x^2(3x) = 3\pi x^3$	B1
	$\frac{dV}{dx} = 9\pi x^2$	B1ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 9\pi x^2 \times "-\frac{1}{4\pi x}" \quad (= -2.25x)$	M1
	$\left(\frac{dV}{dt} =\right) -9 \Rightarrow$ (Rate of decrease $=$) 9 $\left(\text{mm}^3\ \text{s}^{-1}\right)$	A1
		(4)
		(8 marks)

Notes

(a)

B1: $\frac{dA}{dt} = -0.5$ seen or implied from working

B1: $\frac{dA}{dx} = 2\pi x$ seen or implied from working. Must be in terms of x, but allow recovery if in terms of r and later work uses $r = 7$ to achieve a solution.

M1: Attempts to use an appropriate chain rule with their $\frac{dA}{dt}$ and $\frac{dA}{dx}$ e.g. $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = ...$

A1: awrt -0.0114 or $-\frac{1}{28\pi}$ cso (must have the negative sign)

(b)

B1: $V = \pi x^2(3x)$ or $V = 3\pi x^3$

B1ft: $\frac{dV}{dx} = 9\pi x^2$ or ft from their equation for V in one variable

M1: Their $\frac{dV}{dx} \times$ their $\frac{dx}{dt}$. Note the $\frac{dx}{dt}$ must be in terms of x or with $x = 4$ substituted first, M0 if they use their answer to (a).

A1: (Rate of decrease $=$) 9 $\left(\text{mm}^3\ \text{s}^{-1}\right)$ (with or without the negative sign). May be scored following $\frac{dA}{dt} = 0.5$ in part (a)

Question	Scheme	Marks
4(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \to (8\times)3y^2\dfrac{dy}{dx}$	B1
	$-9kx^2y \to ...kxy \pm ... - 9kx^2\dfrac{dy}{dx}$	M1
	$48x^2 - 18kxy - 9kx^2\dfrac{dy}{dx} + 24y^2\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx}(24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \dfrac{dy}{dx} = ...$	M1
	$\dfrac{dy}{dx} = \dfrac{6kxy - 16x^2}{8y^2 - 3kx^2}$ *	A1*
		(4)
(b)	$\dfrac{dy}{dx} = 0, x = \dfrac{5}{2} \Rightarrow \dfrac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0$ or $x = \dfrac{5}{2} \Rightarrow 16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2 y + 8y^3 = 875$	M1
	$15ky - 100 = 0$ or $250 - \dfrac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2\left(\dfrac{20}{3k}\right) + 8\left(\dfrac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = ... \left(= \dfrac{64}{27}\right) \Rightarrow k = ...$	M1
	$k = \dfrac{4}{3}$	A1
		(4)
		(8 marks)

Notes

(a)

B1: For $y^3 \to 3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$. Allow if seen in aside working without the 8.

M1: Correct attempt at implicit differentiation on the $-9kx^2y$. Look for $-9kx^2y \to ...kxy \pm ... -9kx^2\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Collects both of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms together, collects non $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms the other side of the equation, factorises and divides to achieve $\frac{\mathrm{d}y}{\mathrm{d}x} = ...$ Must have two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms, one from the attempt at differentiating $-9kx^2y$ and one from the attempt at differentiating y^3, but condone if an extra $\frac{\mathrm{d}y}{\mathrm{d}x} = ...$ term has been included.

A1*: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$ with no errors

(b)

M1: Uses the information to produce one equation in k and y, e.g. sets the $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to 0 and substitutes $x = \frac{5}{2}$, or substitutes $x = \frac{5}{2}$ into the given equation. Allow one slip substituting.

A1: A correct equation without fraction and with simplified coefficients, so $15ky - 100 = 0$ oe or $250 - \frac{225}{4}ky + 8y^3 = 875$ oe

M1: For a complete method to find k so solves the equations simultaneously to achieve a value for k. May find y first e.g substitutes their $k = \frac{20}{3y}$ into the original equation, solves to find y and substitutes this back into $k = \frac{20}{3y}$ to find k via $250 - 375 + 8y^3 = 875 \Rightarrow y = 5 \Rightarrow k = \frac{20}{3 \times 5} = ...$

A1 $k = \frac{4}{3}$

Alt:

If they do not substitute $x = \frac{5}{2}$ initially then score

M1: Uses numerator of $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to 0 to find y in terms of x and k and substitute into original equation (allowing one slip)

A1: Correct equation:

$$6kxy - 16x^2 = 0 \Rightarrow y = \frac{8x^2}{3kx} \Rightarrow 16x^3 - 9kx^2\left(\frac{8x^2}{3kx}\right) + 8\left(\frac{8x^2}{3kx}\right)^3 = 875 \text{ oe}$$

M1: Substitutes $x = \frac{5}{2}$ and solves to find k

A1: $k = \frac{4}{3}$

Question	Scheme	Marks
5(a)	$1 = 2\sin u \Rightarrow p = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	B1
	$x = 2\sin u \Rightarrow \frac{dx}{du} = 2\cos u$ oe	M1
	$\int \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx = \int \frac{6\sin u + 2}{(4-4\sin^2 u)^{\frac{3}{2}}}\,2\cos u\,du = \int \frac{6\sin u+2}{(4\cos^2 u)^{\frac{3}{2}}}\,2\cos u\,du$	M1
	$= \int \frac{12\sin u}{8\cos^2 u} + \frac{2}{4\cos^2 u}\,du = \int_0^{\frac{\pi}{6}} \left(\frac{3}{2}\sec u\tan u + \frac{1}{2}\sec^2 u\right) du$ *	A1*
		(4)
(b)	$\int \left(\frac{3}{2}\sec u\tan u + \frac{1}{2}\sec^2 u\right) du = \frac{3}{2}\sec u + \frac{1}{2}\tan u$	M1A1
	$\left[\frac{3}{2}\sec u + \frac{1}{2}\tan u\right]_0^{"\frac{\pi}{6}"} = \left(\frac{3}{2}\sec\left("\frac{\pi}{6}"\right) + \frac{1}{2}\tan\left("\frac{\pi}{6}"\right)\right) - \left(\frac{3}{2}\sec 0 + \frac{1}{2}\tan 0\right) = \ldots$	M1
	$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2} = \frac{7\sqrt{3}}{6} - \frac{3}{2} \quad \left(= \frac{7\sqrt{3}-9}{6}\right)$	A1
		(4)
		(8 marks)

Notes

(a)

B1: $p = \frac{\pi}{6}$ Allow if seen anywhere, even in (b). $p = 30$ is B0.

M1: $x = 2\sin u \Rightarrow \frac{dx}{du} = \pm\ldots\cos u$ or any rearrangement of this equation.

M1: **Full** substitution from an integral in terms of x to an integral in terms of u and **uses** the identity $\sin^2 u + \cos^2 u = 1$ in the denominator. Do not be concerned with the limits for this mark.

A1*: Achieves given answer include du (with their p) with no errors and at least one intermediate step with the fractional power simplified. Condone missing du in intermediate lines.

(b)

M1: $\int \left(\frac{3}{2}\sec u\tan u + \frac{1}{2}\sec^2 u\right) du = \ldots\sec u + \ldots\tan u$

A1: $\frac{3}{2}\sec u + \frac{1}{2}\tan u$ ignore any constant c

M1: Depends on having one term of the correct form, attempts to substitute in their p ($\neq 1$) and 0, subtracting either way round. The substitution must be seen or clearly implied, e.g. by correct values for each term in an intermediate step before the answer (allowing missing 0's).

A1: $\frac{7\sqrt{3}}{6} - \frac{3}{2}$ or exact equivalent eg $\frac{7\sqrt{3}-9}{6}$ Allow if $p = 30^{\circ}$ was used.

Question	Scheme	Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}5-1\\3--4\\-2-3\end{pmatrix}=\begin{pmatrix}4\\7\\-5\end{pmatrix}=4\mathbf{i}+7\mathbf{j}-5\mathbf{k}$	M1
	e.g. $\mathbf{r}=\mathbf{i}-4\mathbf{j}+3\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$	M1A1
		(3)
(b)	$\overrightarrow{AC}=\begin{pmatrix}3-1\\p--4\\-1-3\end{pmatrix}=\begin{pmatrix}2\\p+4\\-4\end{pmatrix}=2\mathbf{i}+(p+4)\mathbf{j}-4\mathbf{k}$	M1
	$\begin{pmatrix}2\\p+4\\-4\end{pmatrix}\bullet\begin{pmatrix}4\\7\\-5\end{pmatrix}=8+7p+28+20=0\Rightarrow p=-8$	M1A1
		(3)
(c)	$\lvert AB\rvert=\sqrt{4^2+7^2+(-5)^2}=\sqrt{90}$ or $\lvert AC\rvert=\sqrt{2^2+(-4)^2+(-4)^2}=6$	M1
	Area $\frac{1}{2}\times"\sqrt{90}"\times"6"=9\sqrt{10}$	dM1A1
		(3)
		(9 marks)

Notes

Accept either vector form throughout but extra i, j k in column vectors will lose A mark in (a).
(a) This is now being marked MMA

M1: Attempts to find $\overrightarrow{AB}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

M1: Attempts equation for the line, score for $\overrightarrow{OA}+\lambda\times$their $\overrightarrow{AB}$ or $\overrightarrow{OB}+\lambda\times$their $\overrightarrow{AB}$ No need for **r** = for this mark.

A1: Any correct equation. Must be **r** = … (l = .. is A0)

(b)

M1: Attempts to find $\overrightarrow{AC}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

M1: Takes scalar product of their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$ to form and solve a linear equation in p

A1: $p=-8$

(c)

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$ using their p

dM1: Attempts to find the exact area of the triangle ABC. It is dependent on the previous method mark. There most common method will be $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|$ as in scheme but other methods are possible. E.g.

$\cos\angle ABC=\dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|}\Rightarrow A=\frac{1}{2}\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\sin\angle ABC$. Such a method must be complete, including use of Pythagorean identity to find $\sin\angle ABC$. Other more advanced methods (such as cross products) are also possible. If you see something you feel is worthy of some credit but does not fit the scheme, send to Review.

A1: $9\sqrt{10}$

Question	Scheme	Marks
7(a)	$\frac{dx}{dt}=\cos t+6\cos t\sin t \quad \frac{dy}{dt}=3\cos t-2\sin t$	B1B1
	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{3\cos t-2\sin t}{\cos t+6\cos t\sin t}=\frac{3\cos\pi-2\sin\pi}{\cos\pi+6\cos\pi\sin\pi}=3$ *	M1A1*
		(4)
(b)	When $t=\pi,\ x=-3,\ y=-2$	B1
	$y-"-2"=3(x-"-3")$	M1
	$y=3x+7$	A1
		(3)
(c)	$y=3x+7\Rightarrow 3\sin t+2\cos t=3\left(\sin t-3\cos^2 t\right)+7$ or $y=3\left(x+3\cos^2 t\right)+2\cos t\Rightarrow 3x+7=3x+9\cos^2 t+2\cos t$	M1
	$\Rightarrow 9\cos^2 t+2\cos t-7=0$ *	A1*
		(2)
(d)	$\cos t=\frac{7}{9}$	B1
	$y=3\times\frac{\sqrt{32}}{9}+2\times\frac{7}{9}=\frac{4\sqrt{2}}{3}+\frac{14}{9}$	M1A1
		(3)
		(12 marks)

Notes

(a)

B1: $\left(\frac{dx}{dt}=\right)\cos t+6\cos t\sin t$ or $\cos t+3\sin 2t$

B1: $\left(\frac{dy}{dt}=\right)3\cos t-2\sin t$

M1: Attempts $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ using their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ and substitutes $t=\pi$. (May substitute π before dividing.)

A1*: Achieves $\frac{dy}{dx}=3$ with full working shown and no errors.

(b)

B1: $x=-3,\ y=-2$ which may be seen within their working

M1: Attempts to find the equation of the tangent with gradient 3. If they use $y=mx+c$ they must proceed as far as $c=...$

A1: $y=3x+7$

(c)

M1: A full attempt to solve simultaneously the given parametric equations with their equation of the tangent

A1*: Achieves $9\cos^2 t+2\cos t-7=0$ with no errors

(d)

B1: $\cos t=\frac{7}{9}$ seen or implied. Allow if seen in (c).

M1: Attempts to find the y coordinate Must attempt to evaluate trig terms. If no substitution/working shown, then score for awrt 3.44 following a correct value for cos t

A1: $\frac{4\sqrt{2}}{3}+\frac{14}{9}$ or exact equivalent. Withhold if additional answers are given.

Question	Scheme	Marks
8(a)	$V=\pi\int_0^{10}\left(10xe^{-\frac{1}{2}x}\right)^2 dx =;\ 100\pi\int_0^{10} x^2e^{-x}\ dx$	M1;A1
		(2)
(b)	$\int x^2e^{-x}\ dx = -x^2e^{-x}+2\int xe^{-x}\ dx$	M1
	$=-x^2e^{-x}+2\int xe^{-x}\ dx = -x^2e^{-x}+2\left\{-xe^{-x}+\int e^{-x}dx\right\}$	dM1
	$-x^2e^{-x}-2xe^{-x}+2\int e^{-x}dx = -x^2e^{-x}-2xe^{-x}-2e^{-x}\ (+c)$	A1
		(3)
(c)	Total volume $=2\times\text{"}100\pi\text{"}\int_0^{10} x^2e^{-x}\ dx$	M1
	$\int_0^{10} x^2e^{-x} = \left[-x^2e^{-x}-2xe^{-x}-2e^{-x}\right]_0^{10} = \left(-(10)^2e^{-10}-2\times10e^{-10}-2e^{-10}\right)-(-2)$	M1
	$=2-122e^{-10}\quad(1.9944\ldots)$	A1
	Density $=\dfrac{5000}{\text{"}200\pi\text{"}\times\text{"}1.9944...\text{"}}$	dM1
	awrt 3.99 (g / cm^3)	A1
		(5)
		(10 marks)

Notes

(a)
M1: Forms a correct unsimplified expression for the volume.
A1: Achieves $100\pi\int_0^{10} x^2e^{-x}\ dx$ Condone a missing dx but limits must be present.

(b)
M1: Attempts integration by parts in the right direction to achieve an expression of the form $...x^2e^{-x}\pm...\int xe^{-x}\ dx$ Condone missing dx

dM1: Dependent on the previous method mark. Attempts integration by parts a second time to achieve an expression of the form $...x^2e^{-x}\pm...xe^{-x}\pm...\int e^{-x}\ dx$ Condone missing dx

A1: $-x^2e^{-x}-2xe^{-x}-2e^{-x}\ (+c)$ with or without the constant of integration

For attempts via the DI (tabular) method, look for first two rows of the table to have correct forms for M1, all rows with correct forms and answer extracted for dM1 and A1 for correct answer.

(c)
M1: A correct strategy to find the total volume with their values of k.
M1: Substitutes the limits of 10 and 0 into their part (b) and subtracts. Alternatively allow M1 for limits 20 and 0 used (as a mistaken attempt to double).
A1: $2-122e^{-10}$ or awrt 1.99

dM1: Dependent on second M. Attempts to find the density using $\dfrac{5000}{\text{their Volume}}$. The attempt at the volume need not be correct but an attempt at using (b) must have been made. E.g. if they forget k or forget to double, allow for the attempt with their volume. Must be with 5000 in numerator, or with correct work to reach correct units later.

A1: awrt 3.99 (g / cm^3) oe. Accept exact simplified answers such as $\dfrac{5000}{200\pi(2-122e^{-10})}$

Question	Scheme	Marks
9	For question 9 many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader. **M1:** Will be scored for setting up an algebraic statement in terms of a variable (integer) *k* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage. **A1:** Scored for a correct statement from which it is possible to draw a contradiction. **dM1;** For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions. **A1:** Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technicality a correct assumption as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n*"	
9	(Assume that there is an *n* with n^2-2 is divisible by 4 so) $n^2-2=4k$	M1
	then $n^2=4k+2=2(2k+1)$ (so is even)	A1
	Hence n^2 is even so *n* (=2*m*) is even hence n^2 is a multiple of 4 As n^2 is a multiple of 4 then $n^2-2=4m^2-2=2(2m^2-1)$ cannot be a multiple of 4 (as 2*m* — 1 is odd) so there is a contradiction.	dM1
	So the original assumption has been shown false. Hence "n^2-2 is never divisible by 4" is true for all *n* *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) *k* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2-2=4k$

A1: Reaches $n^2=2(2k+1)$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear.

Accept explanations such as "as n^2 is even then *n* is even hence n^2 is a multiple of 4 so n^2-2 cannot be a multiple of 4 (as 4 does not divide 2)"

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n*"

Question	Scheme	Marks
9 Alt 1	(Assume that n^2-2 is divisible by 4 for some *n*,) so $\frac{n^2-2}{4}$ is an integer. Then if *n* is even $n=2m$ (*m* integer) so $\frac{n^2-2}{4}=\frac{(2m)^2-2}{4}$ (oe with odd)	M1
	$=m^2-\frac{1}{2}$ (which is not an integer)	A1

	Since m^2 is an integer, $m^2 - \frac{1}{2}$ is not, hence n cannot be even, but if n is odd then $\frac{n^2-2}{4} = \frac{(2m+1)^2 - 2}{4} = m^2 + m - \frac{1}{4}$, which is again not an integer (since $m^2 + m$ is)	dM1
	Hence there is a contradiction (as n cannot be an integer) Hence "$n^2 - 2$ is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) m or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this Alt, consider case use of $n = 2m$ or $n = 2m + 1$ in $\frac{n^2-2}{4}$ is sufficient

A1: Reaches $m^2 - \frac{1}{2}$ for n even or $m^2 + m - \frac{1}{4}$ for n odd.

dM1: For a complete argument that leads to a contradiction in both cases. See scheme. Allow if minor details are omitted as long as the overall argument is clear.

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume $n^2 - 2$ is divisible be 4 for all n"

9 Alt 2	(Assume that $n^2 - 2$ is divisible by 4) $\Rightarrow n^2 - 2 = 4k$	M1
	$\Rightarrow n^2 = 4k + 2 \Rightarrow n = 2\sqrt{k + \frac{1}{2}}$ or $n = \sqrt{2}\sqrt{2k+1}$	A1
	So for some integer m $\sqrt{k + \frac{1}{2}} = \frac{\text{m}}{2} \Rightarrow 2k + 1 = \frac{m^2}{2}$ but m^2 is odd if m is odd so $\frac{m^2}{2}$ not an integer, or m^2 is a multiple of 4 if m even, so odd=even or $2k + 1$ is odd, so does not have a factor 2 to combine with the $\sqrt{2}$ outside, hence n must be irrational	dM1
	Hence we have a contradiction. So "$n^2 - 2$ is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2 - 2 = 4k$

A1: Reaches $n = 2\sqrt{k + \frac{1}{2}}$ or $n = \sqrt{2}\sqrt{2k+1}$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Must be a valid attempt to show that $2\sqrt{k+\frac{1}{2}}$ / $\sqrt{2}\sqrt{2k+1}$ is not an integer, and this method is a hard route.

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all n"

9 Alt 3	(Assume that n^2-2 is divisible by 4) then for n even we have (for some integer m) $n^2-2=4m^2-2$ **or** for n odd $n^2-2=4\left(m^2+m\right)-1$	M1
	$4m^2-2$ or $4\left(m^2+m\right)-1$	A1
	Since 4 divides n^2-2 and $4m^2$ thus for n even, 4 must divide 2, a contradiction, so n cannot be even, and also 4 divides $4\left(m^2+m\right)$ so for n odd, 4 divides 1, also a contradiction.	dM1
	Hence we have a contradiction for both cases (and as n must be either even or odd). so "n^2-2 is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) m or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing using n odd or n even to form an expression for n^2-2 of the form $4\times\text{integer}\pm\text{non-mulitple of }4$

A1: Reaches $4m^2-2$ or $4\left(m^2+m\right)-1$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Both cases must be considered with a reason for the contradiction given (not just stated not divisible by 4).

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all n"