

Question Number	Scheme	Marks
1(a)	$f(x) \geq \ln 3$	B1
		(1)
(b)	$g^{-1}(x) = \frac{3-2x}{x-5}$ or $-2 - \frac{7}{x-5}$ oe $x < 5$	M1A1 B1
		(3)
(c)	$g(0) = \frac{3}{2} \Rightarrow f\left(\frac{3}{2}\right) = \ln\left(\frac{21}{4}\right)$	M1A1
		(2)
(d)	$\frac{3+5e^{2a}}{e^{2a}+2} = 4 \Rightarrow 5e^{2a} + 3 = 4e^{2a} + 8 \Rightarrow e^{2a} = \dots$ $e^{2a} = 5 \Rightarrow 2a = \ln 5 \Rightarrow a = \dots$ $a = \frac{1}{2} \ln 5$	M1 dM1 A1
		(3)
		(9 marks)

(a)

B1: $f(x) \geq \ln 3$ o.e. Accept with y in place of $f(x)$.

(b)

M1: Attempts to rearrange to find the inverse. Score for achieving the form

$$y = \frac{\pm 3 \pm 2x}{\pm x \pm 5} \text{ or } x = \frac{\pm 3 \pm 2y}{\pm y \pm 5}. \text{ If they use } y = 5 - \frac{7}{x+2} \text{ score for reaching}$$

$$x = \frac{\pm 7}{\pm 5 \pm y} \pm 2 \text{ oe for } y = \dots$$

A1: $g^{-1}(x) = \frac{3-2x}{x-5}$ o.e. (e.g. see scheme). Allow with just g^{-1} or y or even f^{-1} B1: $x < 5$

(c)

M1: Attempts to find $fg(0)$. May find the function $fg(x)$ and substitute 0 into this. Condone slips. May be implied by awrt 1.7 if no substitution is shown.A1: $\ln\left(\frac{21}{4}\right)$ or exact simplified equivalent and isw.

(d)

M1: Sets $\frac{3+5e^{2a}}{e^{2a}+2} = 4$ and proceeds to make e^{2a} (or condone Ae^{2a}) the subject.

They must achieve the 4 although this may appear later in working.

$$\text{Alternatively, sets } g^{-1}g(e^{2a}) = g^{-1}f\left(\sqrt{e^4 - 3}\right) \Rightarrow e^{2a} = g^{-1}(4) = \frac{-5}{-1} = 5$$

dM1: Takes valid lns of both sides (must be lns of positive numbers) and proceeds to find a value for a . It is dependent on the previous method mark.

A1: $(a =) \frac{1}{2} \ln 5$ or exact equivalent and isw

Question Number	Scheme	Marks
2(a)	$f(1) = \frac{2(1)^2 + 3(1) - 4}{e^1} - \frac{1}{1^2} = \dots \text{ and } f(2) = \frac{2(2)^2 + 3(2) - 4}{e^2} - \frac{1}{(2)^2} = \dots$	M1
	$f(1) = -0.6(321\dots) [< 0] \text{ and } f(2) = 1.(103\dots) [> 0]$ $\Rightarrow \text{There is a sign change and } f(x) \text{ is continuous over the interval hence root (in the interval } [1, 2] \text{)}$	A1
		(2)
(b)	$\frac{2x^2 + 3x - 4}{e^x} - \frac{1}{x^2} = 0 \Rightarrow 2x^4 + 3x^3 = e^x + 4x^2$	M1
	$x^3(2x + 3) = e^x + 4x^2 \Rightarrow x = \sqrt[3]{\frac{e^x + 4x^2}{2x + 3}} \quad *$	A1*
		(2)
(c) (i) (ii)	$x_2 = \sqrt[3]{\frac{e^1 + 4(1)^2}{2(1) + 3}} = \text{awrt } 1.1035 \Rightarrow x_3 = \text{awrt } 1.1484$	M1A1
	1.1813	A1
		(3)
		(7 marks)

(a)

M1: Attempts $f(1)$ and $f(2)$. Values embedded in the expression is sufficient (accept slips if the intention is clear), or if no substitution is seen accept one correct value (truncated or rounded to 1sf) with an attempt at both made.

A1: Both $f(1)$ and $f(2)$ correct (truncated or rounded to 1sf), reason (e.g. "sign change" or < 0 , > 0 shown) and conclusion. Must refer to the function being continuous (over the interval) in some way, though be tolerant with language used.

Note: a narrower interval may be used but if so must contain the root 1.183... to score the M.

(b)

M1: Sets $f(x) = 0$ (may be implied), attempts to multiply both sides by x^2 and e^x and isolates the quartic and cubic terms on one side.

A1*: Factorises the left-hand side and proceeds to the given answer via an intermediate step with no errors seen.

Note: Working backwards is possible. For the M look for cubing and cross multiplying to reach a similar intermediate stage, and all correct for the A mark.

(c)

(i)

M1: Substitutes $x = 1$ into the iterative formula. May be implied by awrt 1.10 or awrt 1.15 if substitution not seen. Condone miscopies if the substitution is shown. NB Correct value for x_3 stated implies this mark.

A1: ($x_3 =$) awrt 1.1484. Must be x_3 and not another term (if labelled).

(ii)

A1: cao 1.1813 as the final answer (cannot be scored without M1 being scored in (c)(i))

Question Number	Scheme	Marks
3(a)	$m = \frac{2.25 - 2}{5}$ $\log_{10} V = 0.05t + 2$	M1 A1
		(2)
(b)	$\log_{10} V = 0.05t + 2 \Rightarrow V = 10^{0.05t+2}$ $a = 100 \text{ or } b = 1.12$ $V = 100 \times (1.12)^t$	M1 A1 A1
		(3)
(c)	$\frac{dV}{dt} = 100 \times \ln 1.12 \times (1.12)^t \quad (= 11.33(1.12)^t)$ $100 \times \ln 1.12 \times (1.12)^T = 50 \Rightarrow (1.12)^T = \frac{50}{100 \ln 1.12}$ $(T =) \log_{1.12} \left(\frac{50}{100 \ln 1.12} \right) = \dots$ $(T =) 13$	B1ft M1 dM1 A1
		(4)
		(9 marks)

(a)

M1: Attempts to find the gradient of the line, must be change in “y”/change in “x”. May be found via simultaneous equations – look for an attempt to solve the correct equations.

A1: $\log_{10} V = 0.05t + 2$ Accept log in place of \log_{10} here and throughout but must be correct variables.

(b)

M1: Correct attempt to make V the subject, or alternatively takes log of $V = ab^t$ and finds an equation in just a or b ie either $2 = \log_{10} a$ or $\frac{1}{20} = \log_{10} b$. If there is no incorrect working allow for obtaining an expression for a or b, e.g. $a = 10^2$ or $b = 10^{0.05}$.

A1: $a = 100$ or $b = \text{awrt } 1.12$ (may be stated or embedded in an equation of the correct form).

A1: $V = 100 \times (\text{awrt } 1.12)^t$. The equation must be given not just values.
SC Award M1A1A0 for candidates who start with an incorrect step of $V = 10^{0.05t} + 10^2$ but recover to $V = 10^2 \times 10^{0.05t} = 100(1.12)^t$ in subsequent work (and similar for variants).

(c)

Blft: Differentiates to the form $100 \ln 1.12 \times (1.12)^t$ follow through their values for a and b from part (b). Accept any unsimplified equivalents and values correct to 3sf.

M1: Sets their $\frac{dV}{dt}$ - which must be a changed function of the form $K(1.12)^t$ (K may be 1) equal to 50 and rearranges to the form $(1.12)^T = \dots$

dM1: Correctly proceeds from an equation of form $(1.12)^T = \dots$ to $T = \dots$ (any valid means, you may need to check their value if no method is shown).

Question Number	Scheme	Marks
4(a)	$R = \sqrt{12} \text{ or } 2\sqrt{3}$ $\alpha = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \dots$ $(f(x) =) \sqrt{12} \sin\left(2x - \frac{\pi}{3}\right)$	B1 M1 A1
		(3)
(b)(i) (ii)	Minimum value = $\frac{18}{\sqrt{12} + 4\sqrt{3}} (= \sqrt{3})$ $\sin\left(6x - \frac{\pi}{3}\right) = 1 \Rightarrow x = \frac{5}{36}\pi$	B1 M1A1
		(3)
		(6 marks)

A1: $(T =) 13$

(a)

B1: $\sqrt{12}$ oee

M1: Attempts to find $\alpha = \tan^{-1}\left(\pm \frac{3}{\sqrt{3}}\right) = \dots$ or $\alpha = \tan^{-1}\left(\pm \frac{\sqrt{3}}{3}\right) = \dots$ or

$$\alpha = \cos^{-1}\left(\pm \frac{\sqrt{3}}{2\sqrt{3}}\right) = \dots \text{ or } \alpha = \sin^{-1}\left(\pm \frac{3}{2\sqrt{3}}\right) = \dots$$

A1: $(f(x) =) \sqrt{12} \sin\left(2x - \frac{\pi}{3}\right)$ (or exact equivalent). Must give the expression but accept if given in (b), not just the values for R and α , but full marks can be scored in subsequent parts from the correct values if the expression is not explicitly stated.

(b)

(i)

B1: $\sqrt{3}$ or exact equivalent. No need to simplify. If multiple answers are given they must clearly identify this as the minimum value to score the mark.

(ii)

M1: Sets their $\sin\left(6x - \frac{\pi}{3}\right) = 1$ and attempts to find a value for x . Follow through on $f(3x)$ for their $f(x)$.

A1: $\frac{5}{36}\pi$ only

Note: Answers only with no working shown is M0 (calculator method).

Question Number	Scheme	Marks
5(a)	$\left(\frac{4000e^0}{19+e^0}\right) = 200$	B1
		(1)
(b)	$\frac{dN}{dt} = \frac{(19+e^{0.2t}) \times 400e^{0.1t} - 4000e^{0.1t} \times 0.2e^{0.2t}}{(19+e^{0.2t})^2}$	M1A1
		(2)
(c)	$(19+e^{0.2t}) \times 400e^{0.1t} - 4000e^{0.1t} \times 0.2e^{0.2t} = 0 \Rightarrow e^{0.2T} = 19$	M1A1
		(2)
(d)	$e^{0.2T} = 19 \Rightarrow T = \frac{\ln 19}{0.2} (=14.7...)$ $N = \frac{4000e^{0.1 \times 14.7}}{19+e^{0.2 \times 14.7}} = 458.8... \Rightarrow 459 \text{ squirrels}$	M1 dM1A1
		(3)
		(8 marks)

(a)

B1: 200

(b)

M1: Attempts to differentiate via the quotient rule (or the product rule). Look for reaching $\frac{(19+e^{0.2t}) \times Ae^{0.1t} - Be^{0.1t} \times e^{0.2t}}{(19+e^{0.2t})^2}$ or

$Ae^{0.1t} \times (19+e^{0.2t})^{-1} + Be^{0.1t} \times -(19+e^{0.2t})^{-2} \times e^{0.2t}$ where $A, B > 0$

Condone missing brackets for the M mark.

A1: Correct unsimplified derivative and isw. Any missing brackets must have been recovered.

(c)

M1: Sets their numerator of the form $\pm Pe^{0.1T} \pm Qe^{0.3T}$ (or unsimplified equivalent) equal to zero and rearranges to $e^{0.2T} = A$

A1: $e^{0.2T} = 19$ and isw. Accept with t instead of T . Allow from answers to (b) with an incorrect denominator but correct numerator or negative of the numerator.

(d)

M1: Takes lns of both sides and proceeds to find a value for T (accept awrt 14.7 if method is not shown). There must be evidence of using logarithms.

Alternatively score for stating or implying $e^{0.1t} = \sqrt{19}$

dM1: Substitutes their positive value for T or $e^{0.1t}$ into the original equation to find a value for N . It is dependent on the previous method mark.

A1: 459 (accept 458) provided the Ms have been scored. The $e^{0.2T} = 19$ must have been genuinely found, not fluked from an incorrect numerator. However, as long as the numerator was correct in (b), this mark may be

Question Number	Scheme	Marks
6(i)	$\left(\frac{dy}{dx} = \right) \frac{4x}{2x^2 + 5}$	M1A1
		(2)
(ii)	$\int \frac{21x}{3x^2 + k} dx = \frac{7}{2} \ln(3x^2 + k) \quad (+c)$ $\left[\frac{7}{2} \ln(3x^2 + k) \right]_1^k < 7 \ln 8 \Rightarrow \frac{7}{2} \ln \left(\frac{3k^2 + k}{3 + k} \right) < 7 \ln 8 \Rightarrow \frac{3k^2 + k}{3 + k} < 64$ $\frac{3k^2 + k}{3 + k} (<) 64 \Rightarrow 3k^2 - 63k - 192 (< 0) \Rightarrow k = \dots$ $k = 23$	M1A1 M1 dM1 A1
		(5)
		(7 marks)

awarded if subsequent working was correct.

(i)

M1: $\frac{\dots}{2x^2 + 5}$. Allow with anything for the ...

A1: $\left(\frac{dy}{dx} = \right) \frac{4x}{2x^2 + 5}$ o.e. and isw

(ii)

M1: Attempts to integrate to an expression of the form $A \ln(3x^2 + k)$ or $B \ln \left(x^2 + \frac{k}{3} \right)$. Allow for cancelling x 's e.g. $\frac{Ax \ln(3x^2 + k)}{x}$ (Allow A or B to be 1)

A1: $\frac{7}{2} \ln(3x^2 + k)$ o.e. with or without the constant of integration. Condone missing brackets for this mark and condone if extra integral symbols are left in.

M1: Attempts to substitute in the limits into their changed expression (which must involve \ln), sets the expression less than $7 \ln 8$ (condone an equation)

and uses correct log laws to remove lns – including correct bracketing used. You may allow $\ln 8$ to be written as a decimal but any \ln or exponential work must be correct. Note the form for their integral must be such that they are able to use log laws to remove lns.

dM1: Rearranges their inequality (may be an equation), proceeds to a three-term quadratic in k and attempts to find a value for k (usual rules apply for solving a quadratic – if using a calculator the value(s) must be correct for their equation). It is dependent on the previous method mark.

A1: 23 cao

Question Number	Scheme	Marks
7	$\left(\frac{dy}{dx} = \right) 2e^{x^2+(3k-2)x} + 2xe^{x^2+(3k-2)x} \times (2x + (3k-2))$ $\Rightarrow 2x^2 + (3k-2)x + 1 = 0 \Rightarrow (3k-2)^2 - 4 \times 2 \times 1$ $9k^2 - 12k - 4 (> 0)$ $9k^2 - 12k - 4 = 0 \Rightarrow k = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 9 \times (-4)}}{2 \times 9} \Rightarrow k = \dots$ $k < \frac{2-2\sqrt{2}}{3} \text{ or } k > \frac{2+2\sqrt{2}}{3}$	M1A1A1 dM1A1 M1A1
		(7 marks)

M1: Attempts the product rule achieving $\left(\frac{dy}{dx}\right) Ae^{(x^2+(3k-2)x)} + e^{(x^2+(3k-2)x)} \times f(x)$

where $f(x)$ is a quadratic function of x , condone missing/misplaced brackets for the M.

A1: One of $2e^{x^2+(3k-2)x}$ or $2xe^{x^2+(3k-2)x} \times (2x + (3k - 2))$ (unsimplified)

A1: Correct unsimplified derivative

dM1: Sets their derivative equal to zero (may be implied), collects terms and attempts $b^2 - 4ac$ on a 3 term quadratic – must be extracted not just part of the quadratic formula. It is dependent on the previous method mark.

A1: $(3k-2)^2-8 (>0)$ or equivalent quadratic expression or multiple thereof.
Need not be simplified.

M1: Must have attempted $\frac{dy}{dx} = 0$ and reached a quadratic in k . Attempts to solve their quadratic in k (including via a calculator) and attempts the outside region for their critical values (accept with endpoints included.). Accept with decimals answer correct to 3 s.f. for this mark.

A1: $k < \frac{2-2\sqrt{2}}{3}$ or $k > \frac{2+2\sqrt{2}}{3}$ or exact equivalent. Allow with “and” instead of “or” when given as two ranges, but use of set notation must use union, not intersection. E.g. $k \in \left(-\infty, \frac{2-2\sqrt{2}}{3}\right) \cup \left(\frac{2+2\sqrt{2}}{3}, \infty\right)$

Alternative method via implicit differentiation scores the same way with M1A1A1 for correct differentiation. M1 for a correct form of derivative statement up to constant/sign errors.

NB: method must be seen for the dM mark, if critical values appear from a derivative statement with no method shown (by calculator) then it is dM0. The discriminant must be considered. But the last mark is not dependent, so may be scored for selecting the correct “outside” region for their cvs.

Question Number	Scheme	Marks
8(a)	$25 = a + -(5 \times -2 + b) (\Rightarrow 25 = a + 10 - b) \Rightarrow a = 15 + b$ *	M1A1*
		(2)
(b)	$9 = a + 10 + b \Rightarrow a = \dots$ or $b = \dots$	M1
	$a = 7, b = -8$	A1A1
		(3)
(c)	$\left(\frac{8}{5}, 7\right)$	B1ftB1
		(2)
(d)	$15 - 5x = -2x^3 + 5x^2 + 4x - 3 \Rightarrow 2x^3 - 5x^2 - 9x + 18 = 0$	M1
	$2x^3 - 5x^2 - 9x + 18 = (x + 2)(2x^2 - 9x + 9)$	dM1A1
	$2x^2 - 9x + 9 = 0 \Rightarrow x = \frac{3}{2}$ (ignore $x = 3$)	ddM1
	$\left(\frac{3}{2}, \frac{15}{2}\right)$	M1A1
		(6)
		(13 marks)

(a)

M1: Substitutes $(-2, 25)$ into $y = a - (5x + b)$. You must **either** see a correct unsimplified equation without the modulus before the answer **or** indication of correct branch with attempt to substitute.
 Note they may use the cubic and substitute $x = -2$ to achieve the 25.
 For methods via squaring send to review if you are unsure if the marks are deserved.

A1*: Achieves the given answer with no errors seen following a correct unsimplified equation.
 If they attempt both equations, they must clearly indicate the choice of solution.

(b)

M1: Substitutes $(2, 9)$ into $y = a + (5x + b)$ and solves simultaneously with the equation from (a) to find a value for a or a value for b . May be implied by a correct value for a or b .

A1: One of $a = 7, b = -8$

A1: Both $a = 7, b = -8$ only

Note if there are attempts at squaring they must reach only the correct values. If unsure, use review.

(c)

B1ft: One of the coordinates of $\left(\frac{8}{5}, 7\right)$. Follow through their a and b for this mark

ie one of $\left(-\frac{b}{5}, a\right)$

B1: $\left(\frac{8}{5}, 7\right)$ Accept as $x = \dots, y = \dots$

(d)

M1: Sets their $y = 15 - 5x$ equal to $-2x^3 + 5x^2 + 4x - 3$ and collects terms to achieve a cubic. Condone a miscopy of the cubic if the intention is clear.

dM1: Attempts to find a quadratic factor (by division or equating coefficients). It is dependent on the previous method mark. For factorisation expect to see $(x \pm 2)(2x^2 + \dots \pm 9)$ or via division $(x \pm 2)(2x^2 + \dots x + \dots)$ Valid non-calculator method must be seen, solutions which go direct to a “factorised” form with no intermediate quadratic factor seen score dM0. Note they may use another linear factor to divide through by, the scheme applies the same way, but must be one of the correct factors.

A1: $2x^2 - 9x + 9$ (or correct factor for their linear term).

ddM1: Attempts to solve their quadratic via a non-calculator method, leading to a value for x . It is dependent on the previous two method marks. Valid non-calculator method must be seen to award this mark.

M1: Proceeds to find both coordinates of Q , accepting answers where the x value was found via a calculator. It is not dependent on the previous Ms but must attempted an initial equation using the cubic and either branch, set up to show what has been solved on the calculator. Correct answer with no working at all seen will score no marks.

A1: $\left(\frac{3}{2}, \frac{15}{2}\right)$ (oe) only. AO if a second set of coordinates is given. Allow for solutions where the cubic or quadratic was solved via calculator. Allow listed as $x = \dots, y = \dots$ and isw but must clear clearly seen as the correct coordinates.

Attempts at squaring, use review if you are unsure how to score unsuccessful attempts.

NB If no quadratic factor or suitable method is seen to solve the cubic then maximum of three mark is available, M1dMOA0ddMOM1A1.
 If the quadratic factor is reached but the quadratic is solved via calculator a maximum of 5 marks may be scored, M1dM1A1ddMOM1A1.
 M0dMOA0ddMOM1A1 is also possible if they give the initial equation and proceed to give the solutions without first gathering terms.
 A non-calculator approach must be shown for full marks.

Question Number	Scheme	Marks
9(a)	$4 \sin \theta \cos \theta = 2 \sin 2\theta$	B1
	e.g. $\Rightarrow 6 \sin^2 \theta \cot 2\theta + 2 \sin 2\theta = (3 - 3 \cos 2\theta) \frac{\cos 2\theta}{\sin 2\theta} + 2 \sin 2\theta$	M1A1
		(3)
(b)	$3 \cot 2\theta - 14 = 6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$	
	e.g. $\Rightarrow 3 \cot 2\theta \sin 2\theta - 14 \sin 2\theta = (3 - 3 \cos 2\theta) \cos 2\theta + 2 \sin^2 2\theta$	M1
	$\Rightarrow -14 \sin 2\theta = -3(1 - \sin^2 2\theta) + 2 \sin^2 2\theta$	M1
	$5 \sin^2 2\theta + 14 \sin 2\theta - 3 = 0$ *	A1*
		(3)
(c)	$(\sin 2x) = \frac{1}{5} \Rightarrow x = \dots$	M1
	$x = \text{awrt } 5.8^\circ, \text{ awrt } 84.2^\circ$	A1A1
		(3)
		(9 marks)

(a)

B1: $4 \sin \theta \cos \theta = 2 \sin 2\theta$ (seen or implied)M1: Attempts to use $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ and $\cot 2\theta = \frac{\cos 2\theta}{\sin 2\theta}$ or alternativeversions of these formulae to get to an expression in just $\sin 2\theta$ and $\cos 2\theta$.Other routes are possible, but any trig identities must be correct up to sign error to reach an expression in just $\sin 2\theta$ and $\cos 2\theta$.A1: $(3 - 3 \cos 2\theta) \frac{\cos 2\theta}{\sin 2\theta} + 2 \sin 2\theta$ or equivalent. isw once a correct expression is seen. Alternatives are possible here.

(b)

M1: Uses their expression from part (a) in the given equation and attempts to multiply both sides by $\sin 2\theta$. Alternatively they may rework part (a) again in this part but must reach an expression in terms of $\sin 2\theta$ and $\cos 2\theta$ only on one line.M1: Cancels $3 \cos 2\theta$ from both sides, attempts to use $\pm \sin^2 2\theta \pm \cos^2 2\theta = \pm 1$ and proceeds to a 3TQ in $\sin 2\theta$ only (terms do not need to be collected on the same side)

A1*: Achieves the given answer with no errors seen (including invisible brackets).

(c)

- M1: Solves the quadratic (usual rules, accept calculator usage) and attempts to use arcsin find a value for x . Accept if radians used or they neglect to divide by 2 as long as a value for x is reached. Condone use of θ throughout this part.
- A1: One of awrt 5.8 or awrt 84.2 (provided $\sin 2x = \frac{1}{5}$ or $2x = \sin^{-1}\left(\frac{1}{5}\right)$ is seen)
- A1: awrt 5.8, awrt 84.2 (provided $\sin 2x = \frac{1}{5}$ or $2x = \sin^{-1}\left(\frac{1}{5}\right)$ is seen) and no others in the given range.