Question Number	Scheme	Marks
1 (a)	$(2,\ -4)$	B1, B1 **(2)**
(b)	$(8,\ 6)$	B1, B1 **(2)**
(c)	$(-2,\ 6)$	B1 **(1)**
		(5 marks)

Notes:

1. Candidates may build up their answer in stages.

E.g. $(6,\ -2) \to (8,\ -2) \to (8,\ 6)$ In such cases you mark the final coordinates only

2. Condone the omission of brackets. So, allow 8, 6 for $(8,\ 6)$

3. Allow for example, $x = 2$ for $(2, ...)$ and $x = 2,\ y = -4$ for $(2,\ -4)$

4. There are no marks for transposed coordinates i.e $(6,\ 8)$ for $(8,\ 6)$

(a)

B1: For either $(2, ...)$ or $(..., -4)$

B1: $(2,\ -4)$

(b)

B1: For either $(8, ...)$ or $(..., 6)$

B1: $(8,\ 6)$

(c)

B1: $(-2,\ 6)$

Question Number	Scheme	Marks
2. (a)	$f(1)=4\times1^3+2\times1^2-12=-6$ AND $f(2)=4\times2^3+2\times2^2-12=28$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b)	$4x^3+2x^2-12=0\Rightarrow x^2(4x+2)=12$	M1
	$\Rightarrow x^2=\dfrac{12}{4x+2}=\dfrac{6}{2x+1}\Rightarrow x=\sqrt{\dfrac{6}{2x+1}}$	A1
		(2)
(c) (i)	$\left(x_2=\sqrt{\dfrac{'6'}{2+1}}=1.4...\right)$, $\quad x_3=1.2519$	M1, A1
(ii)	$\alpha=1.2934$	B1
		(3)
		(7 marks)

(a)

M1: Attempts the value of $f(x)$ at 1 and 2 with at least one correct.

Candidates who state $f(1)<0$, $f(2)>0$ without finding the values will score M0, A0

A smaller interval could be chosen but unlikely. The root must lie within the interval

A1: Both values correct with reason and minimal conclusion (root)

For the reason accept, for example,

- 'Sign change and continuous function'
- '$f(1)=-6<0$, $f(2)=28>0$ and cts'

For the minimal conclusion accept, for example

- Root
- ✓
- QED
- Hence α in the range 1 to 2

(b)

M1: Either writes $4x^3+2x^2-12=0$ OR states $f(x)=0$ with $4x^3+2x^2=12$ o.e.

and 'correctly' reaches one of the following factorised forms

- $x^2(4x\pm2)=\pm12$
- $x^2(2x\pm1)=\pm6$
- $2x^2(2x\pm1)=\pm12$

Alternatively, they reach $4x^3+2x^2=12$ with evidence as stated above in line 1.

- and then divide each term by $2x^2$ giving $2x+1=\dfrac{12}{2x^2}$ o.e

The key step in the above is an attempt that isolates the $2x+1$ or $4x+2$ term

A1: Shows **each** of the following steps in correctly showing $x=\sqrt{\dfrac{6}{2x+1}}$

- States or sets $f(x)=0$
- $x^2(4x+2)=12$ o.e. for instance $2x+1=\dfrac{12}{2x^2}$
- $x^2=\dfrac{6}{2x+1}$ or $x^2=\dfrac{12}{4x+2}$ o.e
- $x=\sqrt{\dfrac{6}{2x+1}}$

....

...

Alt (b)

It is possible to do this backwards

M1: States $x=\sqrt{\dfrac{k}{2x+1}}$ squares and cross multiplies to $x^2(2x+1)=k$

A1: Compares $4x^3+2x^2-12=0$ with $2x^3+x^2-k=0$ o.e and states that $f(x)=0$ when $k=6$

....

(c)

M1: Attempts to use the iteration formula at least once with $x_1=1$. Follow through on their *k*

It cannot be scored form a made-up value for *k*.

Look for $x_2=\sqrt{\dfrac{6}{2\times1+1}}$ but it is implied by $x_2=\sqrt{2}$ or awrt 1.4... for correct *k*

Also implied by $x_3=$ awrt 1.25 for correct *k*

A1: $x_3=$ awrt 1.2519

B1: $\alpha=1.2934$ CAO following evidence of some iteration (and must follow *k* = 6)

Accept as sight of iteration any decimal answer in the range $(1,2)$ for an intermediate value

x_n

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy}=\frac{8y(2y+1)-2(4y^2-3)}{(2y+1)^2}$	M1 A1
	$\frac{dy}{dx}=\frac{(2y+1)^2}{8y^2+8y+6}$	dM1, A1
		(4)
(b)	Sets $\frac{(2y+1)^2}{8y^2+8y+6}=\frac{1}{3}\Rightarrow 4y^2+4y-3=0$	M1, A1
	$\Rightarrow(2y-1)(2y+3)=0\Rightarrow y=\frac{1}{2},x=...$	dM1
	$P=\left(-1,\ \frac{1}{2}\right)$ only	A1
		(4)
		(8 marks)

(a)

M1: Attempts the quotient rule to achieve $\frac{Ay(2y+1)-B(4y^2-3)}{(2y+1)^2}$ $A>0,B>0$.

Product rule attempts may be seen

Condone missing brackets and ignore the LHS, so $\frac{dx}{dy}$ may be $\frac{dy}{dx}$

A1: $\frac{dx}{dy}=\frac{8y(2y+1)-2(4y^2-3)}{(2y+1)^2}$ with both left-hand and right-hand sides correct (unsimplified)

Via the product rule you should see $\frac{dx}{dy}=8y(2y+1)^{-1}-2(4y^2-3)(2y+1)^{-2}$

dM1: Uses $\frac{dy}{dx}=1\div\frac{dx}{dy}$. It is dependent upon the previous M1

A1: $\frac{dy}{dx}=\frac{(2y+1)^2}{8y^2+8y+6}$ o.e such as $\frac{4y^2+4y+1}{8y^2+8y+6}$ ISW after a correct answer.

Allow partially factorised forms such as $\frac{dy}{dx}=\frac{(2y+1)^2}{2(2y+1)^2+4}$ but $\frac{dy}{dx}=\frac{-4y^2-4y-1}{-8y^2-8y-6}$ is A0

....
....

Alt via division

M1: Divides $(4y^2-3)$ by $(2y+1)$ to achieve $2y+\alpha+\frac{\beta}{2y+1}$ and then differentiates to

$2+\frac{\delta}{(2y+1)^2}$

A1: Divides $\left(4y^2-3\right)$ by $\left(2y+1\right)$ to achieve $2y-1-\dfrac{2}{2y+1}$ and then differentiates to

$\dfrac{dx}{dy}=2+\dfrac{4}{\left(2y+1\right)^2}$

dM1: Requires $\dfrac{dx}{dy}=2+\dfrac{\delta}{\left(2y+1\right)^2}$ with attempt to find the reciprocal to reach $\dfrac{dy}{dx}=\dfrac{\left(2y+1\right)^2}{2\left(2y+1\right)^2+\delta}$

A1: Simplifies to $\dfrac{dy}{dx}=\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}$ or $\dfrac{4y^2+4y+1}{8y^2+8y+6}$

....
......

(b)

M1: Sets their $\dfrac{dy}{dx}=\text{"}\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}\text{"}=\dfrac{1}{3}$ and proceeds to a simplified quadratic equation in *y*.

Alternatively sets their $\dfrac{dx}{dy}=\dfrac{8y^2+8y+6}{\left(2y+1\right)^2}=3$ and proceeds to a simplified quadratic equation in *y*.

A1: Correct 3TQ E.g. $4y^2+4y-3=0$ but allow variations such as $4y^2=3-4y$

dM1: Solves quadratic via a correct method, finds a positive *y* value and the associated *x* coordinate.
Allow the quadratic to be solved via a calculator but it must be the simplified 3TQ
It is dependent upon the previous M mark.
This can be awarded if they find both values and cross out the correct answer.

A1: $P=\left(-1,\dfrac{1}{2}\right)$ only. Allow $x=-1,\ y=\dfrac{1}{2}$ ISW after a correct answer

If they find both values $\left(-1,\dfrac{1}{2}\right)$ and $\left(-3,-\dfrac{3}{2}\right)$ AND don't reject $\left(-3,-\dfrac{3}{2}\right)$ they will lose the last mark only

The demand is that solutions relying on calculator technology are not acceptable.

If a candidate uses their calculator to solve a correct $\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}=\dfrac{1}{3}$ o.e they can be awarded M1 for

$y=\dfrac{1}{2}$ and A1 for $x=-1,\ y=\dfrac{1}{2}$ for a total of 1100 in (b)

....

For candidates who have studied WMA14 you may see versions of implicit differentiation

E.g $\quad 2xy+x=4y^2-3\Rightarrow 2y+2x\dfrac{dy}{dx}+1=8y\dfrac{dy}{dx}$

$$\Rightarrow\frac{dy}{dx}=\frac{2y+1}{8y-2x}$$

Then substitute $x=$ into $\dfrac{dy}{dx}=\dfrac{2y+1}{8y-2x}\Rightarrow\dfrac{2y+1}{8y-2\times\dfrac{4y^2-3}{2y+1}}=\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}$

Score M1 for an attempt at both product and chain rule on xy and y^2 terms

A1 for a correct $\frac{dy}{dx}$ in terms of both x and y

dM1 for the substitution and attempt at simplification

A1 Fully correct and simplified

Note that part (b) can be attempted from $\frac{2y+1}{8y-2x} = \frac{1}{3} \Rightarrow x = y - \frac{3}{2}$ o.e which can then be substituted into the equation of the curve.

Question Number	Scheme	Marks
4.(a)	$$\begin{array}{r} 3x-5 \\ x^2+0x+1\overline{\smash{\big)}3x^3-5x^2+7x-5} \\ \underline{3x^3+0x^2+3x} \\ -5x^2+4x \\ \underline{-5x^2+0x-5} \\ 4x \end{array}$$	M1
	$\underline{A=3}$, $B=-5, C=4$	B1, A1
	$D=0$ *	A1*
		(4)
(b)	$\displaystyle\int 3x-5+\frac{4x}{x^2+1}\,\mathrm{d}x=\frac{3x^2}{2}-5x+2\ln(x^2+1)+c$	M1, A1ft
	$\text{Area}=\left(\frac{3\times 3^2}{2}-5\times 3+2\ln(3^2+1)\right)-\left(\frac{3\times 2^2}{2}-5\times 2+2\ln(2^2+1)\right)$	
	$=\frac{5}{2}+\ln 4$	dM1, A1
		(4)
		(8 marks)

(a) **Method via division**

M1: Full attempt to divide producing a linear quotient $(...x+...)$ and linear remainder $(...x+...)$

B1: States $A=3$ which may be awarded from the quotient line in the division sum $3x...$
This is an independent mark and can be awarded without sight of method

A1: $B=-5$ and $C=4$ following the award of the M mark. May be awarded from within the division sum.

A1*: Requires

- fully correct division (see main scheme) with no constant within the remainder which proves that $D=0$
- all previous marks to have been scored
- all constants stated or correct expression seen in (a) or (b)

If the candidates don't put in the $'0'x$ you would need to see something like the following work

$$\begin{array}{r} 3x-5 \\ x^2+1\overline{\smash{\big)}3x^3-5x^2+7x-5} \\ \underline{3x^3\qquad\quad+3x} \\ -5x^2+4x-5 \\ \underline{-5x^2\qquad\quad-5} \\ 4x \end{array}$$

The remainder should be $4x$ and not $4x\pm 5$

To score the A1* line it must be fully correct division

(a) **Alternative method via identity**

M1: Sets $3x^3-5x^2+7x-5\equiv(Ax+B)\left(x^2+1\right)+Cx+D$ and finds values for A, B, and C

B1: States $A = 3$

This is an independent mark and can be awarded without sight of method

A1: $B=-5$ and $C=4$ following the award of the M mark

A1*: This must be proven/shown and not just stated

For example:

Comparing terms in x^2, $\quad -5 = B \quad$ I

Comparing constant terms $\quad -5 = B + D \quad$ II

So $D = 0$

Look for a pair of relevant and correct equations which are correctly solved

All previous marks must have been scored

....

(b)

M1: $\int \frac{Cx}{x^2+1}\,\mathrm{d}x = k\ln(x^2+1)$ where k is a constant. You may see $k\ln u$ where $u=x^2+1$

Allow this to be scored from

$\int \frac{Cx+D}{x^2+1}\,\mathrm{d}x = \int \frac{Cx}{x^2+1}\,\mathrm{d}x + \int \frac{D}{x^2+1}\,\mathrm{d}x == k\ln(x^2+1)+...$ where … is not $=m\ln(x^2+1)$

A1ft: $\int 3x-5+\frac{4x}{x^2+1}\,\mathrm{d}x = \frac{3x^2}{2}-5x+2\ln(x^2+1)$ but follow through on their A, B and C (with D =0)

Also accept versions such as $\frac{(3x-5)^2}{6}+2\ln(u)$ where $u=x^2+1$

dM1: Substitutes 3 and 2 (o.e) into their integrated function (either way around) and correctly combines the two log terms. The limits must be consistent with their variable, so 3 and 2 for x and 10 and 5 for u. It is dependent upon having integrated to a form $\mathrm{f}(x)+\delta\ln(x^2+1)$ where $\mathrm{f}(x)$ is a quadratic expression

A1: CSO $\frac{5}{2}+\ln 4$ Note that $\frac{5}{2}+2\ln 2$ is A0 as the form of the answer has to be $\alpha+\ln\beta$

Question Number	Scheme	Marks
5.(a)	$\dfrac{dy}{dx} = -10\sin 2x - 24\cos 2x$	M1, A1
	$\left.\dfrac{dy}{dx}\right\|_{x=\frac{\pi}{3}} = -10\sin\left(\dfrac{2\pi}{3}\right) - 24\cos\left(\dfrac{2\pi}{3}\right) = -5\sqrt{3} + 12$	A1
		(3)
(b)	$R = 13$	B1
	$\tan\alpha = \dfrac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	M1A1
		(3)
(c)	Sets $-26\sin(2x + 1.176) = 6$	M1
	$\sin(2x + "1.176") = -\dfrac{3}{13}$	A1 ft
	$x = \dfrac{\arcsin\left(-\dfrac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	dM1, A1
		(4)
		(10 marks)

(a)

M1: Correct attempt at differentiation. Score for $\dfrac{dy}{dx} = p\sin 2x + q\cos 2x$

There may be attempts in which the double angle formulae have been used.

For example, $y = 5\cos 2x - 12\sin 2x = 5(\pm 2\cos^2 x \pm 1) - 24\sin x\cos x$

In such cases look for applications of the chain rule and product rules condoning slips in sign and loss of coefficients.

A1: $\dfrac{dy}{dx} = -10\sin 2x - 24\cos 2x$ which may be left un-simplified

If the double angle is used you should see $\dfrac{dy}{dx} = -20\sin x\cos x + 24\sin^2 x - 24\cos^2 x$

A1: Substitutes $x = \dfrac{\pi}{3}$ into a correct $\dfrac{dy}{dx}$ and achieves $-5\sqrt{3} + 12$ o.e.

It must follow the award of M1, A1. ISW after a correct answer

(b)

B1: $R = 13$. Allow for example 13.000 but not 12.99 followed by 13

Note that $\sqrt{169}$ or ± 13 is A0 unless followed by 13

M1: $\tan\alpha = \pm\dfrac{12}{5}, \tan\alpha = \pm\dfrac{5}{12} \Rightarrow \alpha = \ldots$ (Condone $\sin\alpha = 12, \cos\alpha = 5$ in the working)

If R is used to find α accept $\sin\alpha = \pm\dfrac{12}{R}$ or $\cos\alpha = \pm\dfrac{5}{R} \Rightarrow \alpha = \ldots$

A1: $\alpha = \text{awrt } 1.176$ ISW after a correct answer so don't penalise if they then write $\sqrt{13}\cos(2x - 1.176)$

(c) Using their answer to part (b) **Working and method must be shown**. Note: If they had $\alpha = 1.176$ but write the equation of C as $y = 13\cos(2x - 1.176)$ mark (c) as though $\alpha = -1.176$

M1: Differentiates $R\cos(2x+\alpha)$ to $T\sin(2x+\alpha)$ with their α (to at least 2dp) AND sets equal to 6

A1ft: $\sin(2x + '1.176') = \pm\frac{3}{13}$ or decimal equivalent. Condone a sign slip on differentiation and follow through on their α (to at least 3 dp) so score for $\sin(2x + \text{their } \alpha) = \pm\frac{3}{13}$

dM1: Complete attempt to find any **positive** value for x using a correct order of operations.
We now must be using correct differentiation including the sign but you can ft on their α ONLY
The following values are acceptable following intermediate working

E.g. $\sin(2x + \text{awrt "1.18"}) = -\frac{3}{13} \Rightarrow 2x + \text{"1.18"} = \text{awrt } 3.37 \Rightarrow x = \text{awrt } \frac{3.37 - \text{"1.18"}}{2} = \ldots$

E.g. $\sin(2x + \text{awrt } 1.18) = -\frac{3}{13} \Rightarrow 2x + \text{"1.18"} = \text{awrt } 6.05 \Rightarrow x = \frac{6.05 - \text{"1.18"}}{2} = \ldots$

This dependent method mark may be implied by sight of calculations such as

$\sin(2x + \text{awrt } 1.18) = -\frac{3}{13} \Rightarrow x = \text{awrt } \frac{\arcsin\left(-\frac{3}{13}\right) - 1.18}{2}$ o.e., followed by 1.1 or 2.4

A1: awrt 2.44 following the award of M1, A1, dM1. It must be selected and not part of a list of values

Candidates who state $-26\sin(2x+1.176) = 6$ o.e and follow this with the correct answer will score 1,0,0,0

Candidates who state $\sin(2x+1.176) = -\frac{6}{26}$ o.e and follow this with the correct answer will score 1,1,0,0

....

Candidates can use their answer to part (a) but will not score any marks until they put their derivative into a form in which the equation can be easily solved. **There is no follow through marks via this route.**

Use review if you feel unable to award the marks this way

Example I: $6=-10\sin 2x-24\cos 2x$

$\Rightarrow 12\cos 2x+5\sin 2x=-3$

$\Rightarrow \cos\left(2x-0.395\right)=-\dfrac{3}{13}$ M1, A1

$\Rightarrow x=\dfrac{\arccos\left(-\dfrac{3}{13}\right)+0.395}{2},=2.44$ dM1, A1

Example II: $6=-10\sin 2x-24\cos 2x$

Uses a more round-about route

$\Rightarrow 12\cos 2x+3=-5\sqrt{1-\cos^2 2x}$

$\Rightarrow 144\cos^2 2x+72\cos 2x+9=25\left(1-\cos^2 2x\right)$

$\Rightarrow 169\cos^2 2x+72\cos 2x-16=0$ M1, A1

$\Rightarrow \cos 2x=\underline{0.1612},-0.5873$

$\Rightarrow x=\text{awrt } 1.1 \text{ or } 2.4$ dM1

$\Rightarrow x=\text{awrt } 2.44$ A1

Using their answer to part (a) which must be of the form $\dfrac{dy}{dx}=\pm10\sin 2x\pm24\cos 2x$ the marks would be

M1: Equation in a single trig ratio which should be $26\cos\left(2x\pm0.3947..\right)=\pm6$ o.e. following use of $R\cos\left(2x\pm..\right)$ or $26\sin\left(2x\pm1.176\right)=\pm6$ o.e. following use of $R\sin\left(2x\pm..\right)$

Allow accuracy to 2dp or greater for the angle

A1: Correct equation, which is usually

$\cos\left(2x-0.3947..\right)=-\dfrac{6}{26}$ or $\sin\left(2x+1.176\right)=-\dfrac{6}{26}$

Accuracy must be 3 dp or greater for the angle

dM1: Correctly solves $\cos\left(2x\pm0.3947..\right)=-\dfrac{6}{26}$ or $\sin\left(2x\pm1.176\right)=-\dfrac{6}{26}$ and proceeds to a positive value for x.

As in the main method. It requires intermediate working

A1: awrt 2.44 only

Question Number	Scheme	Marks
6. (a)	$\mathrm{f}'(x)=6(2x-3)^2\,\mathrm{e}^{4x-2}+4(2x-3)^3\,\mathrm{e}^{4x-2}$	M1 A1
	$=2(2x-3)^2\,\mathrm{e}^{4x-2}\{3+2(2x-3)\}=2(4x-3)(2x-3)^2\,\mathrm{e}^{4x-2}$	dM1 A1
		(4)
(b) (i)	$x=\frac{3}{2},\frac{3}{4}$	B1 ft
(ii)	Attempts $\mathrm{f}\left(\frac{3}{4}\right)=\left(-\frac{3}{2}\right)^3\mathrm{e}^1=-\frac{27}{8}\mathrm{e},\Rightarrow \mathrm{g}\left(\frac{3}{4}\right)=-27\mathrm{e}$	M1, A1
	$-27\mathrm{e}\leqslant \mathrm{g}(x)\leqslant 0$	A1
		(4)
		(8 marks)

(a)

M1: Attempts the product rule and achieves $\mathrm{f}'(x)=\alpha(2x-3)^2\,\mathrm{e}^{4x-2}+\beta(2x-3)^3\,\mathrm{e}^{4x-2}$ where $\alpha>0,\ \beta>0$

A1: $\mathrm{f}'(x)=6(2x-3)^2\,\mathrm{e}^{4x-2}+4(2x-3)^3\,\mathrm{e}^{4x-2}$ which may be left unsimplified

dM1: Correctly takes out a common factor of $(2x-3)^2\,\mathrm{e}^{4x-2}$ out of $\alpha(2x-3)^2\,\mathrm{e}^{4x-2}+\beta(2x-3)^3\,\mathrm{e}^{4x-2}$

Look for $\mathrm{f}'(x)=\alpha(2x-3)^2\,\mathrm{e}^{4x-2}+\beta(2x-3)^3\,\mathrm{e}^{4x-2}=(2x-3)^2\,\mathrm{e}^{4x-2}\{\alpha+\beta(2x-3)\}$

A1: $2(4x-3)(2x-3)^2\,\mathrm{e}^{4x-2}$ following M1, A1, dM1. The order of the terms is not important.

Allow the dM1 to be implied for candidates who go directly from

$6(2x-3)^2\,\mathrm{e}^{4x-2}+4(2x-3)^3\,\mathrm{e}^{4x-2}\rightarrow 2(4x-3)(2x-3)^2\,\mathrm{e}^{4x-2}$

Marks in part (b) can only be scored following a correct method of differentiation in part (a)

So only allow if $\mathrm{f}'(x)$ was of the form $\alpha(2x-3)^2\,\mathrm{e}^{4x-2}+\beta(2x-3)^3\,\mathrm{e}^{4x-2}$

They cannot be scored following made up values for *P* and *Q*.

They cannot just appear as this would be relying entirely upon a calculator.

(b)(i)

B1ft: $x=\frac{3}{2}$ and $\frac{3}{4}$ following $\mathrm{f}'(x)=2(4x-3)(2x-3)^2\,\mathrm{e}^{4x-2}$

Alternatively, candidates can set

$\mathrm{f}'(x)=6(2x-3)^2\,\mathrm{e}^{4x-2}+4(2x-3)^3\,\mathrm{e}^{4x-2}=0$ and solve to reach $x=\frac{3}{2}$ and $\frac{3}{4}$

If part (a) is not correct follow through on their

- factorised $2(Px+Q)(2x-3)^2\,\mathrm{e}^{4x-2}$ following M1 and some attempt to combine the terms (not necessarily correctly) in part(a), so award for $x=\frac{3}{2}$ and $-"\frac{Q}{P}"$
- solution of $\alpha(2x-3)^2\,\mathrm{e}^{4x-2}+\beta(2x-3)^3\,\mathrm{e}^{4x-2}=0$ **leading** to the solutions $x=\frac{3}{2},\frac{3\beta-\alpha}{2\beta}$

(b)(ii)

M1: Attempts to find $\mathrm{f}\left("\frac{3}{4}"\right)=\left(-\frac{27}{8}\mathrm{e}\right)$ but allow decimal approximation e.g. AWRT -9.17

or attempts to find $\mathrm{g}\left("\frac{3}{4}"\right)=\left(-27\mathrm{e}\right)$ but allow decimal approximation e.g. AWRT -73.4.

The mark is implied by correct or rounded values following the correct method of differentiation

This can only be awarded following a correct method in part (a) and of finding the $\mathrm{f}\left(-"\frac{Q}{P}"\right)$' or $g\left(-"\frac{Q}{P}"\right)$'

A1: Correct exact value for $\mathrm{g}\left(\frac{3}{4}\right)=-27\mathrm{e}$ which must be exact

A1: Correct range for g. Look for $-27\mathrm{e} \leq \mathrm{g} \leq 0$ or equivalent such as $\mathrm{g}\in[-27\mathrm{e},\ 0]$ or $-27\mathrm{e} \leq y \leq 0$

Note that $-27\mathrm{e} \leq \mathrm{g} < 0$ and $-27\mathrm{e} < \mathrm{g} < 0$ are incorrect

Question Number	Scheme	Marks
7 (a)	$\sin 4\theta \equiv 2\sin 2\theta \cos 2\theta$	M1
	$\equiv 2(2\sin\theta\cos\theta)(1-2\sin^2\theta)$	dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$	
	$\equiv \sin\theta\cos\theta(4-8\sin^2\theta)$	A1
		(3)
(b)	$\sec x \sin 4x = 5\sin^3 x \cot x$	
	$\frac{1}{\cos x}\times\cos x\sin x(4-8\sin^2 x) = 5\sin^3 x\cot x$	B1
	$\div \sin x \Rightarrow\ 4-8\sin^2 x = 5\sin^2 x\cot x$	
	$\div \cos^2 x \Rightarrow\ 4\sec^2 x - 8\tan^2 x = 5\tan^2 x \cot x$	M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0$ *	A1*
		(3)
(c)	Uses $\sec^2 x = 1+\tan^2 x$	M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \frac{-5\pm\sqrt{89}}{8} \Rightarrow x = ...$	dM1
	$\Rightarrow x =$ awrt 0.506, 2.08	A1
		(4)
		(10 marks)

(a)

M1: Uses the double angle formula for $\sin 4\theta$ to get $2\sin 2\theta\cos 2\theta$ o.e. such as $\sin 2\theta\cos 2\theta + \cos 2\theta \sin 2\theta$

dM1: Uses the double angle formulae $\sin 2\theta = K\sin\theta\cos\theta$ and $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ to obtain $\sin 4\theta$ in terms of just $\sin\theta$ and $\cos\theta$. If they use one of the other identities for $\cos 2\theta$, either $\pm 2\cos^2\theta \pm 1$ or $\cos^2\theta - \sin^2\theta$, they must subsequently change the $\cos^2\theta$ terms to $1-\sin^2\theta$

A1: Reaches $\sin 4\theta \equiv \sin\theta\cos\theta(4-8\sin^2\theta)$

Withold this mark if there are mixed variables e.g. $\sin 4\theta \equiv 2\sin 2x\cos 2x$ and incorrect notation e.g. $\cos 2\theta \equiv 1-2\sin\theta^2$ used within the body of the proof.

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You may well see more complicated versions of this proof.
For example,

$\sin 4\theta \equiv \sin(3\theta+\theta) = \sin 3\theta\cos\theta + \cos 3\theta\sin\theta$

followed by $\quad = \sin(2\theta+\theta)\cos\theta + \cos(2\theta+\theta)\sin\theta$

$= \sin 2\theta\cos^2\theta + \cos 2\theta\sin\theta\cos\theta + \cos 2\theta\cos\theta\sin\theta - \sin 2\theta\sin\theta\sin\theta$

In this method the M1 is scored for the line above but you can condone sign slips

And the dM1 for using the double angle formulae $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ to obtain $\sin 4\theta$ in terms of just $\sin\theta$ and $\cos\theta$

Do not allow candidates just to write out the expansions for $\sin 3\theta$ and $\cos 3\theta$ without their proof

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(b)

B1: States or uses $\sec x = \dfrac{1}{\cos x}$ in the given equation.

It can be implied. So, writing $\sec x \sin 4x$ as $\sin x\left(4 - 8\sin^2 x\right)$ would be B1

M1: Requires

- $\sin 4x$ to be written as $\sin x \cos x\left('4' - '8'\sin^{'2'} x\right)$ using their values for P, Q and n
- a term in $\sin x$ to be cancelled or factorised out
- an attempt at dividing all terms by $\cos^2 x$ to reach an equation involving at least $\sec^2 x$ and $\tan^2 x$ (you don't need to see $\div\cos^2 x$ stated, it can be implied by their terms)

The last two bullets can be done in just one line and implied by all terms $\div\left(\sin x \cos^2 x\right)$

A1*: Reaches the given equation showing sufficient and necessary lines of work.

Withold this mark for mixed variables and/or incorrect notation within the proof.

(c)

M1: Uses $\sec^2 x = \pm 1 \pm \tan^2 x$ to set up a 3TQ in $\tan x$

A1: Correct 3TQ, $4\tan^2 x + 5\tan x - 4 = 0$.

The terms do not need to be on one side and the = 0 can be implied by further work

dM1: Solves the 3TQ using an appropriate method and proceeds to a value for x via value(s) for $\tan x$

Expect to see $"4"\tan^2 x + "5"\tan x - "4" = 0 \Rightarrow \tan x = ... \Rightarrow x = ...$

Allow calculator methods of solving the quadratic in $\tan x$ to produce a value for $\tan x$

A1: $x =$ awrt 0.506, 2.08 following M1, A1, dM1.

Ignore any extra solutions outside the range $(0, \pi)$.

Withold this mark if there are extra solutions in the range.

Note that the demand of the question is that candidates show their working.

So, from the given $4\sec^2 x - 5\tan x - 8\tan^2 x = 0$

candidates writing $\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$ followed by $x =$ awrt 0.506, 2.08 scores just the first M1, A1

There will be alternatives seen in part (c).

M1: Equation in a single trig ratio A1: Correct equation dM1: Full attempt to solve A1: Correct solutions

Alt (c)

$$4\sec^2 x - 5\tan x - 8\tan^2 x = 0$$

$$\times\cos^2 x \Rightarrow 4 - 5\sin x\cos x - 8\sin^2 x = 0$$

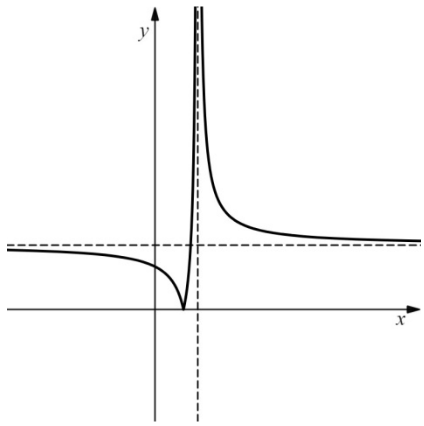
$$\Rightarrow 4 - 8\sin^2 x - 5\sin x\cos x = 0$$

$$\Rightarrow 4\cos 2x - 2.5\sin 2x = 0$$

$$\Rightarrow \tan 2x = \frac{8}{5} \qquad \text{M1, A1}$$

$$\Rightarrow 2x = 1.0121, 4.1538$$

$$\Rightarrow x = \text{awrt } 0.506,\ 2.08 \qquad \text{dM1, A1}$$

Question Number	Scheme	Marks
8. (a)	$g(2a)=\dfrac{3\times 2a-2a}{2a-a}=4,\quad f(4)=\ln(2\times 4-3)=\ln 5$	M1, A1
		(2)
(b)	$y=\ln(2x-3)\Rightarrow e^{y}=2x-3\Rightarrow x=\dfrac{e^{y}+3}{2}$	M1
	$f^{-1}(x)=\dfrac{e^{x}+3}{2}\qquad x>0$	A1, B1
		(3)
(c)	 Shape of curve to right of $x=a$	B1
	Shape of curve to left of $x=a$	B1
	$y=3$ marked at correct place on curve	B1
		(3)
(d) (i)	$\dfrac{3\times 8-2a}{8-a}=10\Rightarrow a=...$	M1
	$\Rightarrow a=7$	A1
(ii)	$\dfrac{3x-2\times 7}{x-7}=-10\Rightarrow x=...$	dM1
	$\Rightarrow x=\dfrac{84}{13}$	A1
		(4)
		(12 marks)

(a)

M1: Complete method to find fg($2a$). E.g. Finds g($2a$) and substitutes into f.

Some may find $fg(x)=\ln\left(2\times\dfrac{3x-2a}{x-a}-3\right)$ and substitute $x=2a$ into this. Condone slips in signs

A1: $\ln 5$. ISW after a correct answer. Allow this to be written down for both marks.

(b)

M1: Complete method to find $f^{-1}(x)$. Score for the correct order of operations so award this mark for work proceeding to $x=\dfrac{e^y \pm 3}{2}$ or $y=\dfrac{e^x \pm 3}{2}$ o.e if they start with $x=\ln(2y-3)$

A1: $f^{-1}(x)=\dfrac{e^x+3}{2}$ o.e. BUT condone/allow $f^{-1}=\dfrac{e^x}{2}+\dfrac{3}{2}$ and $y=\dfrac{1}{2}\left(e^x+3\right)$

B1: States a domain of $x>0$. Do not accept $x>\ln 1$

(c) **Do not accept the curve drawn on Figure 3 unless you can clearly see all parts of the curve.**

Allow the B1 for the equation of the horizontal asymptote if correct

B1: Shape of curve to the right of the vertical asymptote (provided that the whole of the original curve is not copied out). Be tolerant of slips of the pen

B1: Shape of curve to the left of the vertical asymptote. It must be a cusp as opposed to a minimum turning point. Again, be tolerant of slips of the pen and a linear looking section from $x=\dfrac{2}{3}a$ to $x=a$

B1: For the horizontal asymptote marked as $y=3$ and not just 3

(d)(i) **The following two marks are for a correct method and answer of finding the value of the constant *a***

If both equations are used then a correct choice must be made for both marks to be awarded

M1: Sets $\dfrac{3x-2a}{x-a}=10$ o.e., substitutes $x=8$ and solves for a. Note that $3+\dfrac{a}{x-a}=10$ is the same equation

Don't be too worried about the method of solving this equation but the initial equation must be correct

A1: $a=7$ only

(d)(ii) **Marks can only be scored in (ii) only following the award of M1 in (i)**

dM1: Sets $\dfrac{3x-2a}{x-a}=-10$ o.e. and substitutes their a from (d)(i) and solves for x

Don't be too worried about the method of solving this equation

Their a must have been found from solving an equation equivalent to $\dfrac{3x-2a}{x-a}=10$ with $x=8$

A1: $x=\dfrac{84}{13}$ only (exact answer but ISW after a correct answer).

If both solutions are found, $x=8$ and $x=\dfrac{84}{13}$, the $\dfrac{84}{13}$ must be selected and identified as the only solution.

Question Number	Scheme	Marks
9.(a)	$x = 30\mathrm{e}^{-0.2\times 8} = 6.06\,(\mathrm{mg})$	M1, A1
		(2)
(b)	$x = 30\mathrm{e}^{-0.2\times 10} + 20\mathrm{e}^{-0.2\times 2} = 17.5\,(\mathrm{mg})$ *	B1*
		(1)
(c)	$30\mathrm{e}^{-0.2\times(T+8)} + 20\mathrm{e}^{-0.2\times T} = 10$	M1
	$\left(3\mathrm{e}^{-1.6} + 2\right)\mathrm{e}^{-0.2T} = 1$	A1
	$\mathrm{e}^{-0.2T} = \dfrac{1}{3\mathrm{e}^{-1.6} + 2}$	
	$T = 5\ln\left(3\mathrm{e}^{-1.6} + 2\right) = 4.79$	dM1, A1
		(4)
		(7 marks)

(a)

M1: Substitutes $D = 30$ and $t = 8$ into $x = D\mathrm{e}^{-0.2t}$ and proceeds to a value for x

A1: AWRT 6.06 Allow this to be written down for both marks. Units are not important

(b)

B1*: There are many ways to achieve this. Examples of correct expressions are

- $30\mathrm{e}^{-0.2\times 10} + 20\mathrm{e}^{-0.2\times 2}$
- $6.06\mathrm{e}^{-0.2\times 2} + 20\mathrm{e}^{-0.2\times 2}$ (allow with more accurate values than 6.06)
- $26.06\mathrm{e}^{-0.2\times 2}$ (allow with more accurate values than 26.06)

You need to see a correct expression followed by awrt 17.47 or 17.5 (Units not important)

Note that values other than 26.06 in $26.06\mathrm{e}^{-0.2\times 2}$ would still round to 17.5.

For instance, expressions such as $26.1\times\mathrm{e}^{-0.2\times 2}$ and $26.07\times\mathrm{e}^{-0.2\times 2}$ would round to an appropriate value **but will not score this mark**

(c) Condone $T \leftrightarrow t$ but you must be convinced that they are answering the question.

M1: Sets either $30\mathrm{e}^{-0.2\times(T+8)} + 20\mathrm{e}^{-0.2\times T} = 10$ or $6.06\mathrm{e}^{-0.2\times T} + 20\mathrm{e}^{-0.2\times T} = 10$ o.e

Award the mark for a correct equation in T

A1: For a simplified equation in $\mathrm{e}^{-0.2T}$ (that is an equation that has a single $\mathrm{e}^{-0.2T}$ term).

Look for $\left(3\mathrm{e}^{-1.6} + 2\right)\mathrm{e}^{-0.2T} = 1$ or awrt $26.06\mathrm{e}^{-0.2\times T} = 10$

Condone for this mark $26.1\,\mathrm{e}^{-0.2\times T} = 10$, awrt $26.05\mathrm{e}^{-0.2\times T} = 10$ and even awrt $26.07\mathrm{e}^{-0.2\times T} = 10$

dM1: Full method to find T. Condone slips but generally look for

- the candidate making $\mathrm{e}^{-0.2T}$ the subject
- then taking lns
- proceeding to a value for T

A1: awrt 4.79 following correct and accurate work. Note exact simplified answer is $5\ln\left(3\mathrm{e}^{-1.6} + 2\right)$

If they use their answer to part (a) it must be following an accurate answer to 2dp, e.g. awrt $26.06\mathrm{e}^{-0.2\times T} = 10$

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Alternatives in (c) exist where an equation in T is not found directly

E.g Using the answer to (b)

M1, A1: $'17.5'\mathrm{e}^{-0.2\times t} = 10$

dM1: $-0.2t = \ln\left(\dfrac{10}{17.5}\right) \Rightarrow t = \ldots$ then add 2 to the result

A1: $T = 4.79$