Question Number	Scheme	Marks
1(a)	$\mathrm{gf}(1)=\mathrm{g}\left(\dfrac{2(1)}{3\times1+1}\right)=4-\left(\dfrac{2(1)}{3\times1+1}\right)^2$	M1
	$=\dfrac{15}{4}$ oe	A1
		(2)
(b)	$\mathrm{f}(x)\geqslant 0$ or $\mathrm{f}(x)<\dfrac{2}{3}$	B1
	$0\leqslant \mathrm{f}(x)<\dfrac{2}{3}$	B1
		(2)

Notes:

(a)

M1: Correct order of operations so requires an attempt to find $\mathrm{f}(1)$ and then apply g to their $\mathrm{f}(1)$

Allow for an attempt to substitute $x = 1$ into $4-\left(\dfrac{2x}{3x+1}\right)^2$

A1: Correct value. Allow equivalents e.g. 3.75, $3\dfrac{3}{4}$

Correct answer only scores both marks.

(b) Allow the use of y or f but **not** x in place of $\mathrm{f}(x)$ for both marks.

B1: One correct bound e.g. either $\mathrm{f}(x)\geqslant 0$ or $\mathrm{f}(x)<\dfrac{2}{3}$ which may be seen embedded in another inequality.

Allow **exact** equivalents for $\dfrac{2}{3}$ including $0.\dot{6}$

B1: Correct range. Allow equivalent notation e.g. $0\leqslant \mathrm{f}<\dfrac{2}{3}$, $y\in\left[0, \dfrac{2}{3}\right)$, $\left[0, \dfrac{2}{3}\right)$, $\dfrac{2}{3}>y\geqslant 0$

Allow written as separate inequalities e.g. $\mathrm{f}(x)\geqslant 0$, $\mathrm{f}(x)<\dfrac{2}{3}$, $\mathrm{f}(x)\geqslant 0$ and $\mathrm{f}(x)<\dfrac{2}{3}$

but **not** e.g. $\mathrm{f}(x)\geqslant 0$ or $\mathrm{f}(x)<\dfrac{2}{3}$

Allow y or f for $\mathrm{f}(x)$.

Allow **exact** equivalents for $\dfrac{2}{3}$ including $0.\dot{6}$

(c)	$y=\dfrac{2x}{3x+1}\Rightarrow 3xy+y=2x\Rightarrow 3xy-2x=-y\Rightarrow x(3y-2)=-y$ or $x=\dfrac{2y}{3y+1}\Rightarrow 3xy+x=2y\Rightarrow 3xy-2y=-x\Rightarrow y(3x-2)=-x$	M1
	$\Rightarrow \mathrm{f}^{-1}(x)=\dfrac{x}{2-3x}$	A1
		(2)
(d)	$\mathrm{f}^{-1}(x)=\mathrm{f}(x)$ **or** $\mathrm{f}^{-1}(x)=x$ **or** $\mathrm{f}(x)=x$ $"\dfrac{x}{2-3x}"=\dfrac{2x}{3x+1}$ **or** $"\dfrac{x}{2-3x}"=x$ **or** $\dfrac{2x}{3x+1}=x$ $\Rightarrow x(3x+1)=2x(2-3x)$ **or** $x=x(2-3x)$ **or** $2x=x(3x+1)$ $\Rightarrow x=\ldots$	M1
	$x=0,\ \dfrac{1}{3}$	A1
		(2)
		Total 8

(c)

M1: Attempts to change the subject.
If making x the subject, they must proceed as far as getting the two x terms on one side of the equation and the term not in x on the other and factorise out the x.
If making y the subject, they must swap x and y first and then proceed as far as getting the two y terms on one side of the equation and the term not in y on the other and factorise out the y.

A1: Correct expression $\left(\mathrm{f}^{-1}(x)=\right)\dfrac{x}{2-3x}$ oe e.g. $\left(\mathrm{f}^{-1}(x)=\right)\dfrac{-x}{3x-2}$
Accept equivalent expressions and isw after a correct answer is seen.
Do **not** be concerned with the lhs and just look for the correct expression in terms of x.
You can also ignore any reference to a domain as this is **not** required.

(d)

M1: Sets up one of the required equations e.g.

$$\text{their } \mathrm{f}^{-1}(x)=\mathrm{f}(x) \;\; \textbf{or} \;\; \text{their } \mathrm{f}^{-1}(x)=x \;\; \textbf{or} \;\; \mathrm{f}(x)=x$$

and proceeds to obtain at least one value for x.
If using their $\mathrm{f}^{-1}(x)$ it must be a function of x.
Condone copying slips as long as the intention is clear.

A1: Correct values $x=0,\ \dfrac{1}{3}$. Allow exact equivalents for $\dfrac{1}{3}$ including $0.\dot{3}$
Do not isw e.g. if they clearly reject $x = 0$ then score A0
Ignore any attempts to find the y values.

Question Number	Scheme	Marks
2(a)	$R=25$	B1
	$\tan\alpha=\frac{24}{7}\Rightarrow\alpha=...$	M1
	$\alpha=1.287$	A1
		(3)
(b)(i)	$\text{Min}=\frac{5}{90-3\times"25"\times(-1)}$	M1
	$=\frac{1}{33}$	A1
(b)(ii)	$(2x+"1.287")=\pi,...\Rightarrow x=...$	M1
	$\Rightarrow x=\frac{\pi-"1.287"}{2}=0.927$	A1
		(4)
		Total 7

Notes:

(a)

B1: Correct value. May be seen embedded in e.g. $(\text{f}(x)=)25\cos(x+...)$.

Condone e.g. $R=\sqrt{7^2+(-24)^2}=25$

Do not allow ±25 unless the -25 is rejected.

M1: Allow for: $\tan\alpha=\pm\frac{24}{7}$, $\tan\alpha=\pm\frac{7}{24}$, $\cos\alpha=\pm\frac{7}{\text{their }R}$, $\sin\alpha=\pm\frac{24}{\text{their }R}$ leading to a value for α.

If no method is shown it may be implied by their value for α as awrt 1.3 rads or awrt 74°

A1: Awrt 1.287 Isw once the mark is awarded. Must be in radians.

May be seen embedded in e.g. $(\text{f}(x)=)...\cos(x+1.287)$.

Apply isw if correct R and α are seen followed by e.g. $(\text{f}(x)=)25\cos(x-1.287)$.

Mark (b)(i) and (ii) together.

(b)(i)

M1: Award for $\frac{5}{90+3\times\text{Their }R}$.

May be implied by their value (which may be a decimal) and may be seen embedded in e.g. coordinates $\left(\frac{\pi-1.287}{2},\frac{1}{33}\right)$ which would score (i) M1 A1 and (ii) M1

A1: For $\frac{1}{33}$ only.

Note that both marks are available in (b)(i) even if their α is incorrect.

(b)(ii) **Note that (b)(ii) can be marked independently.**

M1: For an attempt to solve $2x\pm"1.287"=\pi$ to obtain a value for x. May be implied by awrt 0.927

Condone a sign slip but the order of operations must be essentially correct e.g.

$2x\pm"1.287"=\pi\Rightarrow2x=\pi\pm"1.287"\Rightarrow x=\frac{\pi\pm"1.287"}{2}$

Condone an attempt to solve e.g. $2x\pm"74°"=180°$ but not mixed units e.g. $2x\pm"1.287"=180$

A1: For awrt 0.927

Question Number	Scheme	Marks
3(a)	$\log_{10} y = \pm\dfrac{2}{3.5}\log_{10} x - 2$	M1
	$\log_{10} y = -\dfrac{4}{7}\log_{10} x - 2$	A1
		(2)

Notes:

M1: Obtains an equation of the form $\log_{10} y = \pm\dfrac{2}{3.5}\log_{10} x - 2$ oe e.g. $\log_{10} y = -\dfrac{4}{7}\log_{10} x + \log_{10} 0.01$

Note $y = -\dfrac{4}{7}x - 2$ or $y = \pm\dfrac{2}{3.5}\log_{10} x - 2$ or $\log_{10} y = -\dfrac{4}{7}x - 2$ score M0

If this is attempted via simultaneous equations e.g. $0 = -3.5m + c$, $-2 = c$ it is scored for reaching a gradient of $\pm\dfrac{2}{3.5}$ which may be implied by awrt 0.57 and then $\log_{10} y = \pm\dfrac{2}{3.5}\log_{10} x - 2$ as above.

Condone log or lg for $\log_{10}$ but not ln or any other incorrect base.

A1: Correct equation $\log_{10} y = -\dfrac{4}{7}\log_{10} x - 2$ or equivalent.

e.g. $7\log_{10} y + 4\log_{10} x + 14 = 0$, $\log_{10} y = -\dfrac{4}{7}\log_{10} x + \log_{10} 0.01$

Condone $\log_{10} y = -\dfrac{2}{3.5}\log_{10} x - 2$ and isw once a correct equation is seen.

Condone log or lg for $\log_{10}$ but not ln

(b)	$\log_{10} y = -\frac{4}{7}\log_{10} x - 2 \Rightarrow$ $y = 10^{\left(-\frac{4}{7}\log_{10} x - 2\right)}$ or $y = 10^{\left(\log_{10} x^{-\frac{4}{7}} - 2\right)}$ or $y = 10^{-\frac{4}{7}\log_{10} x} \times 10^{-2}$ or e.g. $\log_{10} y = \log_{10} x^{-\frac{4}{7}} - 2 \Rightarrow \log_{10} \frac{y}{x^{-\frac{4}{7}}} = -2$ or e.g. $\log_{10} y = -\log_{10} x^{\frac{4}{7}} - 2 \Rightarrow \log_{10} yx^{\frac{4}{7}} = -2$	M1
	$y = 10^{\left(\log_{10} x^{-\frac{4}{7}} - 2\right)}$ or $y = 10^{-\frac{4}{7}\log_{10} x} \times 10^{-2} \Rightarrow y = x^{-\frac{4}{7}} \times 10^{-2}$ or e.g. $10^{\log_{10} \frac{y}{x^{-\frac{4}{7}}}} = 10^{-2} \Rightarrow \frac{y}{x^{-\frac{4}{7}}} = 10^{-2}$ or $10^{\log_{10} yx^{\frac{4}{7}}} = 10^{-2} \Rightarrow yx^{\frac{4}{7}} = 10^{-2}$	dM1
	$y = \frac{1}{100}x^{-\frac{4}{7}}$ oe	A1
			(3)
			Total 5

Do not allow marks for part (a) to be scored in part (b)

Main method: Starting from an equation of the form $\log_{10} y = m\log_{10} x + c$ where m and c are non-zero.

M1: Correctly applies $10^{\text{lhs}} = 10^{\text{rhs}}$ to their $\log_{10} y = m\log_{10} x + c$ including the use of $10^{\log_{10} y} = y$

e.g. $\log_{10} y = m\log_{10} x + c \Rightarrow y = 10^{m\log_{10} x + c}$ or $\log_{10} y = m\log_{10} x + c \Rightarrow y = 10^{m\log_{10} x} \times 10^{c}$

or combines their $\log_{10} y$ and $m\log_{10} x$ correctly to obtain $\log_{10} yx^{m} = \pm c$ or equivalent e.g.

$\log_{10} y = m\log_{10} x + c \Rightarrow \log_{10} y = \log_{10} x^{m} + c \Rightarrow \log_{10} y - \log_{10} x^{m} = c \Rightarrow \log_{10} \frac{y}{x^{m}} = \pm c$

May be implied by their work. Note that e.g.

$\log_{10} y = -\frac{4}{7}\log_{10} x - 2 \Rightarrow y = 10^{-\frac{4}{7}\log_{10} x} - 10^{2}$ scores M0

dM1: **Depends on the previous method mark.**

Complete and correct work to obtain an equation in x and y only or for obtaining values for p and q which must not involve logs.

You can condone sign slips with their constant term but the log work must be correct.

A1: Correct **<u>equation</u>** in the form required $y = \frac{1}{100}x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2}x^{-\frac{4}{7}}$ or e.g.

$y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Note that the correct answer does not imply the method marks and you need to check carefully.

Alternatives for part (b):

Way 2:

M1: Starts with $y = px^q$ and obtains $\log_{10} y = \log_{10} p + \log_{10} x^q$ oe e.g. $\log_{10} y = \log_{10} p + q\log_{10} x$

dM1: **Depends on the previous method mark.**

Correct attempt at p **and** q by comparing their equation of the form $\log_{10} y = m\log_{10} x + c$ from part (a) or e.g. identifies the gradient and y-intercept from the sketch to obtain values for p and q not involving logs e.g. $\log_{10} p = c \Rightarrow p = 10^c$ and $\log_{10} x^q = q\log_{10} x \Rightarrow q = m$

A1: Correct **equation** in the form required $y = \frac{1}{100}x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2}x^{-\frac{4}{7}}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Note that correct values for p and q with no incorrect work seen scores M1dM1

Note that a correct equation with no incorrect work seen scores M1dM1A1

Way 3:

M1: $7\log_{10} y + 4\log_{10} x = -14 \Rightarrow \log_{10} y^7x^4 = -14 \Rightarrow y^7x^4 = 10^{-14}$

Multiplies up to obtain integer coefficients, combines the log terms correctly then applies $10^{\text{lhs}} = 10^{\text{rhs}}$

dM1: $y^7x^4 = 10^{-14} \Rightarrow y = \sqrt[7]{\frac{10^{-14}}{x^4}}$

Complete and correct attempt to obtain y in terms of x.

A1: Correct **equation** in the form required $y = \frac{1}{100}x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2}x^{-\frac{4}{7}}$ or e.g. $y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Way 4:

M1: $(\log_{10} x,\ \log_{10} y) = (0, -2) \Rightarrow x = 1,\ y = 10^{-2} \Rightarrow p = 10^{-2}$

Uses $(0, -2)$ correctly to establish the value of p.

dM1: $(\log_{10} x,\ \log_{10} y) = (-3.5, 0) \Rightarrow y = 1,\ x = 10^{-3.5} \Rightarrow 1 = 10^{-2} \times 10^{-3.5q} \Rightarrow -2 - 3.5q = 0 \Rightarrow q = ...$

Uses $(-3.5, 0)$ correctly with their value of p to obtain a value for q.

A1: Correct **equation** in the form required $y = \frac{1}{100}x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2}x^{-\frac{4}{7}}$ or e.g. $y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Fully correct answer in (b) with **no** working scores full marks in (b).

Partially correct answer in (b) with no working scores no marks.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">4(a)</td><td>$$\frac{49x}{x^2+x-12}+\frac{7x}{x+4}=\frac{49x+7x(x-3)}{(x+4)(x-3)}$$ or e.g. $$\frac{49x(x+4)}{(x^2+x-12)(x+4)}+\frac{7x(x^2+x-12)}{(x+4)(x^2+x-12)}=\frac{49x(x+4)+7x(x^2+x-12)}{(x+4)(x^2+x-12)}$$</td><td>M1A1</td></tr>
<tr><td>$$\frac{49x+7x(x-3)}{(x+4)(x-3)}=\frac{49x+7x^2-21x}{(x+4)(x-3)}=\frac{7x^2+28x}{(x+4)(x-3)}$$ $$=\frac{7x\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}=\frac{7x}{(x-3)}\ *$$</td><td>A1*</td></tr>
<tr><td></td><td>(3)</td></tr>
</table>

Do not allow a mis-read of the numerators of the fractions e.g. $\frac{7x}{x+4}$ as $\frac{7}{x+4}$ or $\frac{49x}{x^2+x-12}$ as $\frac{49}{x^2+x-12}$

M1: Factorises the denominator of the first term to obtain $(x+4)(x\pm3)$ and attempts to combine the two fractions using a common denominator to obtain $\frac{49x}{x^2+x-12}+\frac{7x}{x+4}=\frac{...x+...x(x\pm3)}{(x+4)(x\pm3)}$ oe

Alternatively multiplies numerator and denominator of both fractions by suitable expressions and attempts to combine the two fractions e.g.

$$\frac{49x(x+4)}{(x^2+x-12)(x+4)}+\frac{7x(x^2+x-12)}{(x+4)(x^2+x-12)}=\frac{...x(x+4)+...x(x^2+x-12)}{(x+4)(x^2+x-12)}$$

A1: For a correct unsimplified fraction.
Condone missing brackets if they are recovered.

A1*: Achieves the printed answer with sufficient working shown.
The factorisation required for the cancelling needs to be seen within their fraction so that e.g.

$$\frac{49x}{x^2+x-12}+\frac{7x}{x+4}=\frac{49x+7x(x-3)}{(x+4)(x-3)}=\frac{7x^2+28x}{(x+4)(x-3)}=\frac{7x}{(x-3)}$$ scores M1A1A0*

Do **not** allow just $\frac{7x\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}$ without the printed answer written down.

For reference if they take the more complicated method, the correct working to reach the printed could be as follows:

$$=\frac{49x(x+4)+7x(x^2+x-12)}{(x+4)(x^2+x-12)}=\frac{7x(x^2+8x+16)}{(x+4)(x+4)(x-3)}=\frac{7x(x+4)(x+4)}{(x+4)(x+4)(x-3)}=\frac{7x}{(x-3)}\ *$$

(b)	$\mathrm{f}(x)=\dfrac{7x}{(x-3)}\Rightarrow \mathrm{f}'(x)=\dfrac{(x-3)\times 7-7x\times 1}{(x-3)^2}$	M1
	$\mathrm{f}(x)=\dfrac{7x}{(x-3)}\Rightarrow \mathrm{f}'(x)=-\dfrac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$\mathrm{f}(x)=\dfrac{7x}{(x-3)}=7+\dfrac{21}{x-3}\Rightarrow \mathrm{f}'(x)=-\dfrac{21}{(x-3)^2}$	M1A1
		Total 5

(b)

M1: Attempts to differentiate using the quotient rule or product rule.

For the quotient rule it requires achieving $\mathrm{f}'(x)=\dfrac{\alpha(x-3)-\beta x}{(x-3)^2}\quad \alpha,\beta>0$

If they use a substitution then similar conditions apply e.g. $u=x-3\Rightarrow \mathrm{f}'(u)=\dfrac{\alpha u-\beta(u+3)}{u^2}\quad \alpha,\beta>0$

For the product rule it requires achieving $\mathrm{f}'(x)=A(x-3)^{-1}-Bx(x-3)^{-2}\quad A,B>0$

If they use a substitution then similar conditions apply e.g. $u=x-3\Rightarrow \mathrm{f}'(u)=Au^{-1}-B(u+3)u^{-2}\quad A,B>0$

A1: Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$

Allow $-\dfrac{21}{x^2-6x+9}$ or e.g. $-21(x^2-6x+9)^{-1}$

Isw once a correct simplified answer is seen.

Alternative:

M1: Expresses $\dfrac{7x}{(x-3)}$ as $\alpha+\dfrac{\beta}{(x-3)}$ and differentiates to obtain $\dfrac{\ldots}{(x-3)^2}$ or e.g. $\alpha+\dfrac{\beta}{u}$ where $u=x-3$

and differentiates to obtain $\dfrac{\ldots}{u^2}$

A1: Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$

Allow $-\dfrac{21}{x^2-6x+9}$ or e.g. $-21(x^2-6x+9)^{-1}$

Isw once a correct simplified answer is seen.

Question Number	Scheme	Marks
5(i)	$\cos 6x = 1 - 2\sin^2 3x \Rightarrow \sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x$ $\int \sin^2 3x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6x\right) dx$	M1
	$= \frac{1}{2}x - \frac{1}{12}\sin 6x (+c)$	A1
		(2)
(ii)	$\int x\left(x^2+4\right)^{\frac{3}{2}} dx = \frac{1}{5}\left(x^2+4\right)^{\frac{5}{2}} (+c)$	M1A1
		(2)
		Total 4

Notes:

(i)

M1: Uses $\cos 6x = \pm 1 \pm 2\sin^2 3x$ in an attempt to obtain an integrable form.

An alternative would be to change $\int \sin^2 3x \, dx$ to $\int \left(1 - \cos^2 3x\right) dx$ and then use $\cos 6x = \pm 2\cos^2 3x \pm 1$ in an attempt to obtain an integrable form.

May be implied by an answer of $\alpha x + \beta \sin 6x (+c)$

A1: Correct integration. Condone the omission of "+ c"

Condone spurious notation e.g. $\int \sin^2 3x \, dx == \frac{1}{2}x - \frac{1}{12}\sin 6x \, dx$

(ii)

M1: Obtains $\int x\left(x^2+4\right)^{\frac{3}{2}} dx = k\left(x^2+4\right)^{\frac{5}{2}} (+c)$

May be implied by e.g. $\int x\left(x^2+4\right)^{\frac{3}{2}} dx = \frac{2}{5}x\left(x^2+4\right)^{\frac{5}{2}} \times \frac{1}{2x} (+c)$

Note this may also be implied by an attempt at substitution e.g.

$u = x^2 + 4 \Rightarrow \int x\left(x^2+4\right)^{\frac{3}{2}} dx = \int x\left(x^2+4\right)^{\frac{3}{2}} \frac{dx}{du} du = \int x u^{\frac{3}{2}} \frac{1}{2x} du = \frac{1}{5}u^{\frac{5}{2}} + c$

Score M1 for a complete method leading to $ku^{\frac{5}{2}} (+c)$

A1: Correct integration. Condone the omission of "+ c"

Allow equivalent correct expressions but do not allow $\frac{x\left(x^2+4\right)^{\frac{5}{2}}}{5x}$ without the x's cancelled.

Condone spurious notation e.g. $\int x\left(x^2+4\right)^{\frac{3}{2}} dx = \frac{1}{5}\left(x^2+4\right)^{\frac{5}{2}} dx$

Question Number	Scheme	Marks
6(a)	$\theta = 75,\ t = 0 \Rightarrow 75 = 21 + A \Rightarrow A = ...$	M1
	$A = 54$	A1
		(2)
(b)	$\theta = 21 + 54\mathrm{e}^{-kt} \Rightarrow 25 = 21 + 54\mathrm{e}^{-5k}$	M1
	$54\mathrm{e}^{-5k} = 4 \Rightarrow \mathrm{e}^{-5k} = \dfrac{2}{27} \Rightarrow -5k = \ln\dfrac{2}{27} \Rightarrow k = ...$	M1
	$k = -\dfrac{1}{5}\ln\dfrac{2}{27} = 0.521$	A1
		(3)

Notes:

(a)

M1: Substitutes θ = 75 and t = 0 into the given equation, uses $\mathrm{e}^0 = 1$ and proceeds to find a value for A. Look for $75 = 21 + A \times 1 \Rightarrow A = ...$

A1: For A = 54
Sight of 54 with no incorrect working scores both marks.

(b)

M1: Uses θ = 25 and t = 5 with their A in $\theta = 21 + A\mathrm{e}^{\pm kt}$ so e.g. $25 = 21 + 54\mathrm{e}^{\pm 5k}$ is all that is required.

M1: Proceeds from an equation of the form $\mathrm{e}^{\pm 5k} = \alpha$ where $\alpha > 0$ to $\pm 5k = \ln\alpha \Rightarrow k = ...$ to obtain a value for k. Condone slips but the log work must be correct. Must be using ln and not e.g. $\log_{10}$ which for reference gives k = 0.226
May be implied by the correct exact value for k following through their A.

A1: For awrt 0.521 from correct and sufficient work.
Allow the exact value e.g. $-\dfrac{1}{5}\ln\dfrac{2}{27}$ and apply isw if subsequently rounded incorrectly.
May be seen embedded e.g. $\theta = 21 + 54\mathrm{e}^{-0.521t}$
Minimum for full marks in (b) is: $25 = 21 + 54\mathrm{e}^{-5k} \Rightarrow k = -\dfrac{1}{5}\ln\dfrac{4}{54}$

$25 = 21 + 54\mathrm{e}^{-5k} \Rightarrow \mathrm{e}^{-5k} = \dfrac{4}{54} \Rightarrow k = 0.521$ scores M1dM0A0

$25 = 21 + 54\mathrm{e}^{-5k} \Rightarrow k = 0.521$ scores M1dM0A0

Note that some candidates are using θ = 50 and t = 5 leading to k = 0.124 and this scores a maximum of **M0M1A0** in (b)

(c)	$\theta = 21 + 54\mathrm{e}^{-0.521t} \Rightarrow \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -28.1...\mathrm{e}^{-0.521t}$	M1
	$-28.1...\mathrm{e}^{-0.521T} = -9 \Rightarrow \mathrm{e}^{-0.521T} = \dfrac{9}{28.1...} \Rightarrow -0.521T = \ln\left(\dfrac{9}{28.1...}\right)$ $\Rightarrow T = \ln\left(\dfrac{9}{28.1...}\right) \div -0.521$	dM1
	$= 2.19$	A1
		(3)

(c)

M1: Differentiates to obtain $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \alpha\mathrm{e}^{-kt}$ where $\alpha \neq A$

dM1: **Depends on the previous mark.**

Sets their $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -9$ or sets $-\left(\text{their } \dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right) = 9$ and proceeds to obtain a value for t (or T).

You do not need to be concerned about the processing as long as they reach a value.

For this mark they must have obtained $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \alpha\mathrm{e}^{-kt}$ with $\alpha < 0$ and $k > 0$

Do **not** condone work where candidates ignore/cross out signs to make their equation solvable.

A1: For awrt 2.19

Just look for this value so ignore any units, correct or incorrect, and apply isw once awrt 2.19 is seen.

Must follow M1dM1 previously.

Note candidates who used $\theta = 50$ and $t = 5$ leading to $k = 0.124$ in (b) who proceed correctly in (c) will obtain $T = -2.38$ and can score a maximum of **M1dM1A0** in (c)

Note correct answer with no working scores no marks in (c).

Question Number	Scheme	Marks
7(a)	$y = e^{-x^2}\sin 3x \Rightarrow \dfrac{dy}{dx} = 3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x$	M1A1
	$3e^{-x^2}\cos 3x - 2xe^{-x^2}\sin 3x = 0 \Rightarrow 3\cos 3x - 2x\sin 3x = 0$ $\Rightarrow 3\cos 3x = 2x\sin 3x \Rightarrow \tan 3x = \dfrac{3}{2x}$	dM1
	$x = \dfrac{1}{3}\arctan\left(\dfrac{3}{2x}\right)$ *	A1*
		(4)
(b)(i)	$x_1 = 0.4 \Rightarrow x_2 = \dfrac{1}{3}\arctan\left(\dfrac{3}{2\times 0.4}\right)$	M1
	$(x_2 =)0.4367$	A1
(ii)	$(x_4 =)0.4307$	A1
		(3)

Notes:

(a)

M1: Differentiates using the product rule to obtain $\alpha e^{-x^2}\cos 3x + \beta xe^{-x^2}\sin 3x$, $\alpha, \beta \neq 0$

A1: Correct derivative in any form

dM1: **Depends on the first method mark.**

Sets their $\dfrac{dy}{dx} = 0$ (which may be implied), cancels e^{-x^2} and rearranges to $\pm\tan 3x = ...$

A1*: Correct proof with no errors seen.

Allow e.g. $\tan^{-1}$ or e.g. artan for arctan and allow e.g. $x = \dfrac{1}{3}\arctan\dfrac{3}{2x}$

Ignore any reference or attempts to solve $e^{-x^2} = 0$

(b)

M1: Attempts to substitute $x = 0.4$ into the formula.

Allow if seen embedded in the formula or may be implied by $x_2 =$ awrt 0.44

A1: Awrt 0.4367

A1: 0.4307 cao **not** awrt 0.4307

Both values correct as defined with no working scores M1A1A1

(c)	e.g. $\mathrm{f}(x)=x-\frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ $\mathrm{f}(0.4305)=0.4305-\frac{1}{3}\arctan\left(\frac{3}{2\times0.4305}\right)\left(=6.38\times10^{-5}\right)$ $\mathrm{f}(0.4295)=0.4295-\frac{1}{3}\arctan\left(\frac{3}{2\times0.4295}\right)\left(=-1.141...\times10^{-3}\right)$ or e.g. $\mathrm{f}(x)=3\mathrm{e}^{-x^2}\cos3x-2x\mathrm{e}^{-x^2}\sin3x$ $\mathrm{f}(0.4305)=3\mathrm{e}^{-(0.4305)^2}\cos3(0.4305)-2(0.4305)\mathrm{e}^{-(0.4305)^2}\sin3(0.4305)\left(=-4.968...\times10^{-4}\right)$ $\mathrm{f}(0.4295)=3\mathrm{e}^{-(0.4295)^2}\cos3(0.4295)-2(0.4295)\mathrm{e}^{-(0.4295)^2}\sin3(0.4295)\left(=8.885...\times10^{-3}\right)$	M1
	Sign change therefore x is 0.430 to 3dp	A1
		(2)
		Total 9

(c)

M1: Chooses a suitable function and attempts its value at both 0.4305 and 0.4295

For the attempt we need to see embedded values as in scheme or one value correct to 1sf for their suitable function.

The function they are using must be seen or implied by embedded values.

Allowable functions are:

- $x-\frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ or multiples of this
- $3\mathrm{e}^{-x^2}\cos3x-2x\mathrm{e}^{-x^2}\sin3x$ or multiples of this following through on their $\frac{\mathrm{d}y}{\mathrm{d}x}$
- $3\cos3x-2x\sin3x$ or multiples of this following through on their $\frac{\mathrm{d}y}{\mathrm{d}x}$

Unlikely, but it is also acceptable to pick a tighter interval e.g. 0.4305 and 0.4301

A1: Requires correct differentiation (if $\frac{\mathrm{d}y}{\mathrm{d}x}$ is used)

- both values correct (rounded or truncated to 1sf)
- a valid reason that includes a reference to the sign change e.g. < 0, > 0
- a minimal conclusion which could be ✓, QED, root

Note we condone the omission of a reference to continuity.

Attempts to use repeated iteration score no marks.

Question Number	Scheme	Marks
8(a)	$\tan 3x \equiv \tan(2x+x) \equiv \dfrac{\tan 2x + \tan x}{1-\tan 2x \tan x}$ or e.g. $\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \dfrac{\tan 2x + \tan x}{1-\tan 2x \tan x}$	M1
	$\dfrac{\tan 2x + \tan x}{1-\tan 2x \tan x} \equiv \dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x} \equiv \dfrac{2\tan x + \tan x - \tan^3 x}{1-\tan^2 x - 2\tan^2 x}$ $\dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$ *	A1*
		(3)

Condone use of a different variable e.g. θ for x.
Condone the use of mixed variables for the M marks as long as the intention is clear.
Condone clear use of "tan" written differently e.g. "tg"

M1: Writes $\tan 3x$ as $\tan(2x + x)$ which may be implied and applies the addition formula for $\tan(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only.
Condone one sign slip only.
May also reach this point by writing $\tan 3x$ as $\dfrac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for $\sin(A + B)$ and $\cos(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only.
Condone one sign slip only.

dM1: Replaces $\tan 2x$ with $\dfrac{2\tan x}{1-\tan^2 x}$ to express $\tan 3x$ in terms of $\tan x$ only.

A1*: Reaches the right hand side with sufficient working shown.
For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables or writing $\tan^2 x$ as $\tan x^2$

Attempts to use higher multiples of x e.g. $\tan 3x \equiv \tan(4x - x)$ should be sent to review.

Way 2	$\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x\cos x+\cos 2x\sin x}{\cos 2x\cos x-\sin 2x\sin x}$ $\equiv \dfrac{2\sin x\cos^2 x+\left(\cos^2 x-\sin^2 x\right)\sin x}{\left(\cos^2 x-\sin^2 x\right)\cos x-2\sin^2 x\cos x}$	M1
	$\equiv \dfrac{2\sin x\cos^2 x+\cos^2 x\sin x-\sin^3 x}{\cos^3 x-\sin^2 x\cos x-2\sin^2 x\cos x} \equiv \dfrac{3\sin x\cos^2 x-\sin^3 x}{\cos^3 x-3\sin^2 x\cos x}$	dM1
	$\equiv \dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*

Condone use of a different variable e.g. θ for x.
Condone the use of mixed variables for the M marks as long as the intention is clear.

M1: Writes $\tan 3x$ as $\dfrac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for sin (A + B) and cos(A + B) condoning one sign slip and then applies $\sin 2x = 2\sin x\cos x$ and $\cos 2x \equiv \pm\cos^2 x \pm \sin^2 x$ or equivalent e.g. $\cos 2x \equiv \pm 1 \pm 2\sin^2 x$ or $\cos 2x \equiv \pm 2\cos^2 x \pm 1$ to obtain an expression in terms of $\sin x$ and $\cos x$ only.

dM1: Expands and collects terms to obtain 2 terms in the numerator and 2 terms in the denominator. There must have been no errors in their identities for $\sin 2x$ or $\cos 2x$

A1*: Reaches the right hand side with sufficient working shown.
For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables.

(b)	$\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8 \Rightarrow \tan 3\theta = 2\sec^2 3\theta-8$ $\Rightarrow \tan 3\theta = 2\left(1+\tan^2 3\theta\right)-8$	M1
	$\Rightarrow 2\tan^2 3\theta-\tan 3\theta-6=0$ or e.g. $\Rightarrow 2\tan^2 3\theta-\tan 3\theta=6$	A1
	$\left(2\tan 3\theta+3\right)\left(\tan 3\theta-2\right)=0 \Rightarrow \tan 3\theta=-\dfrac{3}{2},\ 2$ $\tan 3\theta=-\dfrac{3}{2}\Rightarrow 3\theta=...\Rightarrow\theta=...$ or $\tan 3\theta=2\Rightarrow 3\theta=...\Rightarrow\theta=...$	dM1
	$\theta=0.37,\ 0.72,\ 1.42$	A1A1
		(5)
		Total 8

Allow if a different variable used (such as *x*).
For mixed variables allow the M's but only allow the first A
(and final A's) if recovered or interpreted correctly as the same variable.

M1: Writes the lhs as $k\tan 3\theta$ and applies $\sec^2 3\theta=\pm1\pm\tan^2 3\theta$ on the rhs.
A1: Reaches the correct 3TQ in $\tan 3\theta$ with terms collected but not necessarily all on one side.
dM1: Solves their 3TQ in $\tan 3\theta$ by any method (usual rules) including a calculator (you may need to check) to obtain at least one value for $\tan 3\theta$ and uses correct processing to obtain at least one value for θ e.g. finds inverse tan and divides by 3.
A1: Awrt at least one of these answers. Must be in radians and must come from the correct 3TQ.
A1: Awrt these answers and no extras in range. Must be in radians.

Note that there may be more convoluted methods which include writing the lhs as $k\dfrac{\sin 3\theta}{\cos 3\theta}$ **e.g.**

$$\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8\Rightarrow\frac{\sin 3\theta}{\cos 3\theta}=2\sec^2 3\theta-8=\frac{2}{\cos^2 3\theta}-8$$

$$\Rightarrow\sin 3\theta\cos 3\theta=2-8\cos^2 3\theta\Rightarrow\frac{1}{2}\sin 6\theta=2-8\left(\frac{\cos 6\theta+1}{2}\right)$$

$$\Rightarrow\sin 6\theta+8\cos 6\theta=-4$$

$$\Rightarrow\sqrt{65}\sin\left(6\theta+1.4464\right)=-4\Rightarrow\sin\left(6\theta+1.4464\right)=-\frac{4}{\sqrt{65}}\Rightarrow\theta=...$$

$$\theta=0.37,\ 0.72,\ 1.42$$

In such cases, the first M mark will only be scored when they reach an equation they can solve directly e.g. in the above example when they reach $\sqrt{65}\sin\left(6\theta+1.4464\right)=-4$

If you are in doubt whether an approach deserves any credit use Review.

Question Number	Scheme	Marks
9(a)	$x=0 \Rightarrow \frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)=0 \Rightarrow 3y+\frac{\pi}{4}=\pi,\ 2\pi,... \Rightarrow y=...$	M1
	$y=\frac{\pi}{4}$ or $y=\frac{7\pi}{12}$	A1
	$y=\frac{\pi}{4}$ and $y=\frac{7\pi}{12}$	A1
		(3)

Condone any confusion about which point is which and just look for the correct values.
Look for answers given in the body of the question or on Figure 2

M1: Correct strategy to find y at A or B.

Requires an attempt to solve either $3y+\frac{\pi}{4}=\pi$ or $3y+\frac{\pi}{4}=2\pi$

May be implied by at least one correct value and may be seen embedded e.g. $\left(0,\ \frac{\pi}{4}\right),\left(0,\ \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4},\ 0\right),\left(\frac{7\pi}{12},\ 0\right)$

Allow equivalent work e.g. $3y+\frac{\pi}{4}=0 \Rightarrow y=-\frac{\pi}{12} \Rightarrow y=-\frac{\pi}{12}+\frac{\pi}{3},\ -\frac{\pi}{12}+\frac{2\pi}{3}$

May be implied by decimal answers e.g. awrt 0.79 or awrt 1.8

A1: One correct value. Allow simplified or unsimplified and may be seen embedded e.g. $\left(0,\ \frac{\pi}{4}\right),\left(0,\ \frac{7\pi}{12}\right)$

Condone unsimplified for this mark e.g. $y=\frac{1}{3}\left(\pi-\frac{\pi}{4}\right)$ for $\frac{\pi}{4}$ or e.g. $y=\frac{2\pi}{3}-\frac{\pi}{12}$ for $\frac{7\pi}{12}$

A1: Both correct and simplified and may be seen embedded e.g. $\left(0,\ \frac{\pi}{4}\right),\left(0,\ \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4},\ 0\right),\left(\frac{7\pi}{12},\ 0\right)$ and condone unsimplified as long as they are single fractions e.g. $\frac{2\pi}{8}$

Alternative:

$$\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)=0 \Rightarrow \frac{2}{3}\sin 3y\cos\frac{\pi}{4}+\frac{2}{3}\cos 3y\sin\frac{\pi}{4}=0 \Rightarrow \tan 3y=-1 \Rightarrow 3y=\frac{3\pi}{4},\ \frac{7\pi}{4}$$

Score M1 for an attempt to solve either $3y=\frac{3\pi}{4}$ or $3y=\frac{7\pi}{4}$ and then A1A1 as above.

(b)	$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{2}{3}\times3\cos\left(3y+\frac{\pi}{4}\right)\left(=2\cos\left(3y+\frac{\pi}{4}\right)\right)$	B1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\cos\left(3y+\frac{\pi}{4}\right)}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4\cos^2\left(3y+\frac{\pi}{4}\right)}=\frac{1}{4\left(1-\sin^2\left(3y+\frac{\pi}{4}\right)\right)}$	M1
	$=\frac{1}{4\left(1-\left(\frac{3x}{2}\right)^2\right)}$	**d**M1
	$=\frac{1}{4-9x^2}$	A1
		(4)

B1: Correct $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form.

Do not condone missing brackets e.g. $2\cos 3y+\frac{\pi}{4}$ unless recovered or implied by subsequent work.

M1: For **any 2** of the following applied to their $\frac{\mathrm{d}x}{\mathrm{d}y}$:

- finds reciprocal (may have already happened if they find $\frac{\mathrm{d}y}{\mathrm{d}x}$)
- squares
- applies $\cos^2\left(3y+\frac{\pi}{4}\right)=1-\sin^2\left(3y+\frac{\pi}{4}\right)$

Note that this mark (and the next one) may be implied by equivalent work e.g.

$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow\sin\left(3y+\frac{\pi}{4}\right)=\frac{3x}{2}\Rightarrow\cos\left(3y+\frac{\pi}{4}\right)=\frac{\sqrt{4-9x^2}}{2}$

$\frac{\mathrm{d}x}{\mathrm{d}y}=2\cos\left(3y+\frac{\pi}{4}\right)=\sqrt{4-9x^2}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4-9x^2}$

dM1: Uses all 3 of the above correctly to obtain $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ in terms of *x*.

A1: Correct answer.

You can condone e.g. missing trailing brackets within their working e.g. $\frac{1}{4(1-\sin^2\left(3y+\frac{\pi}{4}\right)}$

Special case: allow this mark if the only error in their solution is to obtain $\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{2}{3}\times3\cos\left(3y+\frac{\pi}{4}\right)$

(b) Way 2	$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow 3y+\frac{\pi}{4}=\sin^{-1}\frac{3x}{2}$	B1
	$3y+\frac{\pi}{4}=\sin^{-1}\frac{3x}{2}\Rightarrow 3\frac{dy}{dx}=\frac{1}{\sqrt{1-\frac{9x^2}{4}}}\times\frac{3}{2}$	M1
	$3\frac{dy}{dx}=\frac{1}{\sqrt{1-\frac{9x^2}{4}}}\times\frac{3}{2}\Rightarrow\left(\frac{dy}{dx}\right)^2=\left(\frac{1}{2\sqrt{1-\frac{9x^2}{4}}}\right)^2$	**d**M1
	$=\frac{1}{4-9x^2}$	A1

Way 2:

B1: Correct equation in any form connecting y and $\sin^{-1}\frac{3x}{2}$.

M1: Differentiates to obtain $\frac{dy}{dx}=\frac{k}{\sqrt{1-\left(\frac{3x}{2}\right)^2}}$ or equivalent. Condone missing brackets around $\frac{3x}{2}$.

dM1: Squares and makes $\left(\frac{dy}{dx}\right)^2$ the subject.

A1: Correct answer.

(c)	Identifies that one of the gradients is $\sqrt{"4"}$ or that one of the gradients is $\dfrac{1}{\sqrt{"4"}}$ **or** Identifies that one of the gradients is 2 or that one of the gradients is $\dfrac{1}{2}$	B1ft
	At A: $y-"\frac{\pi}{4}"="2"x$ **and** At B: $y-"\frac{7\pi}{12}"="\frac{1}{2}"x$	M1
	$"2"x+"\frac{\pi}{4}"="\frac{1}{2}"x+"\frac{7\pi}{12}"\Rightarrow x=\ldots$	**d**M1
	$x=\dfrac{2\pi}{9}$	A1
		(4)
		Total 11

Note that in this part the question says to use the answer to part (b) i.e. their $\dfrac{1}{p-qx^2}$

If they use their $\dfrac{1}{p-qx^2}$ then the ft is available as shown.

If candidates "start again" or e.g. use their $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in terms of y with their y values from part (a) then the value(s) must be <u>correct</u>.

B1ft: **Using the answer to part (b):**

Award this mark for **one** gradient identified as $\sqrt{p}$ **or** $\dfrac{1}{\sqrt{p}}$ where p is their value from part (b).

or not using the answer to part (b):

One gradient identified as 2 **or** $\dfrac{1}{2}$

M1: Correct straight line method for the equation of the normal at A **and** the tangent at B.
They must have
- 2 values for y from part (a) (ignore how these were labelled)
- both gradients **positive**

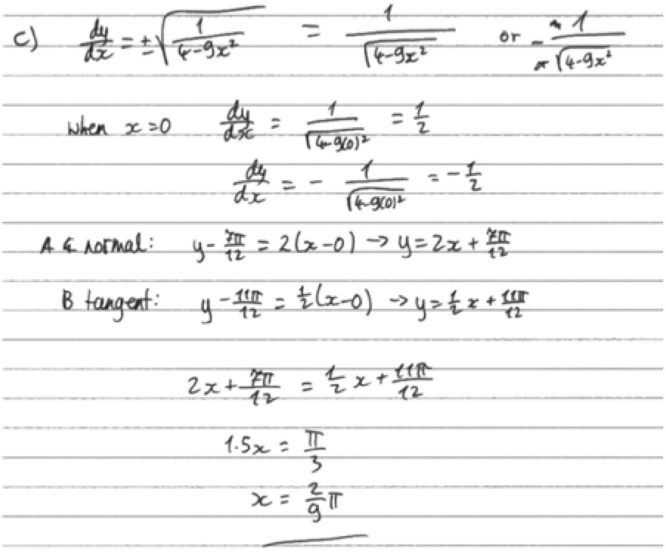
For the **normal at *A*** this requires a gradient of $\sqrt{p}$ or 2 with $x = 0$ and their smaller value of y from part (a)
If they use $y = mx + c$ they must proceed as far as $c = \ldots$

For the **tangent at *B*** this requires a gradient of $\dfrac{1}{\sqrt{p}}$ or $\dfrac{1}{2}$ with $x = 0$ and their larger value of y from part (a)
If they use $y = mx + c$ they must proceed as far as $c = \ldots$

dM1: Correct method for both lines as defined above and solves simultaneously (may be via a calculator) to find the x coordinate of D.
Depends on the previous mark.

A1: Cso. Correct value $x=\dfrac{2\pi}{9}$.
Ignore any attempts to find the y coordinate.
If they use part (b) then full marks can be awarded following an incorrect value for q provided all the other work is correct.

Beware: It is possible to obtain the correct answer in (c) from incorrect values in (a) e.g.



c) $\frac{dy}{dx} = \pm\sqrt{\frac{1}{4-9x^2}} = \frac{1}{\sqrt{4-9x^2}}$ or $-\frac{1}{\sqrt{4-9x^2}}$

when $x=0$ $\frac{dy}{dx} = \frac{1}{\sqrt{4-9(0)^2}} = \frac{1}{2}$

$\frac{dy}{dx} = -\frac{1}{\sqrt{4-9(0)^2}} = -\frac{1}{2}$

A & normal: $y - \frac{7\pi}{12} = 2(x-0) \rightarrow y = 2x + \frac{7\pi}{12}$

B tangent: $y - \frac{11\pi}{12} = \frac{1}{2}(x-0) \rightarrow y = \frac{1}{2}x + \frac{11\pi}{12}$

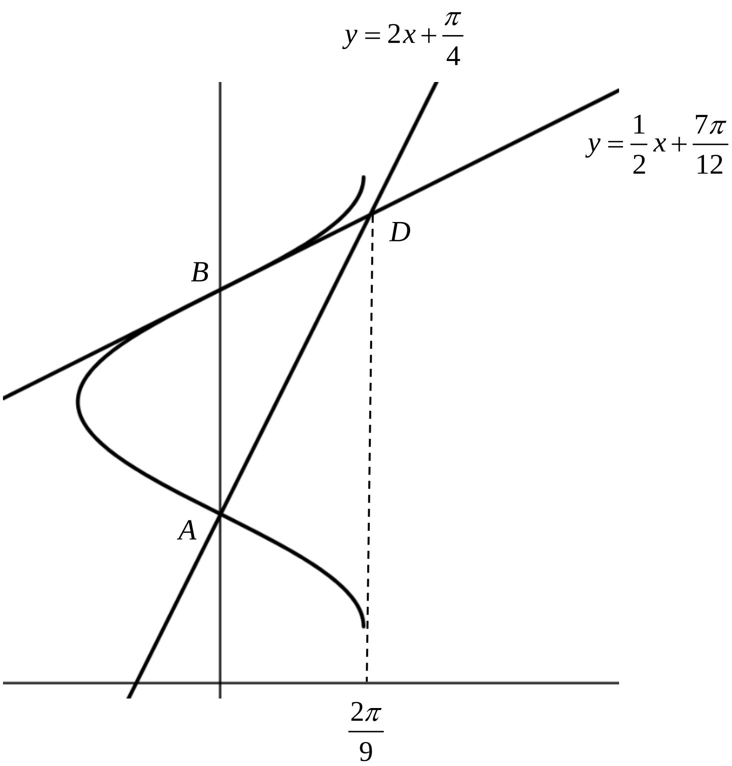
$2x + \frac{7\pi}{12} = \frac{1}{2}x + \frac{11\pi}{12}$

$1.5x = \frac{\pi}{3}$

$x = \frac{2}{9}\pi$

This scores B1M1dM1A0

Diagram for reference:



Question Number	Scheme	Marks
10(a)(i)	$(y=)10+k$	B1
(ii)	$x=\frac{10}{k}$ **or** $y=k$	B1
	$x=\frac{10}{k}$ **and** $y=k$	B1
		(3)

Remember to look for answers written within the question and on Figure 3.
If there is any contradiction, answers in the body of the script take precedence.

(a)(i)

B1: Correct expression. The "$y=$" is not required and condone $x = 10 + k$ or $A = 10 + k$
Allow e.g. $(0, 10+k)$ and condone $(10+k, 0)$
Do **not** allow $(y=)|-10|+k$

(a)(ii)

B1: One correct coordinate. May be seen as $x = \ldots$ or $y = \ldots$ or in coordinate form e.g. $\left(\frac{10}{k}, ...\right)$ or $(..., k)$
Condone missing brackets e.g. $\frac{10}{k}$, ... or ..., k
Do not condone e.g. $P = k$ unless k is clearly identified as the y coordinate.
Do not condone coordinates the wrong way round unless they are correctly assigned first and then apply isw.

B1: Both correct. May be seen as $x = \ldots$ and $y = \ldots$ or as e.g. $\left(\frac{10}{k}, k\right)$ and no other points.
Condone missing brackets e.g. $\frac{10}{k}, k$
Do not condone coordinates the wrong way round unless they are correctly assigned first and then apply isw

(b)	$kx-10+k=2k \Rightarrow x=\ldots$ **or** $-kx+10+k=2k \Rightarrow x=\ldots$	M1
	$kx-10+k=2k \Rightarrow x=\ldots$ **and** $-kx+10+k=2k \Rightarrow x=\ldots$	dM1
	$x \leqslant \dfrac{10-k}{k}$ or $x \geqslant \dfrac{10+k}{k}$ oe	A1
		(3)
	(b) Alternative by squaring.	
	$\lvert kx-10\rvert+k=2k \Rightarrow \lvert kx-10\rvert=k \Rightarrow k^2x^2-20kx+100=k^2$ $\Rightarrow k^2x^2-20kx+100-k^2=0$	M1
	$k^2x^2-20kx+100-k^2=0 \Rightarrow x=\dfrac{20k\pm\sqrt{400k^2-4k^2\left(100-k^2\right)}}{2k^2}$ $=\dfrac{10\pm k}{k}$	dM1
	$x \leqslant \dfrac{10-k}{k}$ or $x \geqslant \dfrac{10+k}{k}$ oe	A1

(b)

M1: Attempts to solve one of these equations to find one of the limits in terms of k.
Condone use of any inequality for either equation e.g. $kx-10+k \geqslant 2k$ where $\geqslant$ is any inequality or "="
May be implied by one correct limit.

dM1: Attempts to solve both of these equations to find both limits.
Condone use of any inequality for either equation e.g. $kx-10+k \geqslant 2k$ where $\geqslant$ is any inequality or "=".
May be implied by both correct limits.

A1: Correct range. Allow equivalent notation e.g. the inequalities written separately or e.g.
$x \leqslant \dfrac{10-k}{k}$, $x \geqslant \dfrac{10+k}{k}$ or $\left(-\infty, \dfrac{10-k}{k}\right], \left[\dfrac{10+k}{k}, \infty\right)$ Condone $x \leqslant \dfrac{10-k}{k}$ and $x \geqslant \dfrac{10+k}{k}$ but **not** $x \leqslant \dfrac{10-k}{k} \cap x \geqslant \dfrac{10+k}{k}$

Do not allow e.g. $\dfrac{10+k}{k} \leqslant x \leqslant \dfrac{10-k}{k}$ unless e.g. $x \leqslant \dfrac{10-k}{k}$, $x \geqslant \dfrac{10+k}{k}$ is seen first then apply isw.

Allow equivalent correct expressions for the limits

e.g. $\dfrac{10}{k}-1, -\dfrac{k-10}{k}, \dfrac{k-10}{-k}$ for $\dfrac{10-k}{k}$ and $\dfrac{10}{k}+1$ for $\dfrac{10+k}{k}$ etc.

Alternative by squaring:

M1: Isolates the $\lvert kx-10\rvert$, squares both sides, obtaining at least 3 terms on the lhs and collects terms to obtain a 3TQ = 0

dM1: Attempts to solve their 3TQ using a correct method to find both limits.

A1: Correct range as above.

(c)	$k > 3$	B1
	Maximum value of k occurs when $y = 3x + 1$ passes through vertex e.g. when $"k" = 3\times"\frac{10}{k}"+1 \Rightarrow k = ...$ **See below for alternative approaches.**	M1
	$k^2 - k - 30 = 0 \Rightarrow (k+5)(k-6) = 0 \Rightarrow k = 6$ So **maximum** k is 6	A1
	$3 < k < 6$	A1
		(4)
		Total 10

B1: For the correct **minimum** value for k.
Condone $k \geqslant 3$ or "minimum value is 3".
This may be seen embedded in an inequality but it must be identified as the **minimum** value.

M1: Correct strategy for the maximum value of k.
E.g. sets $f(x) = 3x + 1$, substitutes their $x = \frac{10}{k}$ and $y = k$ or their y from (a)(ii) and attempts to solve for k.
An equivalent approach would be to substitute their $x = \frac{10}{k}$ and $y = k$ or their y from (a)(ii) into $y = 3x + 1$ and attempt to solve for k
For this mark the value obtained may not be the maximum value for their range.
See below for alternative methods for finding the maximum value of *k*.

A1: For the correct **maximum** value for k
Condone $k \leqslant 6$
This may be seen embedded in an inequality but it must be the **maximum** value.

A1: Correct range in terms of k. Allow equivalent notation e.g. $k > 3$ and $k < 6$ or $(3, 6)$ but **not** $k > 3$ or $k < 6$ or $k > 3$, $k < 6$

Alternative methods for finding the maximum value of *k*:

$$kx-10+k=3x+1,\ -kx+10+k=3x+1$$
$$\Rightarrow x=\frac{11-k}{k-3},\ x=\frac{9+k}{3+k}\Rightarrow\frac{11-k}{k-3}=\frac{9+k}{3+k}$$
$$\Rightarrow 2k^2-2k-60=0\Rightarrow k=-5,\ 6$$

or

$$kx-10+k=3x+1,\ -kx+10+k=3x+1$$
$$\Rightarrow k=\frac{3x+11}{x+1},\ k=\frac{9-3x}{x-1}\Rightarrow\frac{3x+11}{x+1}=\frac{9-3x}{x-1}$$
$$\Rightarrow 3x^2+x-10=0\Rightarrow x=\frac{5}{3},\ -2$$
$$x=\frac{5}{3}\Rightarrow k=\frac{3\left(\frac{5}{3}\right)+11}{\frac{5}{3}+1}=6\ \text{ or }\ \frac{9-3\left(\frac{5}{3}\right)}{\frac{5}{3}-1}=6\ \text{ or }\ 3\left(\frac{5}{3}\right)+1=6$$

or

$$kx-10+k=3x+1\Rightarrow x=\frac{11-k}{k-3}$$
$$\frac{10}{k}=\frac{11-k}{k-3}\Rightarrow k^2-k-30=0\Rightarrow k=-5,\ 6$$

or

$$-kx+10+k=3x+1\Rightarrow k=\frac{9-3x}{x-1}$$
$$k=\frac{9-3\left(\frac{10}{k}\right)}{\left(\frac{10}{k}\right)-1}\Rightarrow k^2-k-30=0\Rightarrow k=-5,\ 6$$

These are some examples – there will be other correct methods.
If you are unsure if a particular approach deserves credit, use review.

In general allow M1 for a complete overall method condoning slips.
A1 can then be scored when $k=6$ is seen as the **maximum** value.