

# Mechanics 3

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## Past Paper Collection

Last updated: January 21, 2025

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<h2 style="margin: 0;">Thursday 9 January 2020</h2>			
Afternoon (Time: 1 hour 30 minutes)		Paper Reference <b>WME03/01</b>	
<h2 style="margin: 0;">Mathematics</h2> <h3 style="margin: 0;">International Advanced Subsidiary/Advanced Level</h3> <h3 style="margin: 0;">Mechanics M3</h3>			
<b>You must have:</b> Mathematical Formulae and Statistical Tables (Blue), calculator			Total Marks <div style="border: 1px solid black; height: 40px; width: 100%;"></div>

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Answers without working may not gain full credit.
- Whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$ , and give your answer to either 2 significant figures or 3 significant figures.

### Information

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- Find the smallest possible value of  $\mu$ .

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### Question 1 continued

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## Q1

**(Total 6 marks)**

- (b) Find the magnitude of the resultant force acting on  $P$  when  $t = 2$  (5)

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### Question 2 continued

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**(Total 10 marks)**

**Q2**





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**(Total 11 marks)**

### Q3



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**(Total 11 marks)**

## Q4



One end of a light inextensible string of length  $4a$  is attached to a fixed point  $A$  on a horizontal ceiling. A particle,  $P$ , of mass  $m$  is attached to the other end of the string. The particle is held in equilibrium at a vertical distance  $3a$  below the ceiling, with the string taut. The particle is then projected with speed  $\sqrt{7ag}$ , in the direction perpendicular to the string, in the vertical plane containing  $A$  and the string, as shown in Figure 2. In the subsequent motion the string remains taut.

- The point  $B$  is the lowest point of the path of  $P$ . The first time  $P$  passes through  $B$  the tension in the string is  $T_1$  and the second time  $P$  passes through  $B$  the tension in the string is  $T_2$

(b) find the ratio  $T_1: T_2$  in its simplest form.

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**Q5**

**(Total 11 marks)**



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## Q6

**(Total 13 marks)**



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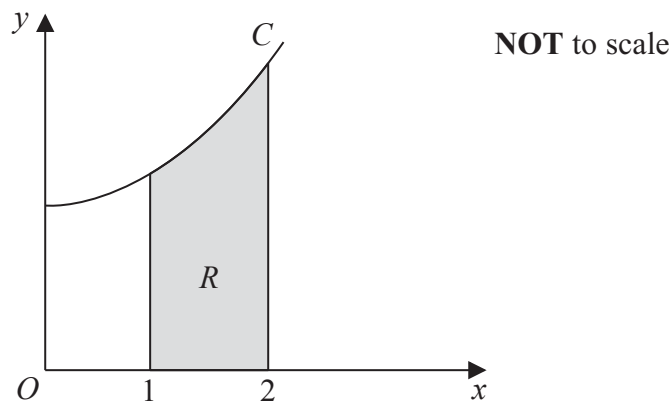


Figure 3

Figure 3 shows part of the curve  $C$  with equation  $y = x^2 + 4$ . The shaded region  $R$  is bounded by  $C$ , the line with equation  $x = 1$ , the  $x$ -axis and the line with equation  $x = 2$

The unit of length on each axis is one centimetre.

A uniform wooden solid,  $S$ , is made in the shape formed by rotating the region  $R$  through  $360^\circ$  about the  $x$ -axis.

(a) Using algebraic integration,

(i) show that the volume of  $S$  is  $\frac{613\pi}{15} \text{ cm}^3$

(ii) find, to 3 significant figures, the distance of the centre of mass of  $S$  from  $O$ .

(8)

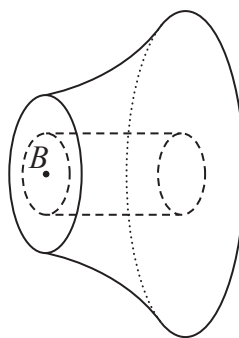


Figure 4

A solid,  $S_1$ , is formed by removing a solid cylinder of radius 3 cm and length 1 cm from  $S$ . A metal cylinder, of radius 3 cm and length 1 cm is placed in the resulting hole to form a new solid  $T$ , as shown in Figure 4. The axis of the metal cylinder coincides with the axis of symmetry of  $S_1$ . The point  $B$  is the centre of the smaller plane face of  $T$ . The mass per unit volume of  $S_1$  is  $M$  and the mass per unit volume of the metal cylinder is  $5M$ .

(b) Find the distance of the centre of mass of  $T$  from  $B$ .

(5)

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**Question 7 continued**

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**Question 7 continued**

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blank**Question 7 continued****Q7****(Total 13 marks)****END****TOTAL FOR PAPER: 75 MARKS**

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**Pearson Edexcel**  
International  
Advanced Level

Centre Number

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Candidate Number

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**Tuesday 13 October 2020**

Afternoon (Time: 1 hour 30 minutes)

Paper Reference **WME03/01**

**Mathematics**  
**International Advanced Subsidiary/Advanced Level**  
**Mechanics M3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

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Q1



A diagram of an inverted triangle. The horizontal base is dashed and labeled with a double-headed arrow and the text "3 m". The vertical height is indicated by a vertical arrow on the right side, labeled "2 m". The triangle is formed by two solid lines meeting at a vertex at the bottom, which is marked with a black dot.

A smooth bead of weight  $12\text{ N}$  is threaded onto a light elastic string of natural length  $3\text{ m}$ . The points  $A$  and  $B$  are on a horizontal ceiling, with  $AB = 3\text{ m}$ . One end of the string is attached to  $A$  and the other end of the string is attached to  $B$ .

(a) Find the tension in the string. (4)

The bead is now pulled down to a point vertically below its equilibrium position and released from rest.

(c) Find the elastic energy stored in the string at the instant when the bead is moving at its maximum speed. (2)

## Q2

**(Total 8 marks)**

A diagram showing a block of mass  $m$  on an inclined plane at an angle of  $30^\circ$ . The block is connected to a spring with a spring constant  $k$  and a natural length of  $0.3\text{ m}$ . The spring is attached to a point  $A$  on the incline. The block is at point  $B$ , and the distance between  $A$  and  $B$  is  $1\text{ m}$ . The block is released from rest at point  $B$ .

A particle  $P$  of mass  $2\text{ kg}$  is attached to one end of a light elastic spring, of natural length  $0.8\text{ m}$  and modulus of elasticity  $12\text{ N}$ . The other end of the spring is attached to a fixed point  $A$  on a rough plane. The plane is inclined at  $30^\circ$  to the horizontal. Initially  $P$  is held at rest on the plane at the point  $B$ , where  $B$  is below  $A$ , with  $AB = 0.3\text{ m}$  and  $AB$  lies along a line of greatest slope of the plane. The point  $C$  lies on the plane with  $AC = 1\text{ m}$ , as shown in Figure 3.

After being released  $P$  passes through the point  $C$ .

(7)

## Q3

**(Total 7 marks)**

- [You may assume that the volume of a sphere of radius  $r$  is  $\frac{4}{3} \pi r^3$ ]

A diagram of a hemisphere. The center of the flat circular base is marked with a black dot and labeled  $O$ . A point on the circular boundary of the base is marked with a black dot and labeled  $A$ .

The loaded solid  $S$  is placed on a horizontal plane. The curved surface of  $S$  is in contact with the plane and  $S$  rests in equilibrium with  $OA$  making an angle  $\alpha$  with the horizontal, where  $\tan \alpha = \sqrt{3}$

- (5)

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**Q4**

**(Total 10 marks)**



Show that

$$(a) \quad v^2 = \frac{4}{x^2} + 5 \tag{5}$$

$$(b) \quad t = \frac{a + \sqrt{bx^2 + c}}{d}, \text{ where } a, b, c \text{ and } d \text{ are integers to be found.} \quad (7)$$

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**Question 5 continued**

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**Q5**

**(Total 12 marks)**

- (2)

(6)

(3)

- (4)

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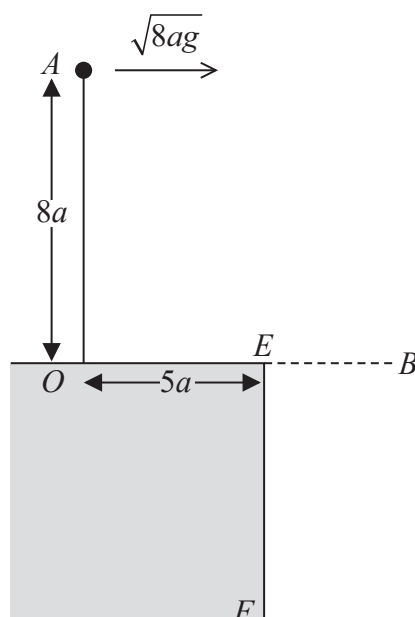
**Question 6 continued**

**Q6**

**(Total 15 marks)**



7.

**Figure 5**

A particle of mass  $m$  is attached to one end of a light inextensible string of length  $8a$ . The other end of the string is fixed to the point  $O$  on the smooth horizontal surface of a desk. The point  $E$  is on the edge of the desk, where  $OE = 5a$  and  $OE$  is perpendicular to the edge of the desk. The particle is held at the point  $A$ , vertically above  $O$ , with the string taut.

The particle is projected horizontally from  $A$  with speed  $\sqrt{8ag}$  in the direction  $OE$ , as shown in Figure 5.

When the particle is above the level of  $OE$  the particle is moving in a vertical circle with radius  $8a$ .

Given that, when the string makes an angle  $\theta$  with the upward vertical through  $O$ , the tension in the string is  $T$ ,

(a) show that  $T = 3mg(1 - \cos \theta)$  (7)

At the instant when the string is horizontal, the particle passes through the point  $B$ .

(b) Find the instantaneous change in the tension in the string as the particle passes through  $B$ . (3)

The particle hits the vertical side  $EF$  of the desk and rebounds. As a result of the impact, the particle loses one third of the kinetic energy it had immediately before the impact.

In the subsequent motion the string becomes slack when it makes an angle  $\alpha$  with the upward vertical through  $O$ .

(c) Show that  $\cos \alpha = \frac{7}{12}$  (7)

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

**Q7**

**TOTAL FOR PAPER = 75 MARKS**

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<b>Advanced Level</b>		<input type="text"/>	<input type="text"/>
<b>Thursday 14 January 2021</b>			
Morning (Time: 1 hour 30 minutes)		Paper Reference <b>WME03/01</b>	
<b>Mathematics</b>			
<b>International Advanced Subsidiary/Advanced Level</b>			
<b>Mechanics M3</b>			
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The region  $R$ , shown shaded in Figure 1, is bounded by the curve with equation  $y = \frac{1}{x}$ , the line with equation  $x = 1$ , the positive  $x$ -axis and the line with equation  $x = a$  where  $a > 1$

(a) Show that the volume of  $S$  is

$$\pi\left(1 - \frac{1}{a}\right) \quad (3)$$

(b) Find the  $x$  coordinate of the centre of mass of  $S$ . (5)

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### Question 1 continued

## Q1

**(Total 8 marks)**



The Earth is modelled as a fixed sphere of radius  $R$ .

(a) Show that the magnitude of the gravitational force on  $P$  is  $\frac{mgR^2}{(x + R)^2}$

**(3)**

A particle is released from rest from a point above the surface of the Earth. When the particle is at a distance  $R$  above the surface of the Earth, the particle has speed  $U$ .

Air resistance is modelled as being negligible.

(b) Find, in terms of  $U$ ,  $g$  and  $R$ , the speed of the particle when it strikes the surface of the Earth.

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**Q2**

**(Total 10 marks)**

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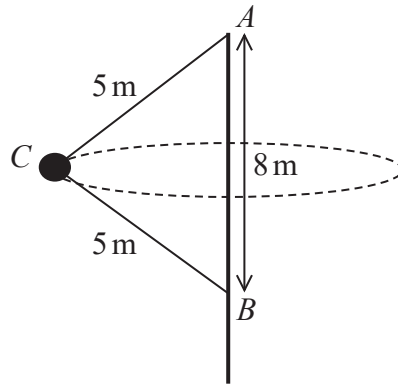


Figure 2

A fairground ride consists of a cabin  $C$  that travels in a horizontal circle with a constant angular speed about a fixed vertical central axis. The cabin is attached to one end of each of two rigid arms, each of length 5 m. The other end of the top arm is attached to the fixed point  $A$  at the top of the central axis of the ride. The other end of the lower arm is attached to the fixed point  $B$  on the central axis, where  $AB$  is 8 m, as shown in Figure 2.

Both arms are free to rotate about the central axis.

The arms are modelled as light inextensible rods.

The cabin, together with the people inside, is modelled as a particle.

The cabin completes one revolution every 2 seconds.

Given that the combined mass of the cabin and the people is 600 kg,

- (a) find
- the tension in the upper arm of the ride,
  - the tension in the lower arm of the ride.

(9)

In a refined model, it is assumed that both arms stretch to a length of 5.1 m.

- (b) State how this would affect the sum of the tensions in the two arms, justifying your answer.

(2)

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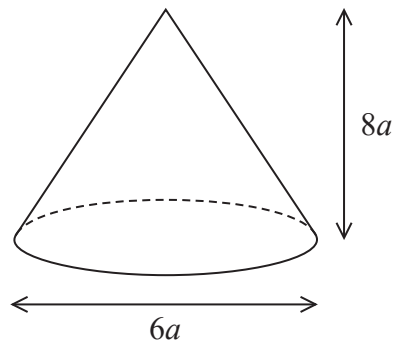
### Question 3 continued

**Q3**

**(Total 11 marks)**

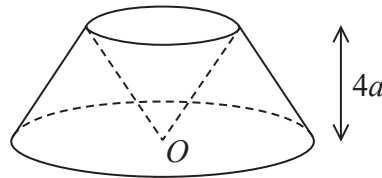


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**Figure 3**

A uniform right solid cone  $C$  has diameter  $6a$  and height  $8a$ , as shown in Figure 3.

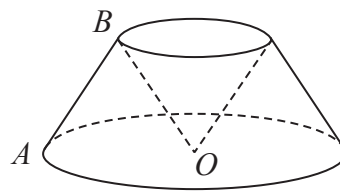
The solid  $S$  is formed by removing a cone of height  $4a$  from the top of  $C$  and then removing an identical, inverted cone. The vertex of the removed cone is at the point  $O$  in the centre of the base of  $C$ , as shown in Figure 4.

**Figure 4**

(a) Find the distance of the centre of mass of  $S$  from  $O$ .

**(5)**

The point  $A$  lies on the circumference of the base of  $S$  and the point  $B$  lies on the circumference of the top of  $S$ . The points  $O$ ,  $A$  and  $B$  all lie in the same vertical plane, as shown in Figure 5.

**Figure 5**

The solid  $S$  is freely suspended from the point  $B$  and hangs in equilibrium.

(b) Find the size of the angle that  $AB$  makes with the downward vertical.

**(4)**


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**Question 4 continued**

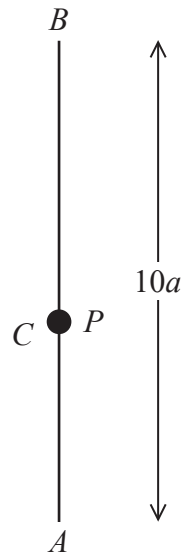
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**Question 4 continued**

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## Q4

5.

**Figure 6**

The fixed points,  $A$  and  $B$ , are a distance  $10a$  apart, with  $B$  vertically above  $A$ .

One end of a light elastic string, of natural length  $2a$  and modulus of elasticity  $2mg$ , is attached to a particle  $P$  of mass  $m$  and the other end is attached to  $A$ .

One end of another light elastic string, of natural length  $4a$  and modulus of elasticity  $6mg$ , is attached to  $P$  and the other end is attached to  $B$ .

The particle  $P$  rests in equilibrium at the point  $C$ , as shown in Figure 6.

(a) Show that each string has an extension of  $2a$ .

**(5)**

The particle  $P$  is now pulled down vertically, so that it is a distance  $a$  below  $C$  and then released from rest.

(b) Show that in the subsequent motion,  $P$  performs simple harmonic motion.

**(4)**

(c) Find, in terms of  $a$  and  $g$ , the speed of  $P$  when it is a distance  $\frac{7}{2}a$  above  $A$ .

**(4)**


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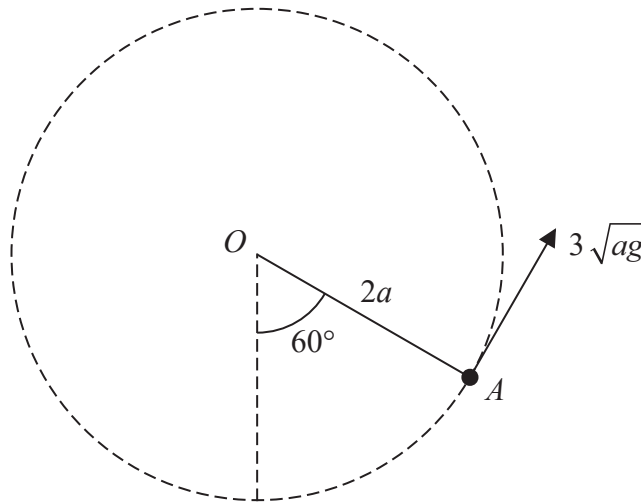
**Question 5 continued**

**Q5**

**(Total 13 marks)**



**6.**



### Figure 7

A particle of mass  $m$  is attached to one end of a light inextensible string of length  $2a$ . The other end of the string is attached to a fixed point  $O$ . The particle is initially held at the point  $A$  with the string taut and  $OA$  making an angle of  $60^\circ$  with the downward vertical.

The particle is then projected upwards with a speed of  $3\sqrt{ag}$ , perpendicular to  $OA$ , in the vertical plane containing  $OA$ , as shown in Figure 7.

In an initial model of the motion of the particle, it is assumed that the string does not break.

Using this model,

- (a) show that the particle performs complete vertical circles. (8)

In a refined model it is assumed that the string will break if the tension in it exceeds  $7mg$ .

Using this refined model,

- (b) show that the particle still performs complete vertical circles. (5)

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**Question 6 continued**

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**Question 6 continued**

**Q6**

**(Total 13 marks)**

A diagram showing a particle of mass  $m$  on an inclined plane. The plane is represented by a solid line making an angle  $\theta$  with a horizontal dashed line. The particle is represented by a black dot at point  $O$ . Point  $A$  is further up the incline. The angle  $\theta$  is indicated by an arc between the dashed horizontal line and the incline.

A particle  $P$  of mass  $0.5\text{kg}$  is attached to one end of a light elastic string of natural length  $2\text{m}$  and modulus of elasticity  $3\text{N}$ . The other end of the string is attached to a fixed point  $O$  on a rough plane. The plane is inclined at an angle  $\theta$  to the horizontal,

The coefficient of friction between  $P$  and the plane is  $\frac{\sqrt{5}}{5}$

The particle  $P$  then receives an impulse of magnitude  $4\text{ Ns}$ , directed up a line of greatest slope of the plane.

(a) Find the extension of the elastic string when  $P$  is at  $A$ . (8)

(b) Show that the particle does not remain at rest at  $A$ . (3)

[illegible]

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**Question 7 continued**



**Q7**

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Time 1 hour 30 minutes	Paper reference	<b>WME03/01</b>	
<b>Mathematics</b> <b>International Advanced Subsidiary/Advanced Level</b> <b>Mechanics M3</b>			
<b>You must have:</b> Mathematical Formulae and Statistical Tables (Yellow), calculator			Total Marks

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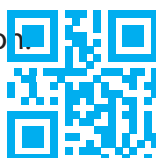
- Use **black** ink or ball-point pen.
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- Whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$ , and give your answer to either two significant figures or three significant figures.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
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- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.



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1.

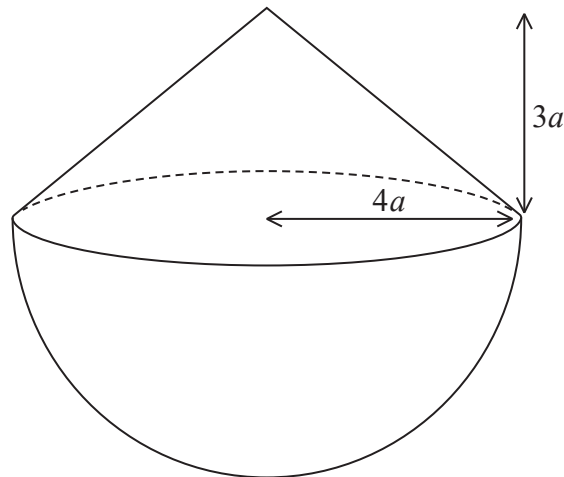


Figure 1

A hollow toy is formed by joining a uniform right circular conical shell  $C$ , with radius  $4a$  and height  $3a$ , to a uniform hemispherical shell  $H$ , with radius  $4a$ . The circular edge of  $C$  coincides with the circular edge of  $H$ , as shown in Figure 1.

The mass per unit area of  $C$  is  $\lambda$  and the mass per unit area of  $H$  is  $k\lambda$  where  $k$  is a constant.

Given that the centre of mass of the toy is a distance  $4a$  from the vertex of the cone, find the value of  $k$ .

(6)

**Question 1 continued**

(Total for Question 1 is 6 marks)

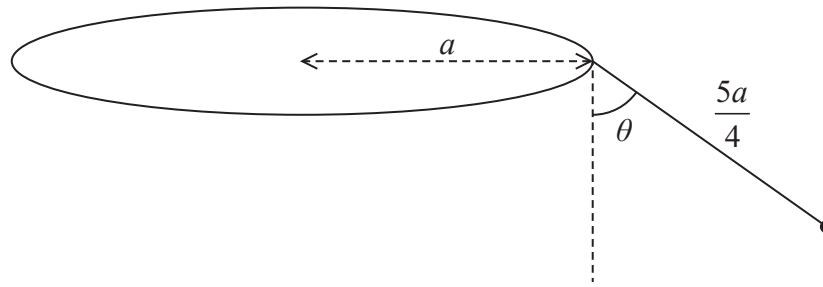


Figure 2 shows a fairground ride that consists of a chair of mass  $m$  attached to one end of a rigid arm of length  $\frac{5a}{4}$ . The other end of the arm is freely hinged to the rim of a thin horizontal circular disc of radius  $a$ . The disc rotates with constant angular speed  $\omega$  about a vertical axis through the centre of the disc. As the ride rotates the arm remains in a vertical plane through the centre of the disc. The arm makes a constant angle  $\theta$  with the vertical, where  $\tan \theta = \frac{3}{4}$ .

(a) Find the tension in the arm in terms of  $m$  and  $g$

**(3)**

(b) Find  $\omega$  in terms of  $a$  and  $g$

**Question 2 continued**

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### Question 2 continued

[illegible]

**Question 2 continued**

(Total for Question 2 is 9 marks)



3. The finite region enclosed by the curve with equation  $y = 3 - \sqrt{x}$  and the lines  $x = 0$  and  $y = 0$  is rotated through  $2\pi$  radians about the  $x$ -axis, to form a uniform solid  $S$ .

Use algebraic integration to

- (a) show that the volume of  $S$  is  $\frac{27}{2}\pi$  (4)

- (b) find the  $x$  coordinate of the centre of mass of  $S$ . (5)

**Question 3 continued**

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### Question 3 continued

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**Question 3 continued**

(Total for Question 3 is 9 marks)



**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

(Total for Question 4 is 9 marks)



5. A particle  $P$  of mass  $0.5\text{ kg}$  moves on the  $x$ -axis under the action of a single force.

At time  $t$  seconds,  $t \geq 0$

- $OP = x$  metres,  $0 \leq x < \frac{\pi}{2}$
- the force has magnitude  $\sin 2x\text{ N}$  and is directed towards the origin  $O$
- $P$  is moving in the positive  $x$  direction with speed  $v\text{ ms}^{-1}$

At time  $t = 0$ ,  $P$  passes through the origin with speed  $2\text{ ms}^{-1}$

(a) Show that  $v = 2 \cos x$

(6)

(b) Show that  $t = \frac{1}{2} \ln(\sqrt{2} + 1)$  when  $x = \frac{\pi}{4}$

(5)

**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 11 marks)**

6. A particle  $P$  of mass  $0.4\text{ kg}$  is attached to one end of a light elastic string, of natural length  $0.8\text{ m}$  and modulus of elasticity  $0.6\text{ N}$ . The other end of the string is fixed to a point  $A$  on a rough horizontal table. The coefficient of friction between  $P$  and the table is  $\frac{1}{7}$

The particle  $P$  is projected from  $A$ , with speed  $1.8 \text{ m s}^{-1}$ , along the surface of the table.

After travelling 0.8 m from  $A$ , the particle passes through the point  $B$  on the table.

- (a) Find the speed of  $P$  at the instant it passes through  $B$ . (5)

The particle  $P$  comes to rest at the point  $C$  on the table, where  $ABC$  is a straight line.

- (b) Find the total distance travelled by  $P$  as it moves directly from  $A$  to  $C$ . (6)

- (c) Show that  $P$  remains at rest at  $C$ . (3)

**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

(Total for Question 6 is 14 marks)





**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 17 marks)

**TOTAL FOR PAPER IS 75 MARKS**

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**Pearson Edexcel International Advanced Level**

**Time** 1 hour 30 minutes

**Paper reference** **WME03/01**

**Mathematics**

**International Advanced Subsidiary/Advanced Level**

**Mechanics M3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Yellow), calculator

**Total Marks**

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

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Turn over ►

- (2)

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**Question 1 continued**

**(Total 6 marks)**

## Q1



A particle  $P$  of mass  $2\text{ kg}$  is moving along the positive  $x$ -axis.

$O$  with speed  $v \text{ m s}^{-1}$  where  $v = \frac{1}{\sqrt{(2x+1)}}$

- (a) Find the magnitude of the resultant force acting on  $P$  when its speed is  $\frac{1}{3} \text{ m s}^{-1}$
- (6)**

(b) Find the value of  $t$  when  $P$  is 7.5 m from  $O$

**(5)**

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### Question 2 continued

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### Question 2 continued

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**Q2**

**(Total 11 marks)**

A particle  $P$  of mass  $m$  is attached to one end of a light elastic spring of natural length  $l$  and modulus of elasticity  $kmg$ , where  $k$  is a constant. The other end of the spring is fixed to horizontal ground.

The point  $E$  is at a height  $\frac{3}{5}l$  above the ground, as shown in Figure 1.

- The particle  $P$  is now moved a distance  $\frac{1}{4}l$  vertically downwards from  $E$  and released from rest. Air resistance is modelled as being negligible.

- (c) Find the speed of  $P$  as it passes through  $E$ . (4)

- (d) Find the time from the instant  $P$  is released to the first instant it passes through  $E$ . (2)

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**Question 3 continued**

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**Question 3 continued**

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**Q3**

**(Total 12 marks)**



- (b) Find the distance  $OB$ . (4)

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**Question 4 continued**

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**Question 4 continued**

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Q4

**(Total 11 marks)**

The diagram shows a vertical line with three points: A at the top, P(m) in the middle, and R(m) at the bottom. A horizontal dashed ellipse is centered on the vertical line. A point P(m) is marked on the right side of the ellipse. A line segment of length  $l$  connects point A to point P(m). Another line segment of length  $l$  connects point P(m) to point R(m). The distance from point A to the center of the ellipse is labeled  $\frac{6}{5}l$ . The distance from the center of the ellipse to point R(m) is labeled  $\frac{l}{5}$ .

A small smooth ring  $R$  of mass  $m$  is threaded on to a thin smooth fixed vertical pole. One end of a light inextensible string of length  $2l$  is attached to a point  $A$  on the pole. The other end of the string is attached to  $R$ . A particle  $P$  of mass  $m$  is attached to the midpoint of the string. The particle  $P$  moves with constant angular speed in a horizontal circle, with both halves of the string taut, and  $AR = \frac{6l}{5}$ , as shown in Figure 2.

(a) Show that the tension in the lower half of the string is  $\frac{5mg}{3}$

**(3)**

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**Question 5 continued**

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**Question 5 continued**

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**(Total 11 marks)**

**Q5**



6.

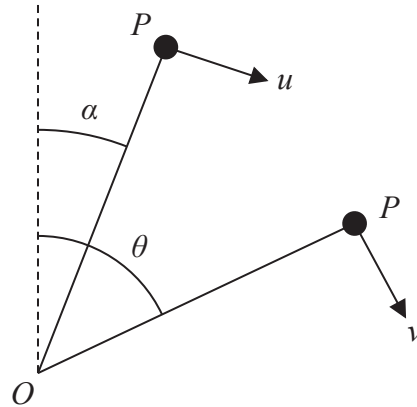


Figure 3

A light rod of length  $a$  is free to rotate in a vertical plane about a horizontal axis through one end  $O$ . A particle  $P$  of mass  $m$  is attached to the other end of the rod. The particle  $P$  is held at rest with the rod making an angle  $\alpha$  with the upward vertical through  $O$ ,

where  $\tan \alpha = \frac{3}{4}$

The particle  $P$  is then projected with speed  $u$  in a direction which is perpendicular to the rod. At the instant when the rod makes an angle  $\theta$  with the upward vertical through  $O$ , the speed of  $P$  is  $v$ , as shown in Figure 3.

Air resistance is assumed to be negligible.

(a) Show that  $v^2 = u^2 + \frac{2ag}{5}(4 - 5\cos\theta)$  (4)

It is given that  $u^2 = \frac{6ag}{5}$  and  $P$  moves in complete vertical circles.

When  $\theta = \beta$ , the force exerted on  $P$  by the rod is zero.

(b) Find the value of  $\cos\beta$  (6)

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**Question 6 continued**

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**Question 6 continued**

[illegible]

**Q6**

**(Total 10 marks)**

7. [You may assume that the volume of a cone of height  $h$  and base radius  $r$  is  $\frac{1}{3} \pi r^2 h$ .]

A uniform solid right circular cone  $C$ , with vertex  $V$ , has base radius  $r$  and height  $h$ .

- (a) Show that the centre of mass of  $C$  is  $\frac{3}{4}h$  from  $V$

(4)

A solid  $F$ , shown below in Figure 4, is formed by removing the solid right circular cone  $C'$  from  $C$ , where cone  $C'$  has height  $\frac{1}{3}h$  and vertex  $V$

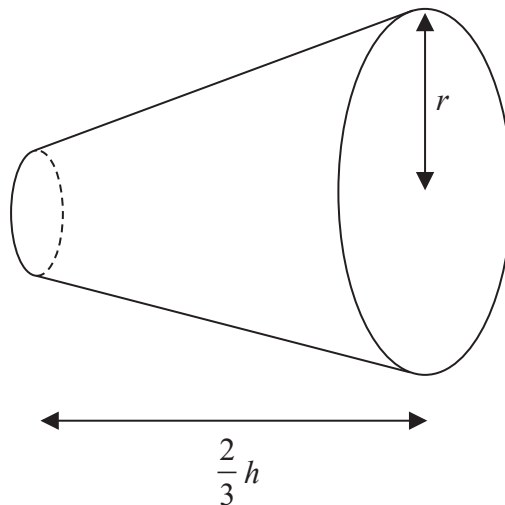


Figure 4

- (b) Show that the distance of the centre of mass of  $F$  from its larger plane face is  $\frac{3}{13}h$

(5)

The solid  $F$  rests in equilibrium with its curved surface in contact with a horizontal plane.

- (c) Show that  $13r^2 \leq 17h^2$

(5)

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**Question 7 continued**

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**Question 7 continued**

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**Q7**

**TOTAL FOR PAPER = 75 MARKS**

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**Pearson Edexcel International Advanced Level**

**Time** 1 hour 30 minutes

**Paper reference** **WME03/01**

**Mathematics**  
**International Advanced Subsidiary/Advanced Level**  
**Mechanics M3**

**You must have:**  
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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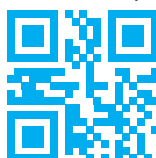
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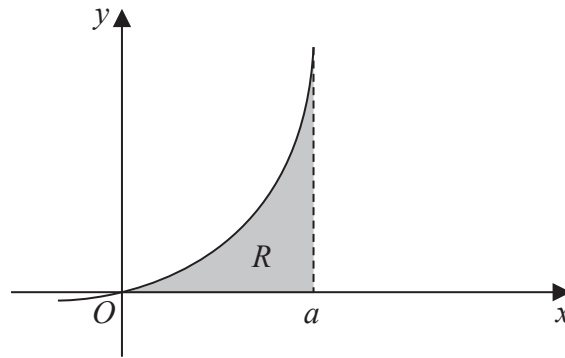
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1.



### Figure 1

A uniform lamina is in the shape of the region  $R$ .

Region  $R$  is bounded by the curve with equation  $y = x(x + a)$  where  $a$  is a positive constant, the positive  $x$ -axis and the line with equation  $x = a$ , as shown shaded in Figure 1.

Find the **y coordinate** of the centre of mass of the lamina.

(7)

### Question 1 continued

**(Total for Question 1 is 7 marks)**

2.

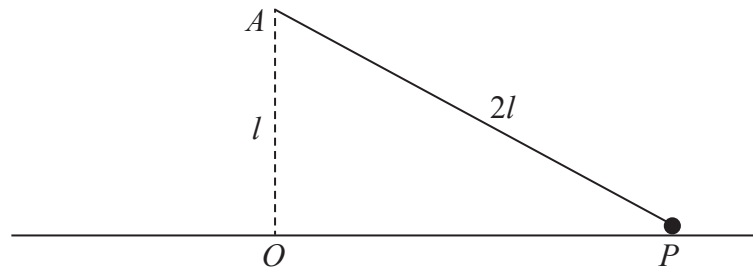


Figure 2

A particle  $P$  of mass  $m$  is attached to one end of a light inextensible string of length  $2l$ . The other end of the string is attached to a fixed point  $A$  above a smooth horizontal floor. The particle moves in a horizontal circle on the floor with the string taut. The centre  $O$  of the circle is vertically below  $A$  with  $OA = l$ , as shown in Figure 2.

The particle moves with constant angular speed  $\omega$  and remains in contact with the floor.

Show that

$$\omega \leq \sqrt{\frac{g}{l}} \quad (8)$$

**Question 2 continued**

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### Question 2 continued

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### Question 2 continued

**(Total for Question 2 is 8 marks)**



3. A particle  $P$  of mass  $m$  kg is initially held at rest at the point  $O$  on a smooth inclined plane. The plane is inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{2}{5}$

The particle is released from rest and slides down the plane against a force which acts towards  $O$ . The force has magnitude  $\frac{1}{3}mx^2$  N, where  $x$  metres is the distance of  $P$  from  $O$ .

- (a) Find the speed of  $P$  when  $x = 2$  (6)

The particle first comes to instantaneous rest at the point  $A$ .

- (b) Find the distance  $OA$ . (2)

### Question 3 continued

[illegible]

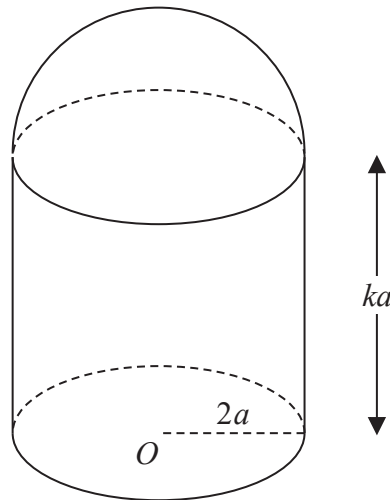
### Question 3 continued

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**Question 3 continued**

(Total for Question 3 is 8 marks)

4.



### Figure 3

A thin uniform right hollow cylinder, of radius  $2a$  and height  $ka$ , has a base but no top. A thin uniform hemispherical shell, also of radius  $2a$ , is made of the same material as the cylinder. The hemispherical shell is attached to the end of the cylinder forming a container  $C$ . The open circular rim of the cylinder coincides with the rim of the hemispherical shell. The centre of the base of  $C$  is  $O$ , as shown in Figure 3.

- (a) Show that the distance from  $O$  to the centre of mass of  $C$  is

$$\frac{(k^2 + 4k + 4)}{2(k + 3)} a \quad (5)$$

The container is placed with its circular base on a plane which is inclined at  $30^\circ$  to the horizontal. The plane is sufficiently rough to prevent  $C$  from sliding. The container is on the point of toppling.

- (b) Find the value of  $k$ . (3)

**Question 4 continued**

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**Question 4 continued**

[illegible]

**Question 4 continued**

(Total for Question 4 is 8 marks)



5. A particle  $P$  is moving along the  $x$ -axis. At time  $t$  seconds the displacement of  $P$  from the origin  $O$  is  $x$  metres, where  $x = 4 \cos\left(\frac{1}{5}\pi t\right)$

- (a) Prove that  $P$  is moving with simple harmonic motion. (3)

- (b) Find the period of the motion. (2)

- (c) State the amplitude of the motion. **(1)**

- (d) Find, in terms of  $\pi$ , the maximum speed of  $P$

The points  $A$  and  $B$  lie on the  $x$ -axis, on opposite sides of  $O$ , with  $OA = 1.5$  m and  $OB = 2.5$  m.

- (e) Find the time taken by  $P$  to move directly from  $A$  to  $B$ . (4)

**Question 5 continued**

[illegible]

**Question 5 continued**

[illegible]

**Question 5 continued**

**(Total for Question 5 is 12 marks)**

6. A particle  $P$  of mass  $1.2\text{ kg}$  is attached to the midpoint of a light elastic string of natural length  $0.5\text{ m}$  and modulus of elasticity  $\lambda$  newtons.

The fixed points  $A$  and  $B$  are  $0.8\text{ m}$  apart on a horizontal ceiling. One end of the string is attached to  $A$  and the other end of the string is attached to  $B$ .

Initially  $P$  is held at rest at the midpoint  $M$  of the line  $AB$  and the tension in the string is 30 N.

- (a) Show that  $\lambda = 50$

**(3)**

The particle is now held at rest at the point  $C$ , where  $C$  is  $0.3\text{ m}$  vertically below  $M$ . The particle is released from rest.

- (b) Find the magnitude of the initial acceleration of  $P$

(6)

- (c) Find the speed of  $P$  at the instant immediately before it hits the ceiling.

(6)

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

(Total for Question 6 is 15 marks)



7.

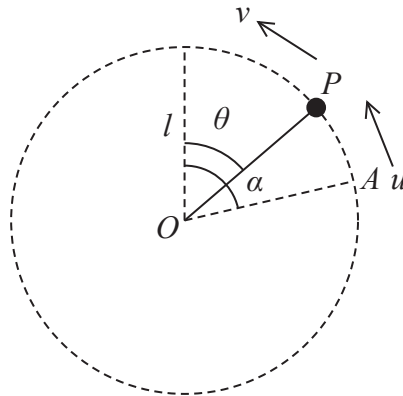


Figure 4

A particle  $P$  of mass  $m$  is attached to one end of a light rod of length  $l$ . The other end of the rod is attached to a fixed point  $O$ . The rod can rotate freely in a vertical plane about  $O$ . The particle is projected with speed  $u$  from a point  $A$ . The line  $OA$  makes an angle  $\alpha$  with the upward vertical through  $O$ , where  $\alpha < \frac{\pi}{2}$

When  $OP$  makes an angle  $\theta$  with the upward vertical through  $O$ , the speed of  $P$  is  $v$ , as shown in Figure 4.

- (a) Show that  $v^2 = u^2 - 2gl(\cos \theta - \cos \alpha)$  (4)

Given that  $\cos \alpha = \frac{2}{5}$  and that  $u = \sqrt{3gl}$

- (b) show that  $P$  moves in a complete vertical circle. (4)

As the rod rotates, the least tension in the rod is  $T$  and the greatest tension is  $kT$

- (c) Find the exact value of  $k$  (9)

**Question 7 continued**

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**Question 7 continued**

[illegible]

**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 17 marks)

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

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Centre Number					Candidate Number				
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**Pearson Edexcel International Advanced Level**

**Time** 1 hour 30 minutes

**Paper reference** **WME03/01**

**Mathematics**

**International Advanced Subsidiary/Advanced Level**

**Mechanics M3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Yellow), calculator

**Total Marks**

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

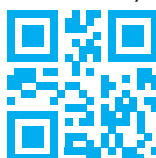
- Use **black** ink or ball-point pen.
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$ , and give your answer to either two significant figures or three significant figures.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
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### Advice

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Turn over ►

1. A particle  $P$  moves in a straight line with simple harmonic motion between two fixed points  $A$  and  $B$ . The particle performs 2 complete oscillations per second. The midpoint of  $AB$  is  $O$  and the midpoint of  $OA$  is  $C$

The length of  $AB$  is 0.6 m.

- (a) Find the maximum speed of  $P$  (4)
- (b) Find the time taken by  $P$  to move directly from  $O$  to  $C$  (2)

### Question 1 continued

**(Total for Question 1 is 6 marks)**



2.

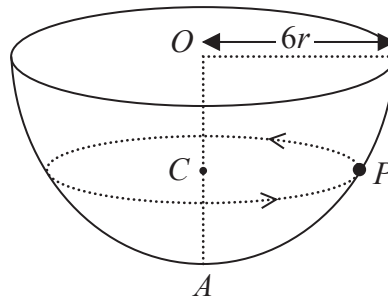


Figure 1

A hemispherical bowl of internal radius  $6r$  is fixed with its circular rim horizontal. The centre of the circular rim is  $O$  and the point  $A$  on the surface of the bowl is vertically below  $O$ . A particle  $P$  moves in a horizontal circle, with centre  $C$ , on the smooth inner surface of the bowl. The particle moves with constant angular speed  $\sqrt{\frac{g}{4r}}$ . The point  $C$  lies on  $OA$ , as shown in Figure 1.

Find, in terms of  $r$ , the distance  $OC$

(9)

### Question 2 continued

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**Question 2 continued**

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**Question 2 continued**

(Total for Question 2 is 9 marks)

**3.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A particle  $P$  is moving along a straight line.

At time  $t$  seconds,  $P$  is a distance  $x$  metres from a fixed point  $O$  on the line and is

moving away from  $O$  with speed  $\frac{50}{2x+3} \text{ m s}^{-1}$

(a) Find the deceleration of  $P$  when  $x = 12$

(5)

Given that  $x = 4$  when  $t = 1$

(b) find the value of  $t$  when  $x = 12$

(5)

**Question 3 continued**

[illegible]

**Question 3 continued**

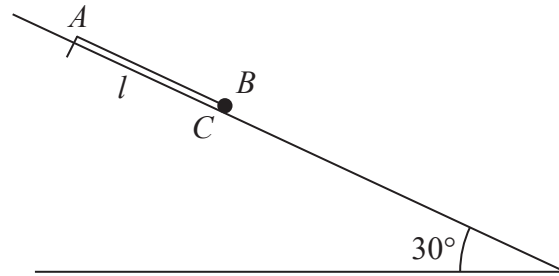
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**Question 3 continued**

(Total for Question 3 is 10 marks)



4.



## Figure 2

One end of a light elastic string, of natural length  $l$  and modulus of elasticity  $\lambda$ , is fixed to a point  $A$  on a smooth plane. The plane is inclined at  $30^\circ$  to the horizontal.

A small ball  $B$  of mass  $m$  is attached to the other end of the elastic string. Initially,  $B$  is held at rest at the point  $C$  on the plane with the elastic string lying along a line of greatest slope of the plane.

The point  $C$  is below  $A$  and  $AC = l$ , as shown in Figure 2.

The ball is released and comes to instantaneous rest at a point  $D$  on the plane.

The points  $A$ ,  $C$  and  $D$  all lie along a line of greatest slope of the plane and  $AD = \frac{5l}{4}$

The ball is modelled as a particle and air resistance is modelled as being negligible.

Using the model,

- (a) show that  $\lambda = 4mg$  (4)
- (b) find, in terms of  $g$  and  $l$ , the greatest speed of  $B$  as it moves from  $C$  to  $D$  (7)

**Question 4 continued**

[illegible]

**Question 4 continued**

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**Question 4 continued**

(Total for Question 4 is 11 marks)



**Question 5 continued**

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**Question 5 continued**

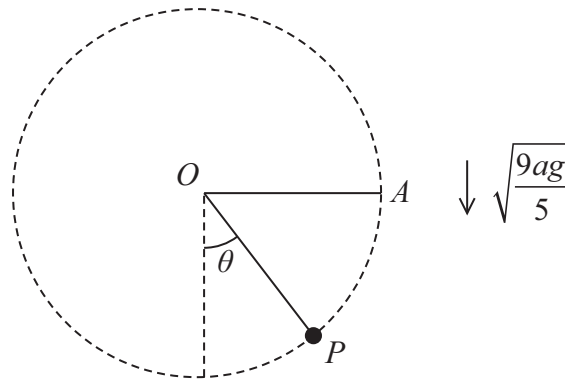
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**Question 5 continued**

(Total for Question 5 is 10 marks)



**6.**



### Figure 4

A particle  $P$  of mass  $m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point  $O$ . The particle is held at the point  $A$ , where  $OA = a$  and  $OA$  is horizontal, as shown in Figure 4.

The particle is projected vertically downwards with speed  $\sqrt{\frac{9ag}{5}}$

When the string makes an angle  $\theta$  with the downward vertical through  $O$  and the string is still taut, the tension in the string is  $S$ .

(a) Show that  $S = \frac{3}{5}mg(5\cos\theta + 3)$  (6)

At the instant when the string becomes slack, the speed of  $P$  is  $v$

(b) Show that  $v = \sqrt{\frac{3ag}{5}}$  (3)

(c) Find the maximum height of  $P$  above the horizontal level of  $O$  (4)

**Question 6 continued**

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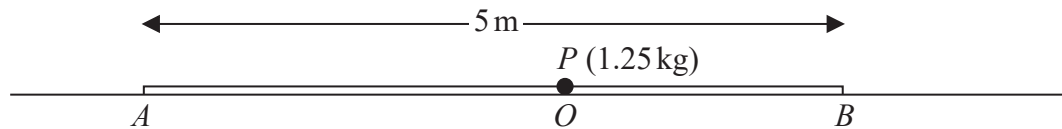
**Question 6 continued**

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**Question 6 continued**

(Total for Question 6 is 13 marks)

7.



### Figure 5

Figure 5 shows two fixed points,  $A$  and  $B$ , which are 5 m apart on a smooth horizontal floor.

A particle  $P$  of mass  $1.25\text{ kg}$  is attached to one end of a light elastic string, of natural length  $2\text{ m}$  and modulus of elasticity  $20\text{ N}$ . The other end of the string is attached to  $A$

A second light elastic string, of natural length 1.2 m and modulus of elasticity  $\lambda$  newtons, has one end attached to  $P$  and the other end attached to  $B$

Initially  $P$  rests in equilibrium at the point  $O$ , where  $AO = 3$  m

- (a) Show that  $\lambda = 15$  (3)

The particle is now projected along the floor towards  $B$

At time  $t$  seconds,  $P$  is a displacement  $x$  metres from  $O$  in the direction  $OB$

- (b) Show that, while both strings are taut,  $P$  moves with simple harmonic motion where  $\ddot{x} = -18x$
- (4)**

The initial speed of  $P$  is  $10\text{ m s}^{-1}$

- (c) Find the speed of  $P$  at the instant when the string  $PB$  becomes slack. (3)

Both strings are taut for  $T$  seconds during one complete oscillation.

- (d) Find the value of  $T$  (6)

[illegible]

**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 16 marks)

**TOTAL FOR PAPER IS 75 MARKS**

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Centre Number		Candidate Number	
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**Pearson Edexcel International Advanced Level**

**Time** 1 hour 30 minutes **Paper reference** **WME03/01**

**Mathematics**  
**International Advanced Subsidiary/Advanced Level**  
**Mechanics M3**

**You must have:**  
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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Turn over ►

1.

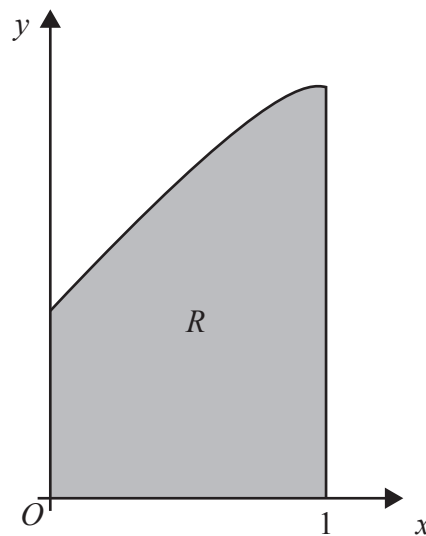


Figure 1

The shaded region  $R$  is bounded by the  $x$ -axis, the line with equation  $x = 1$ , the curve with equation  $y = 1 + \sqrt{x}$  and the  $y$ -axis, as shown in Figure 1. The unit of length on both of the axes is 1 m.

The region  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution which is used to model a uniform solid  $S$ .

Show, using the model and **algebraic integration**, that

(a) the volume of  $S$  is  $\frac{17\pi}{6} \text{ m}^3$  (3)

(b) the centre of mass of  $S$  is  $\frac{49}{85} \text{ m}$  from  $O$ . (5)

**Question 1 continued**

(Total for Question 1 is 8 marks)

2.

**Figure 2**

A light elastic string  $AB$  has natural length  $l$  and modulus of elasticity  $2mg$ .

The end  $A$  of the elastic string is attached to a fixed point. The other end  $B$  is attached to a particle of mass  $m$ . The particle is held in equilibrium, with the elastic string taut and horizontal, by a force of magnitude  $F$ . The line of action of the force and the elastic string lie in the same vertical plane. The direction of the force makes an angle  $\alpha$ , where

$\tan \alpha = \frac{3}{4}$ , with the upward vertical, as shown in Figure 2.

Find, in terms of  $l$ , the length  $AB$ .

**(6)**

**Question 2 continued**

(Total for Question 2 is 6 marks)

3.

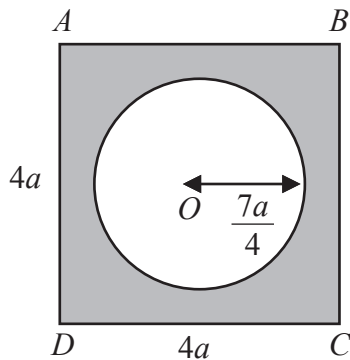


Figure 3

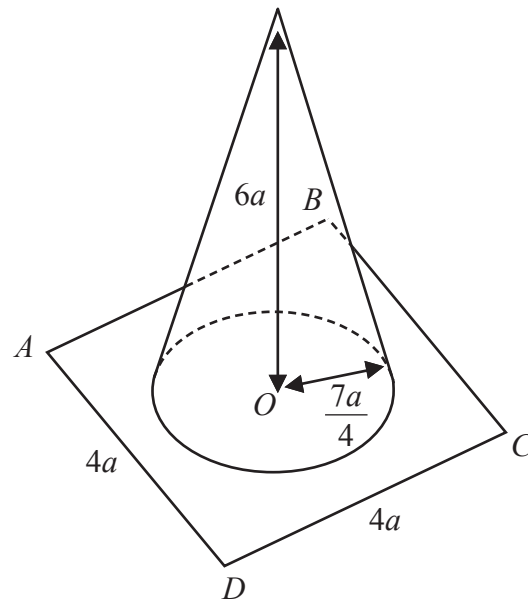


Figure 4

A square  $ABCD$  of side  $4a$  is made from thin uniform cardboard. The centre of the square is  $O$ . A circle with centre  $O$  and radius  $\frac{7a}{4}$  is then removed from the square to form a template  $T$ , shown shaded in Figure 3.

A right conical shell, with no base, has radius  $\frac{7a}{4}$  and perpendicular height  $6a$ .

The shell is made of the same thin uniform cardboard as  $T$ .

The shell is attached to  $T$  so that the circumference of the end of the shell coincides with the circumference of the circle centre  $O$ , to form the hat  $H$ , shown in Figure 4.

[The surface area of a right conical shell of radius  $r$  and slant height  $l$  is  $\pi rl$ .]

(a) Show that the exact distance of the centre of mass of  $H$  from  $O$  is

$$\frac{175\pi a}{(63\pi + 128)} \quad (8)$$

A fixed rough plane is inclined to the horizontal at an angle  $\alpha$ . The hat  $H$  is placed on the plane, with  $ABCD$  in contact with the plane, and  $AB$  parallel to a line of greatest slope of the plane. The plane is sufficiently rough to prevent the hat from sliding down the plane.

Given that the hat is on the point of toppling,

(b) find the exact value of  $\tan \alpha$ , giving your answer in simplest form.

(2)

### Question 3 continued



### Question 3 continued

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**Question 3 continued**

(Total for Question 3 is 10 marks)

4.

**In this question you must show all stages in your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A particle  $P$  is moving along the  $x$ -axis.

At time  $t$  seconds, where  $0 \leq t \leq \frac{2}{3}$ ,  $P$  is  $x$  metres from the origin  $O$  and is moving with velocity  $v \text{ m s}^{-1}$  in the positive  $x$  direction where

$$v = (2x + 1)^{\frac{3}{2}}$$

When  $t = 0$ ,  $P$  passes through  $O$ .

- (a) Find the value of  $x$  when the acceleration of  $P$  is  $243 \text{ m s}^{-2}$  (4)
- (b) Find  $v$  in terms of  $t$ . (6)

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**Question 4 continued**

(Total for Question 4 is 10 marks)

5.

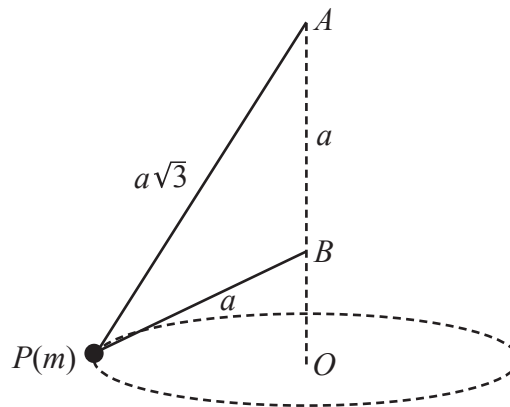


Figure 5

A particle  $P$  of mass  $m$  is attached to one end of a light inextensible string of length  $a\sqrt{3}$ . The other end of the string is attached to a fixed point  $A$ . The particle  $P$  is also attached to one end of a second light inextensible string of length  $a$ . The other end of this string is attached to a fixed point  $B$ , where  $B$  is vertically below  $A$ , with  $AB = a$ .

The particle  $P$  moves in a horizontal circle with centre  $O$ , where  $O$  is vertically below  $B$ .

The particle  $P$  moves with constant angular speed  $\omega$ , with both strings taut, as shown in Figure 5.

- (a) Show that the upper string makes an angle of  $30^\circ$  with the downward vertical and the lower string makes an angle of  $60^\circ$  with the downward vertical. (2)
- (b) Show that the tension in the upper string is  $\frac{1}{2}m\sqrt{3}(2g - a\omega^2)$ . (8)
- (c) Show that  $\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$  (4)

**Question 5 continued**

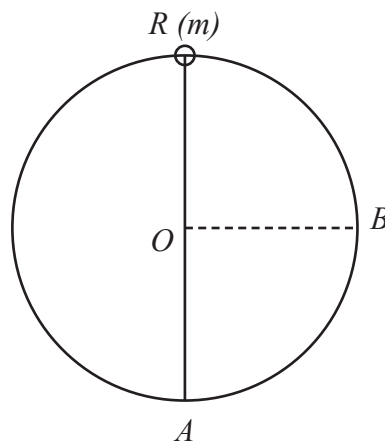
**Question 5 continued**

**Question 5 continued**

(Total for Question 5 is 14 marks)



6.

**Figure 6**

A small smooth ring  $R$  of mass  $m$  is threaded on to a smooth wire in the shape of a circle with centre  $O$  and radius  $l$ . The wire is fixed in a vertical plane. The ring  $R$  is attached to one end of a light elastic string of natural length  $l$  and modulus of elasticity  $mg$ . The other end of the elastic string is attached to  $A$ , the lowest point of the wire. The point  $B$  is on the wire and  $OB$  is horizontal.

The ring  $R$  is at rest at the highest point of the wire, as shown in Figure 6.

The ring  $R$  is slightly disturbed from rest and slides along the wire.

At the instant when  $R$  reaches the point  $B$ , the speed of  $R$  is  $v$  and the magnitude of the force exerted on  $R$  by the wire is  $N$ .

(a) Show that

$$v^2 = 2gl\sqrt{2} \quad (7)$$

(b) Show that

$$N = \frac{1}{2}mg(5\sqrt{2} - 2) \quad (7)$$

**Question 6 continued**

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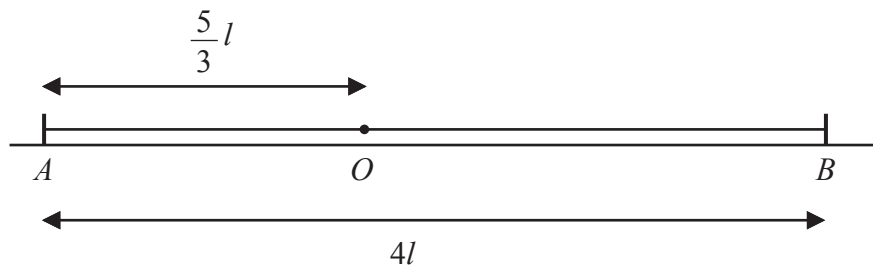
**Question 6 continued**

[illegible]

**Question 6 continued**

(Total for Question 6 is 14 marks)

7.

**Figure 7**

Two points  $A$  and  $B$  lie on a smooth horizontal table where  $AB = 4l$ .

A particle  $P$  of mass  $m$  is attached to one end of a light elastic spring of natural length  $l$  and modulus of elasticity  $2mg$ . The other end of the spring is attached to  $A$ . The particle  $P$  is also attached to one end of another light elastic spring of natural length  $l$  and modulus of elasticity  $mg$ . The other end of the spring is attached to  $B$ .

The particle  $P$  rests in equilibrium on the table at the point  $O$ , where  $AO = \frac{5}{3}l$ , as shown in Figure 7.

The particle  $P$  is moved a distance  $\frac{1}{2}l$  along the table, from  $O$  towards  $A$ , and released from rest.

(a) Show that  $P$  moves with simple harmonic motion of period  $T$ , where

$$T = 2\pi\sqrt{\frac{l}{3g}} \quad (6)$$

(b) Find, in terms of  $l$  and  $g$ , the speed of  $P$  as it passes through  $O$ . (1)

(c) Find, in terms of  $g$ , the maximum acceleration of  $P$ . (1)

(d) Find the exact time, in terms of  $l$  and  $g$ , from the instant when  $P$  is released from rest to the instant when  $P$  is first moving with speed  $\frac{3}{4}\sqrt{gl}$  (5)

**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 13 marks)

**TOTAL FOR PAPER IS 75 MARKS**

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**Pearson Edexcel International Advanced Level**

**Tuesday 6 June 2023**

Morning (Time: 1 hour 30 minutes) **Paper reference** **WME03/01**

**Mathematics**

**International Advanced Subsidiary/Advanced Level**

**Mechanics M3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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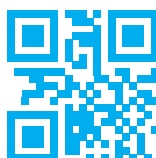
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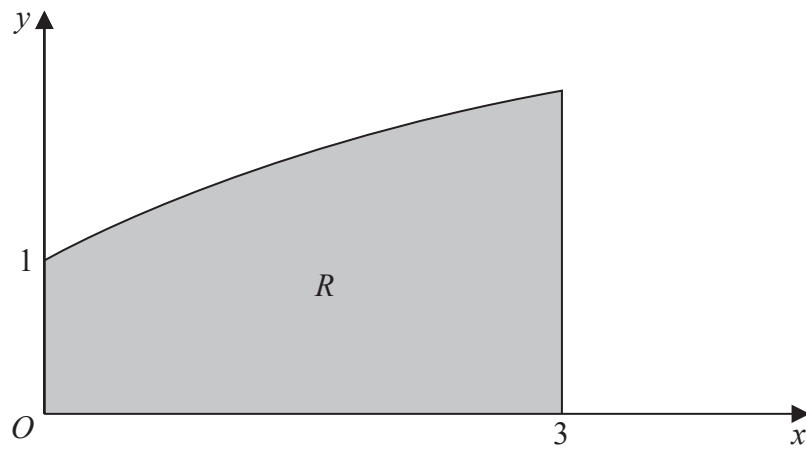


Turn over ►

**1.**

**In this question you must show all stages in your working.**

**Solutions relying on calculator technology are not acceptable.**



### Figure 1

The finite region  $R$ , shown shaded in Figure 1, is bounded by the  $x$ -axis, the line with equation  $x = 3$ , the curve with equation  $y = \sqrt{(x + 1)}$  and the  $y$ -axis.

Find the **y coordinate** of the centre of mass of a uniform lamina in the shape of  $R$ .

(5)

**Question 1 continued**

(Total for Question 1 is 5 marks)

2.

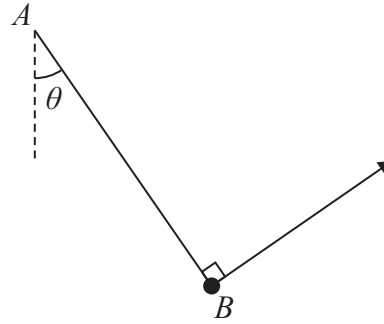


Figure 2

A light elastic string  $AB$  has modulus of elasticity  $2mg$  and natural length  $ka$ , where  $k$  is a constant.

The end  $A$  of the elastic string is attached to a fixed point. The other end  $B$  is attached to a particle of mass  $m$ . The particle is held in equilibrium, with the elastic string taut, by a force that acts in a direction that is perpendicular to the string. The line of action of the force and the elastic string lie in the same vertical plane. The string makes an angle  $\theta$  with the downward vertical at  $A$ , as shown in Figure 2.

Given that the length  $AB = \frac{21}{10}a$  and  $\tan \theta = \frac{3}{4}$ , find the value of  $k$ .

(6)

### Question 2 continued

[illegible]

### Question 2 continued

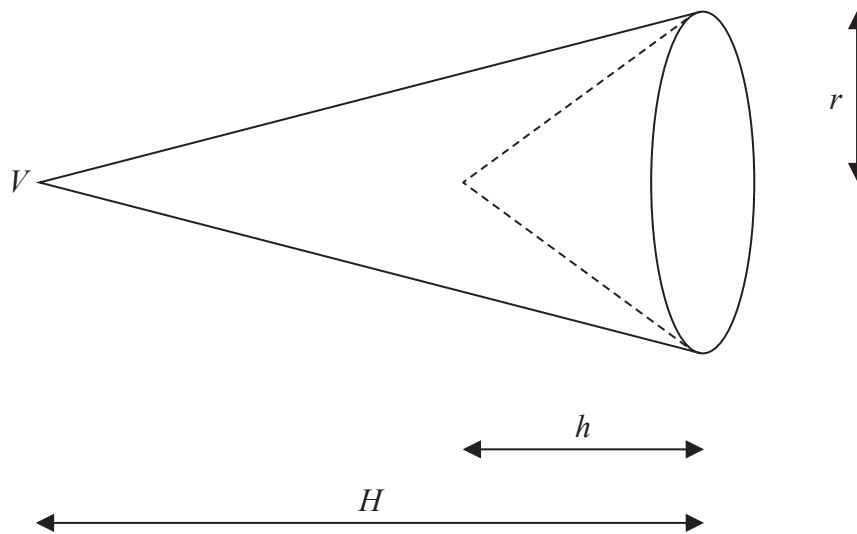
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**Question 2 continued**

(Total for Question 2 is 6 marks)



3. A uniform solid right circular cone  $C$  has base radius  $r$ , height  $H$  and vertex  $V$ . A uniform solid  $S$ , shown in Figure 3, is formed by **removing** from  $C$  a uniform solid right circular cone of height  $h$  ( $h < H$ ) that has the same base and axis of symmetry as  $C$ .



### Figure 3

- (a) Show that the distance of the centre of mass of  $S$  from  $V$  is

$$\frac{1}{4}(3H-h) \tag{5}$$

The solid  $S$  is suspended by two vertical light strings. The first string is attached to  $S$  at  $V$  and the second string is attached to  $S$  at a point on the circumference of the circular base of  $S$ .

The solid  $S$  hangs freely in equilibrium with its axis of symmetry horizontal. The tension in the first string is  $T_1$  and the tension in the second string is  $T_2$ .

- (b) Find  $\frac{T_1}{T_2}$ , giving your answer in terms of  $H$  and  $h$ , in its simplest form.

[illegible]

### Question 3 continued

[illegible]

### Question 3 continued

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Question 3 continued**

(Total for Question 3 is 8 marks)



**Question 4 continued**

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**Question 4 continued**

[illegible]

**Question 4 continued**

(Total for Question 4 is 12 marks)



5. The centre of the Earth is the point  $O$  and the Earth is modelled as a fixed sphere of radius  $R$ .

At time  $t = 0$ , a particle  $P$  is projected vertically upwards with speed  $U$  from a point  $A$  on the surface of the Earth.

At time  $t$  seconds, where  $t \geq 0$

- $P$  is a distance  $x$  from  $O$
- $P$  is moving with speed  $v$
- $P$  has acceleration of magnitude  $\frac{gR^2}{x^2}$  directed towards  $O$

Air resistance is modelled as being negligible.

(a) Show that  $v^2 = \frac{2gR^2}{x} + U^2 - 2gR$

**(6)**

Particle  $P$  is first moving with speed  $\frac{1}{2}\sqrt{gR}$  at the point  $B$ .

(b) Given that  $U = \sqrt{gR}$  find, in terms of  $R$ , the distance  $AB$ .

(c) Find, in terms of  $g$  and  $R$ , the smallest value of  $U$  that would ensure that  $P$  never comes to rest, explaining your reasoning. (3)

**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

(Total for Question 5 is 12 marks)



**Question 6 continued**

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**Question 6 continued**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Question 6 continued**

(Total for Question 6 is 16 marks)



7. A particle  $P$  of mass  $m$  is attached to one end of a light elastic string of natural length  $l$ . The other end of the string is attached to a fixed point on a ceiling. The particle  $P$  hangs in equilibrium at a distance  $D$  below the ceiling.

The particle  $P$  is now pulled vertically downwards until it is a distance  $3l$  below the ceiling and released from rest.

Given that  $P$  comes to instantaneous rest just before it reaches the ceiling,

- (a) show that  $D = \frac{5l}{3}$

- (b) Show that, while the elastic string is stretched,  $P$  moves with simple harmonic motion, with period  $2\pi\sqrt{\frac{2l}{3g}}$
- (6)

- (c) Find, in terms of  $g$  and  $l$ , the exact time from the instant when  $P$  is released to the instant when the elastic string first goes slack.
- (4)**

**Question 7 continued**

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**Question 7 continued**

[illegible]

**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 16 marks)

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	
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**Pearson Edexcel International Advanced Level**

**Friday 19 January 2024**

Afternoon (Time: 1 hour 30 minutes) **Paper reference WME03/01**

**Mathematics**

**International Advanced Subsidiary/Advanced Level**

**Mechanics M3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

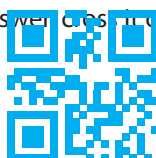
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$ , and give your answer to either two significant figures or three significant figures.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.



**Turn over** ►

1. A spacecraft  $S$  of mass  $m$  moves in a straight line towards the centre,  $O$ , of a planet.

The planet is modelled as a fixed sphere of radius  $R$ .

The spacecraft  $S$  is modelled as a particle.

The gravitational force of the planet is the only force acting on  $S$ .

When  $S$  is a distance  $x$  ( $x \geq R$ ) from  $O$

- the gravitational force is directed towards  $O$  and has magnitude  $\frac{mgR^2}{2x^2}$
- the speed of  $S$  is  $v$

- (a) Show that

$$v^2 = \frac{gR^2}{x} + C$$

where  $C$  is a constant.

(3)

When  $x = 3R$ ,  $v = \sqrt{3gR}$

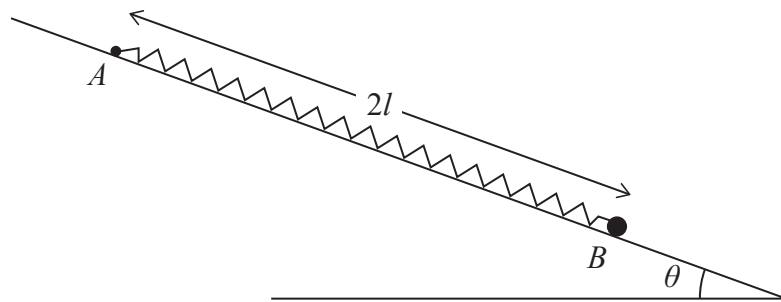
- (b) Find, in terms of  $g$  and  $R$ , the speed of  $S$  as it hits the surface of the planet.

(3)

**Question 1 continued**

(Total for Question 1 is 6 marks)





### Figure 1

The plane is inclined at angle  $\theta$  to the horizontal, where  $\tan \theta = \frac{5}{12}$

Initially  $P$  is held at the point  $B$  on the plane, where  $AB$  is a line of greatest slope of the plane.

The particle is released from rest at  $B$  and first comes to instantaneous rest at the point  $C$  on  $AB$ , where  $AC = 0.7l$

$$\lambda = \frac{100}{91} mg \tag{5}$$

(b) Find the acceleration of  $P$  when it is released from rest at  $B$ . (4)

### Question 2 continued

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### Question 2 continued

[illegible]

**Question 2 continued**

(Total for Question 2 is 9 marks)

The shaded region in Figure 2 is bounded by the  $x$ -axis, the line with equation  $x = 2$  and the curve with equation  $y = \frac{1}{4}x(3-x)$ .

The volume of  $S$  is  $\frac{2}{5}\pi$

- (a) Use the model and algebraic integration to show that the  $x$  coordinate of the centre of mass of  $S$  is  $\frac{31}{24}$

The solid  $S$  is placed with its circular face on a rough plane which is inclined at  $\alpha^\circ$  to the horizontal. The plane is sufficiently rough to prevent  $S$  from sliding.

The solid  $S$  is on the point of toppling.

- (b) Find the value of  $\alpha$

(3)

### Question 3 continued

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### Question 3 continued

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### Question 3 continued

**(Total for Question 3 is 8 marks)**



4.

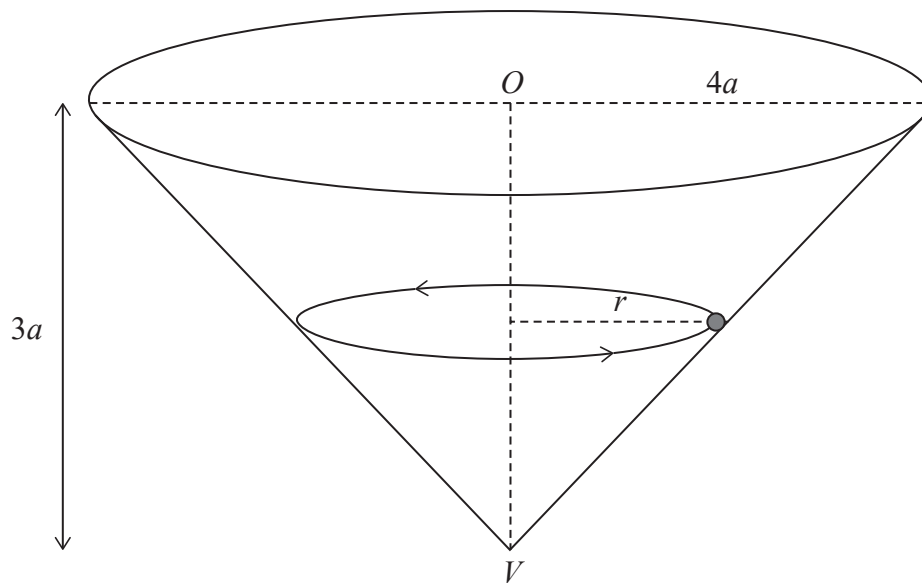
**Figure 3**

Figure 3 shows a thin hollow right circular cone fixed with its circular rim horizontal.

The centre of the circular rim is  $O$ . The vertex  $V$  of the cone is vertically below  $O$ .

The radius of the circular rim is  $4a$  and  $OV = 3a$ .

A particle  $P$  of mass  $m$  moves in a horizontal circle of radius  $r$  ( $0 < r < 4a$ ) on the inner surface of the cone.

The coefficient of friction between  $P$  and the inner surface of the cone is  $\frac{1}{4}$

The particle moves with a constant angular speed.

Show that the maximum possible angular speed is  $\sqrt{\frac{16g}{13r}}$

**(9)**

**Question 4 continued**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Question 4 continued**

[illegible]

**Question 4 continued**

**(Total for Question 4 is 9 marks)**

5. (a) Use algebraic integration to show that the centre of mass of a uniform semicircular disc of radius  $r$  and centre  $O$  is at a distance  $\frac{4r}{3\pi}$  from the diameter through  $O$

[You may assume, without proof, that the area of a circle of radius  $r$  is  $\pi r^2$ ]

(5)

A uniform lamina  $L$  is in the shape of a semicircle with centre  $B$  and diameter  $AC = 8a$ . The semicircle with diameter  $AB$  is removed from  $L$  and attached to the straight edge  $BC$  to form the template  $T$ , shown shaded in Figure 4.

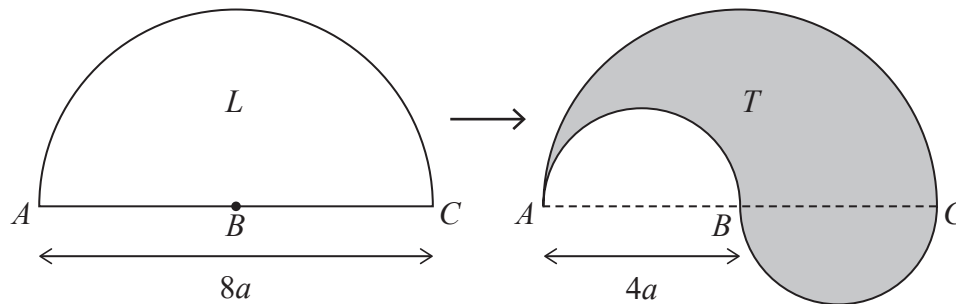


Figure 4

The distance of the centre of mass of  $T$  from  $AC$  is  $d$ .

- (b) Show that  $d = \frac{4a}{\pi}$

(5)

The template  $T$  is freely suspended from  $A$  and hangs in equilibrium with  $AC$  at an angle  $\theta$  to the downward vertical.

- (c) Find the exact value of  $\tan \theta$

(6)

**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

(Total for Question 5 is 16 marks)



6. The fixed point  $A$  is **vertically above** the fixed point  $B$ , with  $AB = 3l$

A light elastic string has natural length  $l$  and modulus of elasticity  $4mg$

One end of the string is attached to  $A$  and the other end is attached to a particle  $P$  of mass  $m$

A second light elastic string also has natural length  $l$  and modulus of elasticity  $4mg$

One end of this string is attached to  $P$  and the other end is attached to  $B$ .

Initially  $P$  rests in equilibrium at the point  $E$ , where  $AEB$  is a **vertical** straight line.

- (a) Show that  $AE = \frac{13}{8}l$

The particle  $P$  is now held at the point that is a distance  $2l$  vertically below  $A$  and released from rest.

At time  $t$ , the vertical displacement of  $P$  from  $E$  is  $x$ , where  $x$  is measured vertically downwards.

- (b) Show that  $\ddot{x} = -\frac{8g}{l}x$  (4)

- (c) Find, in terms of  $g$  and  $l$ , the speed of  $P$  when it is  $\frac{1}{8}l$  below  $E$ . (3)

- (d) Find the length of time, in each complete oscillation, for which  $P$  is more than  $1.5l$  from  $A$ , giving your answer in terms of  $g$  and  $l$
- (3)**

**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

(Total for Question 6 is 14 marks)

7.

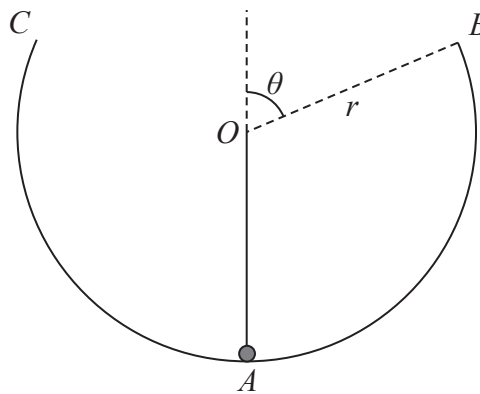


Figure 5

A thin smooth hollow spherical shell has centre  $O$  and radius  $r$ . Part of the shell is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point  $A$  is the lowest point of the bowl, as shown in Figure 5.

The point  $B$  is on the rim of the bowl, with  $OB$  at an angle  $\theta$  to the upward vertical,

where  $\tan \theta = \frac{12}{5}$

A small ball is placed in the bowl at  $A$ . The ball is projected from  $A$  with horizontal speed  $u$  and moves in the vertical plane  $AOB$ . The ball stays in contact with the bowl until it reaches  $B$ .

At the instant when the ball reaches  $B$ , the speed of the ball is  $v$ .

By modelling the ball as a particle and ignoring air resistance,

(a) use the principle of conservation of mechanical energy to show that

$$v^2 = u^2 - \frac{36}{13}gr \quad (3)$$

(b) show that  $u^2 \geq \frac{41}{13}gr$  (4)

The point  $C$  is such that  $BC$  is a diameter of the rim of the bowl.

Given that  $u^2 = 4gr$

(c) use the model to show that, after leaving the inner surface of the bowl at  $B$ , the ball falls back into the bowl before reaching  $C$ . (6)

**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 13 marks)

**TOTAL FOR PAPER IS 75 MARKS**

Please check the examination details below before entering your candidate information

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Centre Number		Candidate Number	
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**Pearson Edexcel International Advanced Level**

**Wednesday 12 June 2024**

Afternoon (Time: 1 hour 30 minutes) **Paper reference WME03/01**

**Mathematics**

**International Advanced Subsidiary/Advanced Level**

**Mechanics M3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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### Question 1 continued

**(Total for Question 1 is 8 marks)**

2.

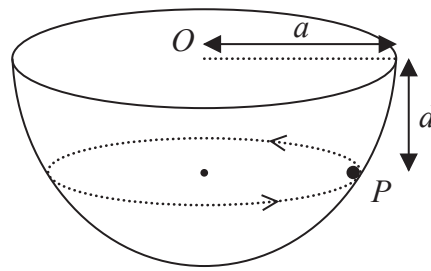


Figure 2

A thin hemispherical shell, with centre  $O$  and radius  $a$ , is fixed with its open end uppermost and horizontal.

A particle  $P$  of mass  $m$  moves in a horizontal circle on the smooth inner surface of the shell. The vertical distance of  $P$  below the level of  $O$  is  $d$ , as shown in Figure 2.

- (a) Find, in terms of  $m$ ,  $g$ ,  $d$  and  $a$ , the magnitude of the force exerted on  $P$  by the inner surface of the hemisphere.

(3)

The particle moves with constant speed  $v$ .

- (b) Find  $v$  in terms of  $g$ ,  $a$  and  $d$ .

(5)

**Question 2 continued**

(Total for Question 2 is 8 marks)

3. A particle  $P$  is moving along the  $x$ -axis.

At time  $t$  seconds, where  $t \geq 0$ , the displacement of  $P$  from the origin  $O$  is  $x$  metres and  $P$  is moving with velocity  $v \text{ m s}^{-1}$  in the positive  $x$  direction.

The acceleration of  $P$  is  $\frac{3\sqrt{x+1}}{4} \text{ m s}^{-2}$  in the positive  $x$  direction.

When  $t = 0$ ,  $x = 15$  and  $v = 8$

- (a) Show that  $v = (x+1)^{\frac{3}{4}}$
- (4)**

- (b) Find  $t$  in terms of  $v$ . (5)

### Question 3 continued

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### Question 3 continued

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### Question 3 continued

**(Total for Question 3 is 9 marks)**

4. In a harbour, the water level rises and falls with the tides with simple harmonic motion.

On a particular day, the depths of water in the harbour at low and high tide are 4m and 10m respectively.

Low tide occurs at 12:00 and high tide occurs at 18:20

- (a) Find, in  $\text{m h}^{-1}$ , the speed at which the water level is rising on this particular day at 13:35

(6)

A ship can only safely enter the harbour when the depth of water is at least 8.5 m.

- (b) Find the earliest time after 12:00 on this particular day at which it is safe for the ship to enter the harbour, giving your answer to the nearest minute.

(4)

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**Question 4 continued**

(Total for Question 4 is 10 marks)

5. A uniform right solid circular cone  $C$  has radius  $r$  and height  $4r$ .

- (a) Show, using algebraic integration, that the distance of the centre of mass of  $C$  from its vertex is  $3r$ .

[You may assume that the volume of  $C$  is  $\frac{4}{3}\pi r^3$ ]

(4)

A uniform solid  $S$ , shown below in Figure 3, is formed by **removing** from  $C$  a uniform solid right circular cylinder of height  $r$  and radius  $\frac{1}{2}r$ , where the centre of one end of the cylinder coincides with the centre of the plane face of  $C$  and the axis of the cylinder coincides with the axis of  $C$ .

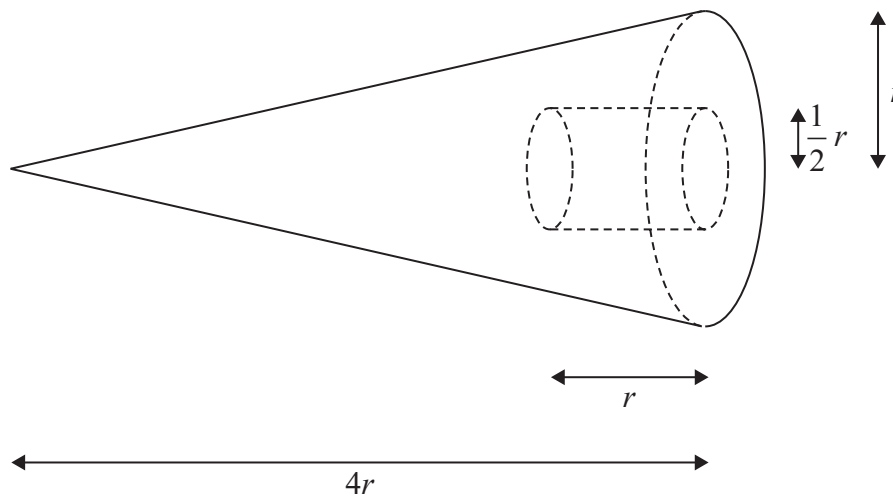


Figure 3

- (b) Show that the distance of the centre of mass of  $S$  from the vertex of  $C$  is  $\frac{75}{26}r$

(5)

A rough plane is inclined at an angle  $\alpha$  to the horizontal.

The solid  $S$  rests in equilibrium with its plane face in contact with the inclined plane.

Given that  $S$  is on the point of toppling,

- (c) find the exact value of  $\tan \alpha$

(3)

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**Question 5 continued**

[illegible]

**Question 5 continued**

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**Question 5 continued**

(Total for Question 5 is 12 marks)



6.

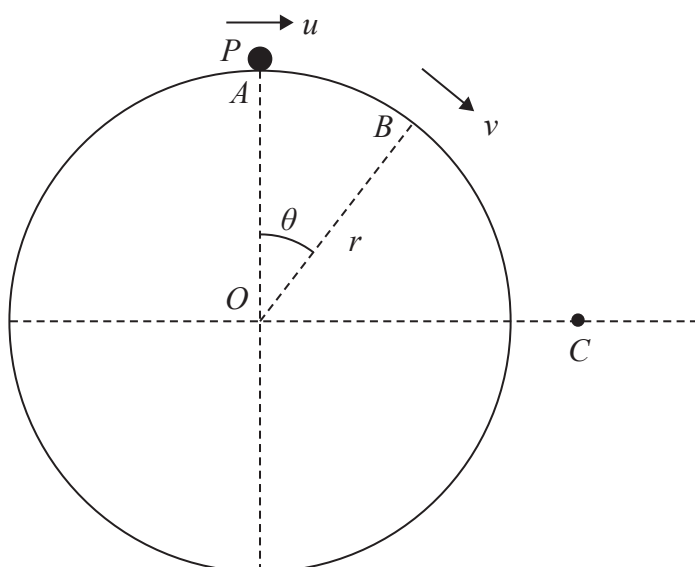


Figure 4

A fixed solid sphere has centre  $O$  and radius  $r$ .

A particle  $P$  of mass  $m$  is held at rest on the smooth surface of the sphere at  $A$ , the highest point of the sphere.

The particle  $P$  is then projected horizontally from  $A$  with speed  $u$  and moves on the surface of the sphere.

At the instant when  $P$  reaches the point  $B$  on the sphere, where angle  $AOB = \theta$ ,  $P$  is moving with speed  $v$ , as shown in Figure 4.

At this instant,  $P$  loses contact with the surface of the sphere.

(a) Show that

$$\cos \theta = \frac{2gr + u^2}{3gr} \quad (7)$$

In the subsequent motion, the particle  $P$  crosses the horizontal through  $O$  at the point  $C$ , also shown in Figure 4.

At the instant  $P$  passes through  $C$ ,  $P$  is moving at an angle  $\alpha$  to the horizontal.

Given that  $u^2 = \frac{2gr}{5}$

(b) find the exact value of  $\tan \alpha$ .

(6)

**Question 6 continued**

**Question 6 continued**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Question 6 continued**

(Total for Question 6 is 13 marks)

7. A particle  $P$  of mass  $m$  is attached to one end of a light elastic string of natural length  $l$  and modulus of elasticity  $2mg$ . The other end of the string is attached to a fixed point  $A$  on a smooth horizontal table. The particle  $P$  is at rest at the point  $B$  on the table, where  $AB = l$ .

At time  $t = 0$ ,  $P$  is projected along the table with speed  $U$  in the direction  $AB$ .

At time  $t$

- the elastic string has not gone slack
- $BP = x$
- the speed of  $P$  is  $v$

(a) Show that

$$v^2 = U^2 - \frac{2gx^2}{l} \quad (4)$$

- (b) By differentiating this equation with respect to  $x$ , prove that, before the elastic string goes slack,  $P$  moves with simple harmonic motion with period  $\pi\sqrt{\frac{2l}{g}}$

Given that  $U = \sqrt{\frac{gl}{2}}$

- (c) find, in terms of  $l$  and  $g$ , the exact total time, from the instant it is projected from  $B$ , that it takes  $P$  to travel a total distance of  $\frac{3}{4}l$  along the table. (6)

**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

(Total for Question 7 is 15 marks)

**TOTAL FOR PAPER IS 75 MARKS**