

1a	Equation of motion	M1	Dimensionally correct. Condone sign error.
	$F - R = 1500a$	A1	Correct unsimplified equation in F or P
	Use of $P = Fv : \left(\frac{30000}{20} - R = 1500 \times 0.6 \right)$	M1	Must be trying to use 30 kW but condone error in zeros
	$R = 600$	A1	Correct answer only
		4	
1b	Equation of motion	M1	Dimensionally correct. Need all relevant terms. Condone sign errors and sin/cos confusion. Allow with F .
	$\frac{30000}{V} - 1500g \times \frac{1}{8} - 500 = -1500 \times 0.2$	A1	Unsimplified equation with F substituted and at most one error
		A1	Correct unsimplified equation with F substituted. If F is never substituted, A0A0
	$V = 14.7 \quad (15)$	A1	3 sf or 2 sf
		4	
		(8)	
2	1 st equation e.g. Equation for change in KE	M1	Dimensionally correct. Must be subtracting but condone sign error.
	$\frac{1}{2} \times 0.5(x^2 + y^2 - (5^2 + 3^2)) = 22$ $(x^2 + y^2 = 122) \quad (1^2 + (2\lambda + 3)^2 = 122)$	A1	Correct unsimplified equation seen or implied (They might have used impulse-momentum first and done some work before substituting x and y .)
	2 nd equation e.g. Impulse-momentum equation	M1	Dimensionally correct. Must be subtracting but condone sign error.
	$0.5(x\mathbf{i} + y\mathbf{j}) - 0.5(5\mathbf{i} + 3\mathbf{j}) = (-2\mathbf{i} + \lambda\mathbf{j})$ $((x-5)\mathbf{i} + (y-3)\mathbf{j}) = -4\mathbf{i} + 2\lambda\mathbf{j}$	A1	Correct unsimplified equation
NB: epen has M1A1A1 for the final 3 marks but this should be marked DM1DM1A1			
	Form a quadratic equation in λ	DM1	e.g. $1^2 + (3 + 2\lambda)^2 = 122$ Dependent on the 2 preceding M marks
	Solve for 2 values of λ	DM1	e.g. solve $4\lambda^2 + 12\lambda - 112 = 0$ or $(3 + 2\lambda)^2 = 121$ Dependent on the preceding M1
	$\Rightarrow \lambda = 4 \quad \text{or} \quad \lambda = -7$	A1	Correct only and no errors seen (watch out for $x = -1$ used)
alt	Form a quadratic in y	DM1	e.g. $1 + y^2 = 122 \quad (y^2 = 121)$ Dependent on the 2 preceding M marks
	Solve for 2 values of y and use these to obtain 2 values of λ	DM1	Dependent on the preceding M1
	$\Rightarrow \lambda = 4 \quad \text{or} \quad \lambda = -7$	A1	
		7	

3a		area	distance from AE		B1	Mass ratio correct
	rectangle	$48a^2$	$4a$		B1	Distances from AE (or parallel axis) correct
	triangle	$18a^2$	$8a - 2a (= 6a)$			
	lamina	$30a^2$				
	$M(AE)$			M1	Allow use of a parallel axis. The moments equation should include a but condone if the mass ratio does not include a factor of a^2 . Dimensionally correct.	
	$48a^2 \times 4a - 18a^2 \times 6a = 30a^2 \bar{x}$			A1	Correct unsimplified equation for their axis. Accept as part of a vector equation.	
	$\bar{x} = \frac{84}{30}a = \frac{14}{5}a$ *			A1*	Obtain given answer from correct working (including correct use of a)	
					If they take moments about BD they get $d = 5.2a$ Allow B1B1M1A1A0 if they get this far.	
					5	
3b	Find trig ratio of a relevant angle			M1	Correct use of trig.	
	$\tan \theta^\circ = \frac{3a}{2.8a}$			A1	Correct equation for the required angle. (DO NOT ISW: If they obtain 47 and then use $90 - 47 = 43$ they score M1A0A0)	
	$\theta = 47$			A1	The Q asks for a whole number of degrees. 0.82 radians scores M1A1A0	
					3	
					(8)	

4a	Use $t = 2$ and $3t^2 + 2t = t^3 + kt$ $(12 + 4 = 8 + 2k)$	M1	Allow verification.
	$k = 4$ *	A1*	Obtain given answer from correct working. Verification requires a clear conclusion.
		2	
4b	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1	Differentiate the vector \mathbf{v} Majority of powers going down
	$\mathbf{a} = (6t + 2)\mathbf{i} + (3t^2 + 4)\mathbf{j}$	A1	Correct only
	Use $ \mathbf{F} = m \mathbf{a} $	DM1	Correct use of Pythagoras and N2L Dependent on the preceding M1
	$ \mathbf{F} = 1.5 \times \sqrt{14^2 + 16^2} = 3\sqrt{113}$	A1	Or $\frac{3}{2}\sqrt{452}$ or 32 or better (31.89....)
		4	
4c	Use of $\mathbf{r} = \int \mathbf{v} dt$	M1	Majority of powers going up
	$\mathbf{r} = (t^3 + t^2 (+A))\mathbf{i} + \left(\frac{1}{4}t^4 + \frac{4}{2}t^2 (+B)\right)\mathbf{j}$	A1	Allow without constant of integration
	Correct use of $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$ to find \mathbf{r} when $t = 2$	DM1	$\left(\mathbf{r} = (t^3 + t^2 + 3)\mathbf{i} + \left(\frac{1}{4}t^4 + \frac{4}{2}t^2 + 4\right)\mathbf{j}\right)$ Dependent on the preceding M1 Use of $\mathbf{r} = -3\mathbf{i} - 4\mathbf{j}$ is M0
	$\mathbf{r} = 15\mathbf{i} + 16\mathbf{j}$	A1	Correct answer only. Accept column vector
		4	
		(10)	

5a	Use of $F_{\max} = \mu R : F_{\max} = \frac{2}{7} \times 1.5g \cos \theta$	M1	(3.87...) Condone trig confusion. Trig substitution not required. Allow M1 if there is a clear statement for F_{\max} “correct” and then used in a calculation including the gain in GPE
	Use of $WD = 2.5 F_{\max}$	M1	Trig substitution not required. M0 if they have included the gain in GPE
			If the method for F is incorrect but involves the use of μ to obtain F and then they use the “work done” formula correctly allow M0M1
	$WD = 9.69 \quad (9.7)(J)$	A1	3 sf or 2 sf not $\frac{126}{13}$
		3	
5b	Work-energy equation	M1	The Q asks for work-energy. Need all terms and dimensionally correct. Condone sign errors and sin / cos confusion
	If their answer to (a) included the GPE then it must be used for the total work done here to score the M1		
	$\frac{1}{2} \times 1.5U^2 = WD + 1.5 \times 9.8 \times 2.5 \times \sin \theta$	A1ft	Unsimplified equation with at most one error.
		A1ft	Correct unsimplified equation Follow their WD against friction
	$U = 5.64 \quad (5.6)$	A1	3 sf or 2 sf
		4	
5c	Work-energy equation for A to A	M1	The Q asks for work-energy. Need all terms and dimensionally correct.
	$\frac{1}{2} \times 1.5v^2 = \frac{1}{2} \times 1.5U^2 - 2WD$	A1ft	Correct unsimplified equation. Follow their WD against friction and their U
	$v = 2.43 \quad (2.4)(\text{ms}^{-1})$	A1	3 sf or 2 sf
		3	
5c alt	Work-energy equation for B to A	M1	The Q asks for work-energy. Need all terms and dimensionally correct.
	$\frac{1}{2} \times 1.5v^2 = 1.5 \times 9.8 \times 2.5 \times \sin \theta - WD$	A1ft	Correct unsimplified equation. Follow their WD
	$v = 2.43 \quad (2.4)(\text{ms}^{-1})$	A1	3 sf or 2 sf
		3	
		(10)	

6a			
	M(A)	M1	Or equivalent method to form an equation in W only. Equation(s) must be dimensionally correct and contain all relevant terms. Condone sin / cos confusion and sign error(s)
	$50 \times 3 \cos 30^\circ + W \times 6 \cos 30^\circ = 60\sqrt{3} \times 4 \sin 30^\circ$	A1	Unsimplified equation with at most one error.
		A1	Correct unsimplified equation
	$W = 15$ *	A1*	Correct answer only
		4	
6b	First equation e.g. Resolve vertically	M1	Or resolve parallel to pole
	$(\pm)V + 50 + 15 = T \cos 30^\circ$ ($V = 25$)	A1	Or: $P + 50 \cos 60^\circ + 15 \cos 60^\circ = 60\sqrt{3} \times \frac{\sqrt{3}}{2}$
	Second equation e.g. Resolve horizontally	M1	Or resolve perpendicular to the pole
	$(\pm)H = T \cos 60^\circ (= 30\sqrt{3} = 51.96 \dots)$	A1	Or: $50 \cos 30^\circ + 15 \cos 30^\circ = 60\sqrt{3} \cos 60^\circ + Q$
	NB: One of the equations could be a second moments equation		
	$ R = \sqrt{25^2 + (30\sqrt{3})^2}$	DM 1	Dependent on the 2 preceding M marks $(\sqrt{57.5^2 + 3 \times 6.25})$
	$= 5\sqrt{133} (57.662 \dots)$ (N)	A1	58 N or better
		6	Full marks available using $\pm V, \pm H, \pm P, \pm Q$
6b alt	Form vector triangle for the vertical forces, the thrust and the resultant Correct triangle	M1 A1	
	Use cosine rule	M1	
	$R^2 = T^2 + (50 + W)^2 - 2T(50 + W)\cos 30^\circ$	A1	Correct unsimplified equation
	$R^2 = (60\sqrt{3})^2 + (65)^2 - 2 \times 60\sqrt{3} \times 65 \cos 30^\circ$	DM 1	Substitute values and solve for $ R $
	$ R = 5\sqrt{133} (57.662 \dots)$ (N)	A1	58 N or better
		6	
		(10)	

7a			
	Use CLM	M1	Need all terms and dimensionally correct. Condone sign errors. Might see them using equal (and opposite) impulses.
	$6mu - 3kmu = 3mu + kmv \quad ((3-3k)u = kv)$	A1	Correct unsimplified equation
	$\Rightarrow v = \frac{(3-3k)}{k}u \quad *$	A1*	Obtain given answer from full and correct working
		3	
7b	Use of Impulse = change in momentum	M1	Must be subtracting. Can be for either particle.
	$ I_Q = I_P = 3mu - 3m \cdot 2u = 3mu$ or $ kmv - (-3mku) = \left km \cdot \frac{3-3k}{k}u + 3mku \right = 3mu$	A1	Correct only (Do not need to state that $ I_Q = I_P $ if find $ I_P $)
		2	
7c	Use impact law:	M1	Seen or implied. If stated in (a) must be used here. Must be used correctly but condone sign errors
	$\frac{v-u}{5u} = e$ or $\frac{3-3k}{k}u - u = 5ue$	A1	Correct unsimplified equation
	NB: the second and third M mark are not dependent on the first M mark		
	Use $v > u$ or $e > 0$ to form an inequality in k	M1	Could use $e \dots 0$ followed by $v \neq u$
	Use $e, 1$ to form an inequality in k	M1	
	$\frac{3-3k}{k} > 1$ and $3-3k, 6k \Rightarrow \frac{1}{3}, k < \frac{3}{4}$	A1	Correct answer only.
		5	
		(10)	

8a	Condone use of θ or a mixture of θ and α throughout but final answer should be in one variable.		
	Equation for horizontal distance	M1	Complete method using <i>suvat</i> . Condone sine / cosine confusion
	$x = u \cos \alpha t$	A1	Correct only
	Equation for vertical distance	M1	Complete method using <i>suvat</i> . Condone sine / cosine confusion and sign error
	$y = u \sin \alpha t - \frac{1}{2} g t^2$	A1	Correct only
	$t = \frac{x}{u \cos \alpha} \Rightarrow$ $y = u \sin \alpha \cdot \frac{x}{u \cos \alpha} - \frac{g}{2} \left(\frac{x}{u \cos \alpha} \right)^2$	DM1	Substitute for t to obtain y in terms of x and α Dependent on the 2 preceding M marks
	$\Rightarrow y = x \tan \alpha - \frac{g x^2}{2 u^2} (1 + \tan^2 \alpha) *$	A1*	Obtain given answer from full and correct working. Need some evidence for the final step. $\frac{1}{\cos^2 \alpha} = 1 + \tan^2 \alpha$ is not sufficient.
		6	
8b	Conservation of energy:	M1	Method specified in the question. Need all terms and dimensionally correct. Condone sign errors
	$\frac{1}{2} m \times 25^2 = \frac{1}{2} m U^2 + m g \times 20$	A1	Correct unsimplified equation
	$U = 15.3 \text{ (15)}$	A1	3 sf or 2 sf only
		3	
8c	Use part (a) or work from first principles to form an equation in $\tan \theta$	M1	$\left(-20 = 30 \tan \theta - \frac{9.8 \times 900}{2 U^2} (1 + \tan^2 \theta) \right)$
	Obtain $18.9 \tan^2 \theta - 30 \tan \theta - 1.07 = 0$ $\left(\frac{4410}{233} \tan^2 \theta - 30 \tan \theta - \frac{250}{233} = 0 \right)$	A1ft	Or 3 term equivalent Follow their U Can be implied by a correct final answer
	$\Rightarrow \theta = 58.3^\circ \text{ or } 58^\circ$	A1	3 sf or 2 sf only
		3	
		(12)	