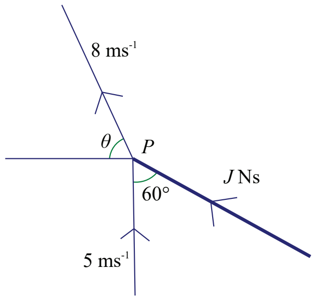
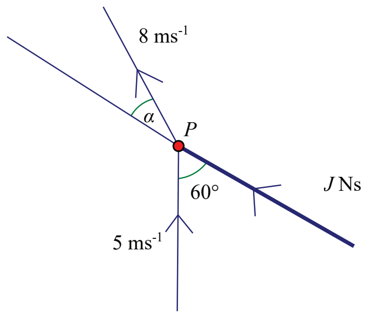
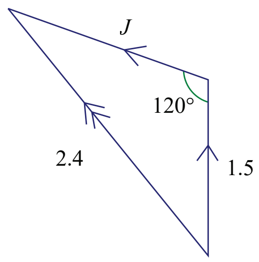
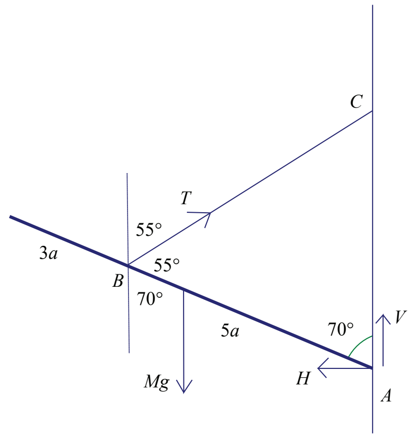
Q	Solution	Mark	Notes
1	Driving force $(F) = \frac{3500}{V}$	B1	Use of $P = Fv$
	Equation of motion: $F - 20V + 480g\sin\theta = 0$	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion
	$\frac{3500}{V} - 20V + 40g = 0$	A1	Correct unsimplified equation in V.
	$20V^2 - 392V - 3500 = 0$	M1	Form a 3 term quadratic equation $(=0)$ in V
	$V = 26.3 \quad (26)$	A1	3 sf or 2 sf Not $\frac{49+22\sqrt{14}}{5}$ (follows use of 9.8)
		(5)	
		[5]	

2a			Allow column vectors throughout
	Use $\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1	Differentiate – at least 3 powers going down by 1
	$\mathbf{a} = (10t - 3t^2)\mathbf{i} + (6t^2 - 8)\mathbf{j}$	A1	
	$\mathbf{F} = 1.5 \times ((20-12)\mathbf{i} + (24-8)\mathbf{j})$	DM1	Substitute $t = 2$ and use $\mathbf{F} = m\mathbf{a}$ Dependent on preceding M1
	$= 12\mathbf{i} + 24\mathbf{j}$	A1	Ignore magnitude of **F** if found
		(4)	
2b	$5t^2 - t^3 = 0 \quad \Rightarrow t = 5$	B1	(Not moving when $t = 0$ so no need to mention $t = 0$)
	Use of $\mathbf{r} = \int \mathbf{v}\,dt$	M1	Integrate to find **r** – at least 3 powers going up by 1.
	$\mathbf{r} = \left(\frac{5}{3}t^3 - \frac{1}{4}t^4\right)\mathbf{i} + \left(\frac{1}{2}t^4 - 4t^2\right)\mathbf{j}$	A1	Condone if no constant of integration seen (since $t = 0, \mathbf{r} = \mathbf{0}$)
	$\mathbf{r} = \left(\frac{625}{12}\right)\mathbf{i} + \left(\frac{425}{2}\right)\mathbf{j}$	A1	Final answer $52\mathbf{i} + 210\mathbf{j}$ or better $(52.08\dot{3}\mathbf{i} + 212.5\mathbf{j})$
		(4)	
		[8]	

<table>
<tr><td>3a</td><td><table><tr><td></td><td>square</td><td>triangle</td><td>circle</td><td>T</td></tr><tr><td>mass</td><td>36</td><td>8</td><td>π</td><td>$28-\pi$</td></tr><tr><td>c from AD</td><td>$3a$</td><td>$\frac{7}{3}a$</td><td>$4a$</td><td>d</td></tr></table></td><td></td><td></td></tr>
<tr><td></td><td>Mass ratio</td><td>B1</td><td></td></tr>
<tr><td></td><td>Distances from AD or a parallel axis</td><td>B1</td><td></td></tr>
<tr><td></td><td>M(AD or parallel axis):</td><td>M1</td><td>Moments equation. Need all terms and dimensionally correct. Condone sign errors.</td></tr>
<tr><td></td><td>$36\times 3a-8\times\frac{7}{3}a-\pi\times 4a=(28-\pi)d$</td><td>A1</td><td>Correct unsimplified equation for their parallel axis</td></tr>
<tr><td></td><td>$\left(108a-\frac{56}{3}a-4\pi a=(28-\pi)d\right)$</td><td></td><td></td></tr>
<tr><td></td><td>$d=\frac{324-56-12\pi}{3(28-\pi)}a=\frac{4(67-3\pi)}{3(28-\pi)}a$ *</td><td>A1*</td><td>Obtain given answer from correct working</td></tr>
<tr><td></td><td></td><td></td><td>Distance from BC is $\frac{(236-6\pi)a}{3(28-\pi)}$
Allow 4/5 if seen.</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>3b</td><td>M(A): $W\times\frac{4(67-3\pi)}{3(28-\pi)}a=kW\times 6a$
0r resolve vertically and use M(G), where G is the centre of mass,
$T_A+kW=W$
$T_A\left(\frac{4(67-3\pi)}{3(28-\pi)}a\right)=kW\left(6a-\frac{4(67-3\pi)}{3(28-\pi)}a\right)$</td><td>M1</td><td>Complete method to form an equation in k and W only
Dimensionally correct but condone use of incorrect distsance(s)</td></tr>
<tr><td></td><td></td><td>A1</td><td>Correct unsimplified equation
$\left(\text{NB } \frac{4(67-3\pi)}{3(28-\pi)}a=3.088a\right)$</td></tr>
<tr><td></td><td>$k=0.51$</td><td>A1</td><td>Q asks for 2dp</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td></td><td></td><td>[8]</td><td></td></tr>
</table>

4			Resolving parallel and perpendicular to the original direction of motion
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ parallel or perpendicular to original direction
	$J\cos 30° = 2.4\cos\theta$ or $J\cos 60° = 2.4\sin\theta - 1.5$	A1	One correct unsimplified equation
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ to form second equation
		A1	2nd correct unsimplified equation
	The first 4 marks are available for a correct equation in vector form.		$\begin{pmatrix} -2.4\cos\theta \\ 2.4\sin\theta \end{pmatrix} = \begin{pmatrix} -J\cos 30° \\ J\cos 60° + 1.5 \end{pmatrix}$
	$2.4^2 = \frac{3J^2}{4} + \frac{J^2}{4} + 1.5J + 1.5^2$ $\left(J^2 + 1.5J - 3.51 = 0\right)$	DM1	Form an equation in J only Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268….)
		(6)	
	NB Use of initial velocity parallel to final velocity or final velocity parallel to impulse is a method error, not a misread		
			See over for alternatives

4 Alt 1			Resolving parallel and perpendicular to the direction of the impulse.
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ in any direction
	$J = 0.3(8\cos\alpha - 5\cos 60°)$ Or $5\sin 60° = 8\sin\alpha$	A1	Correct unsimplified equation $\begin{pmatrix} 2.4\cos\alpha = J + 1.5\cos 60° \\ 2.4\sin\alpha = 1.5\sin 60° \end{pmatrix}$
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ in perpendicular direction
		A1	Correct unsimplified equation
	$2.4^2 = \left(J + \frac{3}{4}\right)^2 + \left(\frac{3}{2}\right)^2 \times \frac{3}{4}$ $\left(J^2 + 1.5J - 3.51 = 0\right)$	DM1	Form an equation in J only Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268….)
	Could have a mixture of the first 2 alternatives. M1A1M1A1 for 2 independent equations. DM1A1 for solving		
		(6)	
4 Alt 2			Using vector triangle.
	Impulse momentum triangle	M1	Form dimensionally correct vector triangle (for impulse or momentum)
	Use of cosine rule	M1	Use of cosine rule in momentum or velocity triangle
	$2.4^2 = J^2 + 1.5^2 - 3J\cos 120°$	A1 A1	unsimplified equation in v or mv with at most one error Correct unsimplified equation
	$J^2 + 1.5J - 3.51 = 0$	DM1	Form a simplified equation in J Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268….)
		(6)	
		[6]	

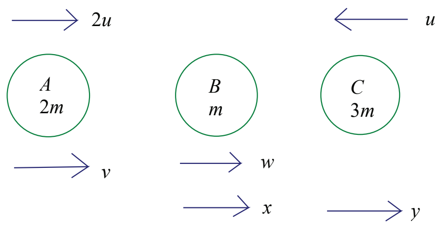
5a			
	Moments about A:	M1	Need all terms and dimensionally correct. Condone sign errors, incorrect angles and sin/cos confusion Or complete method to form equation in T (and M).
	$5a \times T\sin 55° = 4a\cos 20° \times Mg$	A1	Correct unsimplified equation in T (and M).
	$T = \dfrac{4\cos 20°}{5\sin 55°}Mg\left(=0.918Mg\right)$	A1	Or equivalent (Exact or $0.92Mg$ or better)
		(3)	
5b	Resolve vertically	M1	Need all terms. Condone sign errors, incorrect angle and sin/cos confusion
	$\updownarrow$: $Mg = V + T\cos 55°$ $\left(V = 0.47...Mg\right)$	A1	Correct unsimplified equation in T or their T
	Resolve horizontally	M1	Condone consistent sin/cos confusion
	$H = T\sin 55°$ $\left(H = 0.75...Mg\right)$	A1	Correct unsimplified equation in T or their T
	Resultant $\lambda = \sqrt{(0.4736..)^2 + (0.7517..)^2}$	M1	Substitute for T and use Pythagoras
	$= 0.89$	A1	The Q asks for 2 sf
		(6)	
			See over for further alternative

5b alt	Moments about *B*	M1	Dimensionally correct. Need all terms. Condone sign errors and sin/cos confusion
	$Mga\cos 20° + 5aH\cos 70° = 5aV\cos 20°$	A1	Correct unsimplified equation
	Moments about *C*	M1	Dimensionally correct. Condone sign errors and sin/cos confusion
	$5aH = 4aMg\cos 20°$	A1	Correct unsimplified equation
	Resultant $\lambda = \sqrt{(0.4736..)^2 + (0.7517..)^2}$	M1	Use Pythagoras
	$= 0.89$	A1	The Q asks for 2 sf
	M1A1M1A1 for 2 independent equations M1A1 to solve for λ		

6a	GPE lost	M1	Need all terms. Condone sign errors and sin/cos confusion
	$= 3g \times 2 - 2g \times 2\sin\theta$ $(= 6g - 4g \times \frac{5}{13})$	A1	Correct unsimplified. Accept $\pm$
	$= \frac{58}{13}g = 43.7\,(44)\,(\text{J})$	A1	Must be positive. Exact multiple of g or 3 sf or 2 sf
		(3)	
6b	Normal reaction $= 2g\cos\theta \left(= \frac{24}{13}g\right)$	B1	Condone $\frac{1176}{65}$
	$F_{\max} = \frac{3}{8} \times R \left(= \frac{9g}{13}\right)$	M1	Use $F = \mu R$ with their R $\left(\frac{441}{65}\right)$
	Work done $= 2 \times F_{\max}$	M1	Their $F_{\max}$
	$\left(= \frac{18g}{13}\right) = 13.6(\text{J})\ \ 14(\text{J})$	A1	Exact multiple of g or 3 sf or 2 sf. Not $\frac{882}{65}$
		(4)	
6c	Total KE gained = GPE lost - total WD against friction	M1	Must be using work-energy. Dimensionally correct. Required terms and no extras. Condone sign errors.
	$\frac{1}{2}(2+3)v^2 = (their(a)) - (their(b))$ $\left(\frac{5}{2}v^2 = \frac{58}{13}g - \frac{18}{13}g = \frac{40}{13}g\right)$	A2ft	Follow their (a) and (b) -1 each error
	$v = \sqrt{\frac{16}{13}g} = 3.47\,(\text{m s}^{-1})$ or $3.5\,(\text{m s}^{-1})$	A1	3 sf or 2 sf (need to substitute for g)
		(4)	
6d	KE lost = GPE gained + WD against friction	M1	Must be using work-energy. Dimensionally correct. Required terms and no extras. Condone sign errors.
	$\frac{1}{2} \times 2 \times \frac{16}{13}g = 2g \times d\sin\theta + \frac{3}{8} \times 2g \times \frac{12}{13}d$ $\frac{1}{2} \times 2 \times v^2 = 2g \times d\sin\theta + d \times F_{\max}$ $\frac{16}{13}g = \left(\frac{10}{13}g + \frac{9}{13}g\right)d$	A2ft	Follow their (c) and their $F_{\max}$ -1 each error
	$d = \frac{16}{19}$	A1	g cancels. 0.84 or better (0.8421....)
		[15]	

7a	$-12 = 12 - gt$	M1	Use *suvat* to find time taken
	$t = \frac{24}{g} (= 2.45)$	A1	
	$AB = 6t$	M1	Horizontal distance
	$= 14.7\ (15)\ (\text{m})$	A1	3 sf or 2 sf Not $\frac{720}{49}$ (follows use of 9.8) Not $\frac{144}{g}$ (do not accept g in the denominator)
		(4)	
7b	Vertical component of velocity $= (\pm)8$	B1	
	$v^2 = u^2 + 2as$	M1	Complete method using *suvat* to find h
	$\Rightarrow 8^2 = 12^2 - 2gh$	A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf Not $\frac{200}{49}$ (follows use of 9.8) Not $\frac{40}{g}$ (do not accept g in the denominator)
		(4)	
7b alt	$\mathbf{v} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ g \end{pmatrix} t \ \Rightarrow 12 - gt = (\pm)8$	B1	Correct expression for critical value(s) of t
	$h = 12t - \frac{1}{2}gt^2$	M1	Complete method using *suvat* to find h
	$= \frac{48}{g} - \frac{8}{g}$ or $= \frac{240}{g} - \frac{200}{g}$	A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf
		(4)	
7b alt	Conservation of energy	M1	Need all terms and dimensionally correct
	$mgh + \frac{1}{2}m \times 10^2 = \frac{1}{2}m(12^2 + 6^2)$	A(B)1 A1	Unsimplified equation with at most one error Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf
		(4)	
			See over for (c)

7c	$\begin{pmatrix} 6 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 0$	M1	Complete method to find vertical component at *C*.
	$\Rightarrow v = 3$	A1	
	$\mathbf{v} = 6\mathbf{i} + 3\mathbf{j} \ \left(\text{m s}^{-1}\right)$	A1	Must be a vector in terms of **i** and **j**
	If see $\begin{pmatrix} 6 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 0$ leading to $\mathbf{v} = 6\mathbf{i} - 3\mathbf{j}$ mark as a misread: M1A0A0		
		(3)	
		[11]	
	Accept working in column vectors throughout apart from the final A1		

8a			
	Use CLM: $4mu = 2mv + mw$	M1	Need all terms. Condone sign errors. Dimensionally correct but allow with m cancelled
	$(4u = 2v + w)$	A1	Correct unsimplified. Signs correct for their v, w
	Use Impact law	M1	Used the right way round. Condone sign errors.
	$w - v = 2ue$	A1	Correct unsimplified. Signs consistent with CLM equation.
	$\Rightarrow 4u = 2(w - 2ue) + w$	DM1	Solve for v or w. Dependent on previous 2 M marks
	$3w = 4u + 4ue, \quad w = \frac{4}{3}u(1+e)$ *	A1*	Obtain **given result** from correct working
	$v = \frac{2}{3}u(2-e)$	A1	Or equivalent. Must be positive
		(7)	
8b	$2 > e$ so A moving towards centre	B1	Correct statement about direction of travel for A
	$mw - 3mu = mx + 3my$ $y - x = e\left(u + \frac{4u}{3} + \frac{4eu}{3}\right)$	M1	Use CLM and impact law correctly to form simultaneous equations in x and y.
	$\frac{4}{3}eu - \frac{5}{3}u = x + 3y$ $3y - 3x = e(7u + 4ue)$	A1	Both equations correct unsimplified in u, e, x and y
	$4x = \frac{4}{3}ue - \frac{5}{3}u - 7ue - 4ue^2$	DM1	Solve for x
	$x = -\frac{5}{12}u - \frac{17}{12}ue - ue^2$	A1	Allow for a correct constant multiple of x
	$e > 0, u > 0$ so B moving towards centre from opposite direction, hence they collide.*	A1*	Obtain **given answer** from correct working
		(6)	
	Alternative for last 3 marks;		
	C moving towards centre implies B moving towards centre, so collision. C moving away from centre , so $y > 0$, $x = w - 3u - 3y = -\frac{8u}{3} + \frac{4eu}{3} - 3y$	DM1	Consider direction of C

	$= -\frac{u}{3}(8-4e)-3y$	A1	
	<0 because $e \leq 1$ and $y>0$ hence B moving towards centre from opposite direction, and they will collide.*	A1*	Obtain **given answer** from correct working
		[13]	