M2\_2020\_01\_MS

Question Number	Scheme	Marks
1.	Use of $56 = FV$	B1
	Equation of motion	M1
	$F + 75g\sin\alpha - 40 = 75 \times \frac{1}{3}$	A1
	$\left(\frac{56}{V} = 65 - 49 = 16\right)$ $V = \frac{56}{16} = 3.5$	A1
	$V = \frac{56}{16} = 3.5$	A1
	Notes	[5]
B1 M1	Require all terms. Dimensionally correct.(Omission of g is an accuracy error.) Condone sine / cosine confusion and sign errors	
A1	Unsimplified equation with at most one error. In $F$ or in $V$ . Two signs inconsistent is 2 errors.	
A1	Correct unsimplified equation. In $F$ or in $V$ .	
A1	Max 3 s.f Not $\frac{7}{2}$ Not $3\frac{1}{2}$	

Question Number	M2 Scheme	01MS Marks
2	Work energy equation KE lost = WD + PE gain	M1
	$\frac{1}{2} \times 2 \times 16 = WD + 2g \times 2.5 \sin \theta$	A1
	(WD = 9)	A1
	Use of $F = \mu \times 2g \cos \theta$	B1
	Use of Work done $= 2.5F$	B1
	$9 = 2.5 \times \mu \times 2g \cos \theta \implies \mu = 0.19$	A1 (6) [6]
M1 A1	Must be using work-energy. Require all terms. Dimensionally correct. Allow their WD, but must be WD, not $F$ Condone sine/cosine confusion and sign errors Unsimplified equation with at most one error	
A1	Correct unsimplified equation NB: $16 = WD + 7$ seen scores 3 marks	
B1	$(F = \mu \times 19.398)$ Allow ± This mark is available if they use a <i>suvat</i> approach	
B1	Allow ±	
A1	Or 0.186. Max 3 sf. Not $\frac{3\sqrt{3}}{28}$	

Question Number	Scheme	Marks
3	Use of $m\mathbf{v} = \mathbf{I} + m\mathbf{u}$	M1
	Component of momentum parallel to original direction = $6 \times 0.75 + \sqrt{24} \cos 60 \ \left(= 4.5 + \sqrt{6}\right)$	A1 A1
	Use of Pythagoras: $\left(\frac{3}{4}v\right) = \sqrt{\left(4.5 + \sqrt{6}\right)^2 + 18}$	M1
	$v = 10.9 \text{ (m s}^{-1}), 11 \text{ (m s}^{-1})$	A1
	Alternative for the 1st 5 marks:	
	Vector triangle for impulses or velocities	M1
	Use of cosine rule	M1
	$\left(\frac{3}{4}v\right)^2 = 4.5^2 + 24 - 2 \times 4.5 \times \sqrt{24} \times \cos 120^\circ$	A1
		A1
	$v = 10.9 \text{ (m s}^{-1}), 11 \text{ (m s}^{-1})$	A1
	Change in direction = $\tan^{-1} \frac{3\sqrt{2}}{4.5 + \sqrt{6}}$	M1
	$=31.4^{\circ} (31^{\circ})$	A1 (7)
	Notes	[7]
M1 A1 M1 A1 A1	Need to consider both components. Or equivalent Or equivalent. Correct LHS Or better	
M1	Must be using correct triangle - need 120° seen or implied Correct unsimplified	
A1 A1	Or better Or equivalent use of trig. With their components to find <b>the required angle</b>	
M1	Eg angle = $\cos^{-1}\left(\frac{4.5^2 + (mv)^2 - 24}{2 \times 4.5 \times (mv)}\right)$ Or from scalar product,	
	$\cos^{-1}\left(\frac{6\times9.27}{6\times10.9}\right)$	
A1	0.548 radians (0.55 radians) or better. Do not ISW	

Question Number	M Scheme	2_2020_01_M Marks
4(a)	Moments about AC	M1
	$18 \times \frac{3a}{2} - 2\pi \times \frac{8a}{3\pi} + 2\pi \left(3a + \frac{8a}{3\pi}\right) = 18\overline{y}$	A1
	NB: valid to use $18 \times \frac{3a}{2} - 2\pi \times d + 2\pi (3a+d) = 18\overline{y}$ for $d \neq 0$ without	
	stating value for $d$ Use of $d = 0 \Rightarrow M0$	
	$(27a + 6\pi a = 18\overline{y})$	A1
	The same incorrect distance used twice in place of $\frac{8a}{3\pi}$ is one error	
	The same incorrect area for the semicircle used twice is one error. $27a + 6\pi a = 18\overline{y} \implies \overline{y} = \frac{9 + 2\pi}{6}a  *$	A1 (4)
4b	$M\overline{x} + kM \times 6a = (1+k)M\overline{x}_T$	M1
	$3a+6ak=(1+k)\overline{x}_T$ o.e.	Al
	$M \overline{y} = (1+k)M \overline{y}_T$	M1
	$(1+k)\overline{y}_T = \frac{9+2\pi}{6}a$	A1
NB	For their second equation they could use $\tan \phi$ and their $\overline{x}_T$ or $\overline{y}_T$ to form	
ND	an expression for $\overline{y}_T$ or $\overline{x}_T$	
	$\tan\phi = \frac{3}{2} = \frac{\overline{x}_T}{\overline{y}_T} \implies \frac{3}{2} = \frac{6(3a+6ak)}{(9+2\pi)a}$	DM1
	$\Rightarrow k = \frac{\pi}{12} - \frac{1}{8}$ or equivalent	A1 (6)
	Notes	
M1	All terms. Dimensionally correct. Condone sign errors	<u> </u>
A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation	
A1	Obtain <b>given answer</b> from sufficient correct exact working. Must see a sepa for $\overline{y}$ .	arate conclusion
NB	e.g. $\overline{y}_T = \frac{2(3+6k)a}{3(1+k)}$	
DM1	Form equation in $k$ and solve for $k$ .Dependent on the previous 2 M marks.	
A1	k = 0.137 (0.14) or better See over for alternative solution to 4(b)	

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Question Number	Scheme	Marks
4(b) alt	Distance of original c of m from vertical through A	M1
	$\left(\frac{9+2\pi}{6}a-2a\right)\times\sin\phi\left(=\frac{\sqrt{13}\left(2\pi-3\right)a}{26}\right)$	A1
	Distance of additional particle from vertical through A	M1
	$6a \times \cos\phi \left( = \frac{12a}{\sqrt{13}} \right)$	A1
	$mg \times \frac{\sqrt{13}(2\pi - 3)a}{26} = kmg \times \frac{12a}{\sqrt{13}}$	DM1
	k = 0.137  (0.14)	A1 (6)
	Notes	[10]
M1	Notes	[10]
M1 A1	Notes           Or equivalent	[10]
		[10]
A1	Or equivalent	[10]
A1 M1 M1	Or equivalent Distance of additional particle from vertical through <i>A</i>	[10]
A1 M1 M1 A1	Or equivalent Distance of additional particle from vertical through <i>A</i> Or equivalent Moments about <i>A</i>	[10]
A1 M1 M1 A1 DM1	Or equivalent Distance of additional particle from vertical through <i>A</i> Or equivalent Moments about <i>A</i> Dependent on the 2 previous M marks	[10]

Question Number	Scheme	Marks
r.	Use of $\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ : $\mathbf{a} = 6t\mathbf{i} + 2\mathbf{j}$	M1
5a	$t = 0 \implies \mathbf{a} = 2\mathbf{j} \pmod{(\mathbf{m}\mathbf{s}^{-2})}$	A1 (2)
5b	$11(2t-4) = (3t^2 - 4)$	M1
	$3t^2 - 22t + 40 = 0 \implies \left(t = \frac{10}{3}\right)t = 4$	M1
	$v = 44i + 4j$ , speed = $\sqrt{44^2 + 4^2}$	DM1
	$=4\sqrt{122} (m s^{-1})$	A1 (4)
5c	Use of $\mathbf{r} = \int \mathbf{v} dt$	M1
	$\mathbf{r} = \left(t^3 - 4t\right)\mathbf{i} + \left(t^2 - 4t\right)\mathbf{j}$	A1
	Set $\mathbf{r} = 0$ and solve for $t$	M1
	$t^3 - 4t = 0 \implies t = 0, 2, (-2)$	
	$t^2 - 4t = 0 \implies t = 0, 4$	A 1 * (4)
	the only common value is $t = 0$ , so does not return to $O$ .*	A1* (4)
	Notes	[10]
5a M1 A1	Powers going down Must see vector answer but ISW if go on to state the magnitude.	
5b M1	Use of velocity parallel to $11\mathbf{i} + \mathbf{j}$ 11 must be on the correct side.	
DM1	Select the larger root (dependent on the previous 2 M1 marks and on 2 posi use Pythagoras. Condone if they find both speeds	tive roots) and
A1	Any equivalent simplified surd form $(\sqrt{1952})$ ISW	
	44.18 implies M1 if correct surd form not seen. Both values for speed given is A0	
M1	Powers going up	
A1 M1	If a constant of integration is introduced, they must conclude it is equal to the zero vector Consider both components	
A1*	Or equivalent clear explanation of <b>given result</b> . Condone if they ignore $t = 0$ . Do not need to see the roots. But do need to see the factorised form for each component if using this method.	

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Scheme	Marks
$\begin{array}{c} R \\ \hline \\ A \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	
Resolve vertically $\Upsilon R + N \cos \alpha = W$ Take moments about 4	M1 A1 M1
$7aN = 4a \cos \alpha \times W$ Obtain equation in <i>R</i> , <i>W</i> and $\alpha$	A1 DM1
$N = W \times \frac{4}{7} \cos \alpha \implies$ $R = W - \frac{4}{7} W \cos^2 \alpha$	A1* (6)
$= W \left( 1 - \frac{4}{7} \cos^2 \alpha \right)  *$	
Alternative equations $R \sin \alpha + F \cos \alpha = W \sin \alpha$ $N + R \cos \alpha = W \cos \alpha + F \sin \alpha$	
$W.3a\cos\alpha + F.7a\sin\alpha = R.7a\cos\alpha$	M1 A1
First 4 marks for alternative methods	M1 A1 DM1 A1* (6)
Correct unsimplified equation	
Correct unsimplified equation Solve for $R$ in terms of $W$ . Dependent on the 2 preceding M marks Obtain <b>given answer</b> from correct working	
Parallel to the rod Perpendicular to the rod Moments about <i>C</i>	
Equation in $R$ . All terms needed. Condone sin/cos confusion and sign errors	
Correct unsimplified equation	
Sufficient additional equations to solve for $R$ in terms of $W$ . Dimensionally c terms needed. Condone sin/cos confusion and sign errors Correct unsimplified equation	correct. All
	Resolve vertically $\uparrow R + N \cos \alpha = W$ Take moments about A $7aN = 4a \cos \alpha \times W$ Obtain equation in R, W and $\alpha$ $N = W \times \frac{4}{7} \cos \alpha \Rightarrow$ $R = W - \frac{4}{7} W \cos^2 \alpha$ $= W \left(1 - \frac{4}{7} \cos^2 \alpha\right) *$ Alternative equations $R \sin \alpha + F \cos \alpha = W \sin \alpha$ $N + R \cos \alpha = W \cos \alpha + F \sin \alpha$ $W \cdot 3a \cos \alpha + F \cdot 7a \sin \alpha = R \cdot 7a \cos \alpha$ First 4 marks for alternative methods Correct unsimplified equation Solve for R in terms of W. Dependent on the 2 preceding M marks Obtain <b>given answer</b> from correct working Parallel to the rod Perpendicular to the rod Moments about C Equation in R. All terms needed. Condone sin/cos confusion and sign errors Correct unsimplified equation Sufficient additional equations to solve for R in terms of W. Dimensionally of the solve for R in terms of W. Dimensionally of the constant of the constant of the constant of the constant of th

Question Number	Scheme	Marks
6b	$R = W \left( 1 - \frac{4}{7} \times \frac{9}{10} \right) = \frac{17W}{35}$ Resolve horizontally $F = N \sin \alpha = \frac{4}{7} \times \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{10}} W$ $\left( = \frac{6}{35} W \right)$ Use of $F \le \mu R$ $\Rightarrow \mu \ge \frac{6}{17}$	B1 M1 A1 M1 A1 (5)
	17 Notes	[11]
6b B1 M1 A1	Seen or implied Obtain equation in Correct unsimplified equation in $F$ and $W$ (trig. substituted) (0.171W)	
M1 A1	Correct method to find the critical value. Condone with any symbol. 0.35 or better (0.3529) from correct working Final answer. Do not ISW	

Question Number	Scheme	Marks
7a	NB: sine/cosine confusion is not condoned in projectile questions	M1
	Use of conservation of energy	
	$\frac{1}{2}m \times 25^2 = \frac{1}{2}m \times 15^2 + mgh$	A1
	$\Rightarrow h = 20 \text{ or } 20.4 \text{ (m)}$	A1 (3)
7b	Vertical distance	M1
	$20.4 = 25 \sin \alpha \times 3 - 4.5 \times 9.8$	Alft
	$\alpha = 59^{\circ} \text{ or } 59.3^{\circ}$	A1 (3)
7c	Horizontal component of speed is constant	M1
	$\Rightarrow 25 \cos \alpha = 15 \cos \beta$	Alft
	$\beta = 32^{\circ} \text{ or } 31.8^{\circ}$	A1 (3)
7. alt	Vertical distance	MI
7c alt	$20.4 = -15\sin\beta \times 3 + 4.5 \times 9.8$	M1 A1ft
	$\beta = 32^{\circ} \text{ or } 31.8^{\circ}$	A1 (3)
	Notes	(5)
7a M1	Need energy equation with all 3 terms. Must be dimensionally correct. Con-	done sign errors.
A1	Correct unsimplified equation	
A1	Max 3 sf Not $\frac{1000}{49}$ nor $\frac{200}{g}$	
AI	49   g	
7b M1	Use of <i>suvat</i> to find $\alpha$	
A1ft	Correct unsimplified equation in their h	
A1	0.554 rads. Max 3 sf From CWO	
7c M1	Or horizontal distance travelled	
A1ft	Correct unsimplified in $\alpha$ or their $\alpha$	
A1	0.554 rads. Max 3 sf From CWO	
7c alt		
M1	Use of <i>suvat</i> to find $\beta$	
A1ft	e.g. using $s = vt - \frac{1}{2}gt^2$ . Correct unsimplified equation in their <i>h</i>	
A1	0.554 rads. Max 3 sf From CWO	

Question Number	Scheme	Marks
7d	Min speed = horizontal component = $25 \cos \alpha (= 15 \cos \beta)$ = 13 or 12.8 (m s <sup>-1</sup> )	M1 A1 (2)
7e	By considering vertical component of speed at <i>B</i> : $15\sin 31.8^\circ - gT = -15\sin 31.8^\circ$ T = 1.6  or  1.61  (s)	M1 A1ft A1 (3)
	Notes	[14]
7d M1 A1 7e M1 A1ft A1	<ul> <li>Follow their angle. Must show working if using incorrect angle.</li> <li>Max 3 sf From CWO</li> <li>Complete method using <i>suvat</i> to find <i>T</i></li> <li>Correct unsimplified equation in <i>T</i> - follow their angles.</li> <li>Max 3 sf From CWO</li> </ul>	

Question Number	Scheme	Marks
8	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	Change in KE $\frac{4m}{2}(4u^2 - v^2) + \frac{3m}{2}(9u^2 - w^2) = \frac{473}{24}mu^2$	M1 A1
	$(48v^2 + 36w^2 = 43u^2)$ Equation for CLM Need all terms. Dimensionally correct. Condone sign errors.	M1
	8mu - 9mu = -4mv + 3mw $(u = 4v - 3w)$	A1 M1
	Impact law w + v = 5eu	A1
	$48v^{2} + 36\left(\frac{4v-u}{3}\right)^{2} = 43u^{2} \text{ Or } 48\left(\frac{u+3w}{4}\right)^{2} + 36w^{2} = 43u^{2}$	DM1
	Or $\frac{48}{49}(1+15e)^2 + \frac{36}{49}(20e-1)^2 = 43$	
	$112v^{2} - 32uv - 39u^{2} = 0$ = $(4v - 3u)(28v + 13u)$ Or $63w^{2} + 18uw - 40u^{2} = 0$ = $(21w + 20u)(3w - 2u)$	DM1
	Or $25200e^2 = 2023$ <b>Notes</b>	
	The first 8 marks are available if they have ignored the information about Work with their directions. Ignore the diagram if that is to the candidate's	
M1 A1 M1 A1 M1 A1	Need all terms. Dimensionally correct. Accept $\pm$ Correct unsimplified equation in $v$ , $w$ or their $v$ , $w$ Need all terms. Dimensionally correct. Condone sign errors. Correct unsimplified equation with their correct signs Must be used the right way round Or equivalent in their $w$ , $v$ . Signs for $v$ , $w$ consistent with CLM eqn	
DM1 DM1	Form equation for $v$ or $w$ or $e$ Dependent on M marks scored for the equations used. Solve for $v$ or $w$ or $e$ . Dependent on the preceding M	

	M	2_2020_01_MS
Question Number	Scheme	Marks
Number		
8	3u = 3u = 2u	A1
8	$v = \frac{3u}{4}  w = \frac{2u}{3}$ $\frac{3u}{4} + \frac{2u}{3} = 5eu, \ e = \frac{17}{60}$	A 1
	$\frac{3u}{4} + \frac{2u}{3} = 5eu, \ e = \frac{17}{60}$	A1
	Use of $I - m(v - u)$	DM1
	Use of $I = m(v-u)$ $4m\left(2u + \frac{3u}{4}\right) = 11mu$	A1 (12)
	$A_m\left(2u+\frac{3u}{2}\right)-11mu$	
	$4m\left(2u+\frac{1}{4}\right)^{-11mu}$	
		[12]
A1	Notes           v or w correct	
l		
A1	$\frac{3u}{4} + \frac{2u}{3} = 5eu$	
DM1	Must be attempting to subtract corresponding values for $u$ and $v$ Dependent on the first 4 M marks	
A1	Or $3m\left(3u + \frac{2u}{3}\right)$ from correct solution only	
	$\left(\frac{3}{3}\right)$ from context boundaries only	
l		
l		
l		