Question Number	Scheme	Notes	Marks
1.	"...show all stages of your working. Solutions relying entirely on calculator technology are not acceptable."		
	Move to the alternative scheme if they use exponentials		
	$\cosh 2x - 7\sinh x = 5 \Rightarrow 1 + 2\sinh^2 x - 7\sinh x = 5$ Replaces cosh2x with $1+2\sinh^2 x$. Or $\cosh 2x$ with $2\cosh^2 x - 1$ or $\cosh^2 x + \sinh^2 x$ **and then** $\cosh^2 x$ with $1+\sinh^2 x$. There must be no incorrect identities used.		**M1**
	$2\sinh^2 x - 7\sinh x - 4 = 0$	Correct 3TQ (implied by a correct solution for sinh x)	**A1**
	$(2\sinh x + 1)(\sinh x - 4) = 0 \Rightarrow \sinh x =$	Attempt to solve 3TQ in sinh$x$ (usual rules) Solving method must be seen - note that e.g., 1100111 is a possible mark profile	**M1**
	$\sinh x = -\frac{1}{2}, 4$	Both correct values	**A1**
	$x = \ln\left(-\dfrac{1}{2} + \sqrt{\left(-\dfrac{1}{2}\right)^2 + 1}\right), \quad \ln\left(4 + \sqrt{(4)^2 + 1}\right)$ Obtains at least one **correct** exact log form of x for one of their real values of sinh x. This mark may also be gained by using the correct exponential form of sinh$x$ and solving appropriately (may be slips but must reach 3TQ in e^x - does not have to be written in =0 form). Then solved by usual rules but allow if consistent calculator answer - and must take lns)		**M1**
	$x = \ln\left(-\dfrac{1}{2} + \sqrt{\dfrac{5}{4}}\right)$ or e.g., $\ln\dfrac{-1+\sqrt{5}}{2}, \quad \ln\left(4+\sqrt{17}\right)$ A1: One correct exact value of x. Allow equivalent processed exact answers. A1: Both values correct and exact and no additional incorrect answers. Allow equivalent processed exact answers. **Condone missing brackets.**		**A1** **A1** **(7)**
			Total 7

	Alternative by exponential definitions (<u>in most cases</u> will score a max of 2/7)		
	e.g., $\left(\dfrac{e^{2x} + e^{-2x}}{2}\right) - 7\left(\dfrac{e^x - e^{-x}}{2}\right) = 5$	Uses correct identities	**M1**
	$e^{4x} - 7e^{3x} - 10e^{2x} + 7e^x + 1 = 0$	Correct quartic in e^x (any multiple - terms may not all be on one side but must be collected)	**A1**
	$(e^{2x} + e^x - 1)(e^{2x} - 8e^x - 1) = 0 \Rightarrow e^x = \ldots \quad \dfrac{-1+\sqrt{5}}{2}, \; 4+\sqrt{17}$ M1: Solves their quartic as far as $e^x = \ldots$ A1: Both correct exact values There must be a recognisable attempt to write a 5 term quartic as the product of two 3TQs in e^x (factorisation for the constant and for e^{4x} can have sign errors only) **followed by** an explicit method (usual rules) on at least one 3TQ.		**M1** **A1**
	$\Rightarrow x = \ldots$ Converts correctly to give at least one value for x as a valid exact natural logarithm. This mark requires a recognisable attempt to write a 5 term quartic as the product of two 3TQs in e^x as before but condone no explicit method on 3TQ		**M1**
	$x = \ln\left(-\dfrac{1}{2} + \sqrt{\dfrac{5}{4}}\right)$ or e.g., $\ln\dfrac{-1+\sqrt{5}}{2}, \quad \ln\left(4+\sqrt{17}\right)$ A1: One correct exact value of x. Allow equivalent processed exact answers A1: Both values correct and exact and no additional incorrect answers. Allow equivalent exact answers. **Condone missing brackets**.		**A1** **A1** **(7)**

Question Number	Scheme	Notes	Marks
2(a)	"...show all stages of your working. Solutions relying entirely on calculator technology are not acceptable."		
	$x^2+4x+13=(x+2)^2+9$ or $(x+2)^2+3^2$	Correctly completes the square	**B1**
	$\int \frac{1}{(x+2)^2+9}\mathrm{d}x=\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$ M1: $k\arctan(\pm ax\pm b)$ $a,b\neq 0$, A1: Correct k can be 1. Condone e.g., "artan" if work confirms that "arctan" was intended		**M1** **A1**
	$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}=\frac{1}{3}(\arctan 1-\arctan 0)$	Correct use of correct limits but arctan 0 or 0 need not be shown. Could go straight to a consistent value	**dM1**
	$\frac{\pi}{12}$	Correct value	**A1**
	May see "mini" substitutions here (& in (b)) such as $u=x+2$: limits must change correctly		**(5)**
Example by Sub--stitution	$x^2+4x+13=(x+2)^2+9$ or $(x+2)^2+3^2$	Correctly completes the square	**B1**
	$x+2=3\tan t\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=3\sec^2 t$	$x=-2, \tan t=0,\ t=0;\ x=1, \tan t=1,\ t=\frac{\pi}{4}$	
	$\int\frac{3\sec^2 t}{9\tan^2 t+9}\mathrm{d}t=\frac{1}{3}\int\mathrm{d}t=\frac{1}{3}t$	M1: Substitutes $x+2=3\tan t$, uses $\tan^2 t+1=\sec^2 t$ and integrates to kt A1: Correct expression. Ignore limits	**M1** **A1**
	$\left[\frac{1}{3}t\right]_0^{\frac{\pi}{4}}=\frac{1}{3}\times\frac{\pi}{4}-\frac{1}{3}\times 0,\ =\frac{\pi}{12}$	Either changes limits correctly and substitutes correctly or reverts to x correctly and uses original limits correctly. $\frac{1}{3}\times 0$ or 0 need not be seen - could go straight to a consistent value	**dM1** **A1**
(b)	$4x^2-12x+34=4\left(x-\frac{3}{2}\right)^2+25$ or $(2x-3)^2+5^2$ oe May be seen within root M1: $4(x\pm p)^2\pm q$ or $(2x\pm p)^2\pm q$ oe $(p,q\neq 0)$ A1: $4\left(x-\frac{3}{2}\right)^2+25$ oe e.g., $4\left(\left(x-\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2\right)$		**M1** **A1**
	$\frac{1}{2}\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{25}{4}}}\mathrm{d}x=\frac{1}{2}\operatorname{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)$ or $\frac{1}{2}\operatorname{arsinh}\left(\frac{2x-3}{5}\right)$ or e.g., $\frac{1}{2}\ln\left(x-\frac{3}{2}+\sqrt{\left(x-\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2}\right)$ M1: $k\operatorname{arsinh}\mathrm{f}(x)$ or $p\ln\left(\mathrm{f}(x)+\sqrt{(\mathrm{f}(x))^2+q}\right)$ oe $f(x)=\pm ax\pm b$ $a,b\neq 0$ A1: Correct (If log form ignore any embedded multiplying constant within the ln bracket). k can be 1. Condone e.g., "arcsinh" if recovered. Above may be obtained via substitutions e.g., $x-\frac{3}{2}=\frac{5}{2}\sinh t\left(\to kt,=\frac{1}{2}t\right)$ or (unlikely) $x-\frac{3}{2}=\frac{5}{2}\tan t\left(\to k\left\{=\frac{1}{2}\right\}\ln(\sec t+\tan t)\right)$		**M1** **A1**
	$\left[\frac{1}{2}\operatorname{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)\right]_{-1}^{4}$ or $\frac{1}{2}[t]_{\operatorname{arsinh}(-1)}^{\operatorname{arsinh}1}=\frac{1}{2}(\operatorname{arsinh}(1)-\operatorname{arsinh}(-1))$	Correct use of correct limits (changed correctly if necessary)	**ddM1**
	$=\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)-\ln\left(-1+\sqrt{2}\right)\right)$ or $=\frac{1}{2}\times 2\operatorname{arsinh}(1)=\ln\left(1+\sqrt{2}\right)$ Uses the correct logarithmic form of arsinh twice (or once if uses $\operatorname{arsinh}(1)-\operatorname{arsinh}(-1)=2\operatorname{arsinh}(1)$). This mark may also be gained by twice using the correct exponential form of sinh$x$ and proceeding appropriately to a numerical expression. May be slips but must reach 3TQs in e^x (do not have to be written in =0 form). Then solved by usual rules but allow if consistent calculator answers		**ddM1 Requires first 2 M marks**
	Score ddM1ddM1 together for correctly using correct limits on an appropriate log form, e.g., $\left[\frac{1}{2}\ln\left(\frac{2x-3}{5}+\sqrt{\left(\frac{2x-3}{5}\right)^2+1}\right)\right]_{-1}^{4}=\ldots$ or $\left[\frac{1}{2}\ln\left(2x-3+\sqrt{(2x-3)^2+25}\right)\right]_{-1}^{4}=\ldots$ $\left\{\frac{1}{2}\left(\ln\left(5+5\sqrt{2}\right)-\ln\left(-5+5\sqrt{2}\right)\right)\right\}$		
	$=\frac{1}{2}\ln\frac{1+\sqrt{2}}{-1+\sqrt{2}}=\frac{1}{2}\ln\left(\frac{1+\sqrt{2}}{-1+\sqrt{2}}\times\frac{-1-\sqrt{2}}{-1-\sqrt{2}}\right)=\frac{1}{2}\ln\left(3+2\sqrt{2}\right)$ $\left[\text{or } \ln\left(1+\sqrt{2}\right)\right]$ Not just values of constants unless form seen. Must include explicit surd work but it could be minimal and is not necessary with answer $\ln\left(1+\sqrt{2}\right)$. Condone missing brackets.		**A1** **(7)** **Total 12**

Question Number	Scheme	Notes	Marks
3(a)	$\left\{\frac{\mathrm{d}x}{\mathrm{d}\alpha}=4\sinh\alpha,\frac{\mathrm{d}y}{\mathrm{d}\alpha}=2\cosh\alpha\Rightarrow\right\}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cosh\alpha}{4\sinh\alpha}$ **or** $\frac{2x}{16}-\frac{2yy'}{4}=0\Rightarrow y'=\frac{x}{4y}\left\{=\frac{4\cosh\alpha}{8\sinh\alpha}\right\}$ **or** $\left\{y=\frac{\sqrt{x^2-16}}{2}\Rightarrow\right\}y'=\frac{x}{2\sqrt{x^2-16}}\left\{=\frac{4\cosh\alpha}{2\sqrt{16\cosh^2\alpha-16}}=\frac{4\cosh\alpha}{8\sinh\alpha}\right\}$ M1: Differentiates x and y (correct forms) & divides correctly or differentiates implicitly to obtain $px-qyy'=0$ oe and reaches $y'=...$ or explicitly to obtain $y'=\frac{kx}{\sqrt{x^2-16}}$ A1: Any correct derivative **in terms of α** e.g., $\frac{1}{2}\coth\alpha$		M1 A1
	$(y-2\sinh\alpha)=\frac{\cosh\alpha}{2\sinh\alpha}(x-4\cosh\alpha)$ or $2\sinh\alpha=\frac{\cosh\alpha}{2\sinh\alpha}(4\cosh\alpha)+c\Rightarrow c=...\left\{2\sinh\alpha-\frac{4\cosh^2\alpha}{2\sinh\alpha}\right\}\Rightarrow y=\frac{\cosh\alpha}{2\sinh\alpha}x+2\sinh\alpha-\frac{4\cosh^2\alpha}{2\sinh\alpha}$ Correct straight line using their **tangent** gradient **in terms of α**		M1
	$2y\sinh\alpha-4\sinh^2\alpha=x\cosh\alpha-4\cosh^2\alpha$ or e.g., $y\sinh\alpha+4\left(\cosh^2\alpha-\sinh^2\alpha\right)-x\cosh\alpha=0$ or e.g., $y=\frac{\cosh\alpha}{2\sinh\alpha}x-\frac{4}{2\sinh\alpha}\Rightarrow 2y\sinh\alpha-x\cosh\alpha+4=0$ * Achieves the given answer (terms as printed in any order on one side) with an intermediate step. A0 if see e.g., sin for sinh.		A1*
			(4)
(b)	$A\left(0,\dfrac{-2}{\sinh\alpha}\right)$ oe	M1: $y=\pm\frac{p}{\sinh\alpha}$ or $y=\pm\frac{p}{q\sinh\alpha}$ A1: $y=\frac{-2}{\sinh\alpha}$ or $y=\frac{-4}{2\sinh\alpha}$ oe For the A1 if x is given it must be 0. Could use exponentials. Allow M1A0 if just $\left(\pm\frac{p}{q\sinh\alpha},0\right)$. "sin" is 00 unless above are seen	M1 A1
			(2)
(c)	**For work in terms of *a* and/or *e* score the marks once appropriate values are seen or implied. May use e.g., *F* for *S*. May see use of** $c^2=(ae)^2=a^2+b^2$		
	e.g., $b^2=a^2\left(e^2-1\right)\Rightarrow 4=a^2e^2-16$ or $4=16e^2-16\Rightarrow e^2=\frac{5}{4}\left(e=\frac{\sqrt{5}}{2}\right)\Rightarrow a^2e^2=20$ **Uses** a **correct** eccentricity formula with $a=4$ (or 16) & $b=2$ (or 4) to obtain a value for a^2e^2 or ae. Or finds a value for e and multiplies by a or a value for e^2 and $\times a^2$		M1
	$ae=\sqrt{20}$ or $2\sqrt{5}$ $\left\{\Rightarrow S\left(2\sqrt{5},0\right),\ B\left(0,10\sinh\alpha\right)\right\}$	Correct value for ae - could be implied	A1
	Grad $AS=\dfrac{\left(\frac{2}{\sinh\alpha}\right)}{2\sqrt{5}}$ **or** Grad $BS=-\frac{10\sinh\alpha}{2\sqrt{5}}$ **or** $\overrightarrow{AS}=\begin{pmatrix}2\sqrt{5}\\ \frac{2}{\sinh\alpha}\end{pmatrix}$ **or** $\overrightarrow{BS}=\begin{pmatrix}2\sqrt{5}\\ -10\sinh\alpha\end{pmatrix}$ At least 1 correct gradient or vector (allow as "coordinates" and could be 3D). If an ambiguous fraction within fraction given, look for recovery later. Not a ft mark.		B1
	$\dfrac{\frac{2}{\sinh\alpha}}{2\sqrt{5}}\times-\dfrac{10\sinh\alpha}{2\sqrt{5}}$ or $\overrightarrow{AS}.\overrightarrow{BS}=\begin{pmatrix}2\sqrt{5}\\ \frac{2}{\sinh\alpha}\end{pmatrix}.\begin{pmatrix}2\sqrt{5}\\ -10\sinh\alpha\end{pmatrix}=\left(2\sqrt{5}\right)^2+\dfrac{2}{\sinh\alpha}\times(-10\sinh\alpha)$ (M1) $=-1$ or $=0$ so AS and BS are perpendicular/"shown" etc. (A1) M1: Writes the product of their AS and BS gradients correctly (or writes $m_1=-\frac{1}{m_2}$) or obtains a consistent scalar product expression e.g. $\overrightarrow{\mathbf{AS}}.\overrightarrow{\mathbf{BS}}$. Must see more than just -1 or 0. May be using exponentials/cosech. A1: Product = -1 (or $m_1=-\frac{1}{m_2}$) or scalar product = 0 with no errors and conclusion.		M1 A1 (last mark requires all previous marks)
	ALT: $AS^2+BS^2=\left(\frac{2}{\sinh\alpha}\right)^2+\left(2\sqrt{5}\right)^2+(10\sinh\alpha)^2+\left(2\sqrt{5}\right)^2$ $AB^2=\left(10\sinh\alpha+\frac{2}{\sinh\alpha}\right)^2$ (B1 - either AS^2+BS^2 (or AS^2 and BS^2) or AB^2 correct, M1 attempts both/all three, A1 **shows** equal e.g., $AB^2=\left(\frac{2}{\sinh\alpha}\right)^2+(10\sinh\alpha)^2+2\times\frac{2}{\sinh\alpha}\times10\sinh\alpha$ and conclusion)		Alt by Pyth.
	A0 if any sin for sinh seen. Allow work with SA and SB but A1 needs reference to AS and BS. Ignore $e=\pm\frac{\sqrt{5}}{2}$ and "S(0, 2rt5)" if recovered.		**(5)**
			Total 11

Question Number	Scheme	Notes	Marks
4	$I_n = \int \sec^n x \, \mathrm{d}x \qquad n \geq 0$ If d(...) notation is used it must be resolved before the relevant marks are awarded		
4(a) **Integrates first**	$\int \sec^n x \, \mathrm{d}x = \int \sec^{n-2} x \sec^2 x \mathrm{d}x$	Splits $\sec^n x$ into $\sec^{n-2} x \sec^2 x$ at the start	**M1**
	$\int \sec^n x \, \mathrm{d}x = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x \tan^2 x \, \mathrm{d}x$	dM1: Uses integration by parts to obtain $\sec^{n-2} x \tan x \pm k\int \sec^{n-2} x \tan^2 x \, \mathrm{d}x$ oe A1: Any correct expression	**dM1** **A1**
	$\int \sec^n x \, \mathrm{d}x = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x(\sec^2 x - 1) \, \mathrm{d}x$	Replaces $\tan^2 x$ with $\sec^2 x - 1$ Requires 1st mark	**B1**
	$\int \sec^n x \, \mathrm{d}x = \sec^{n-2} x \tan x - (n-2)\int \sec^n x \, \mathrm{d}x + (n-2)\int \sec^{n-2} x \, \mathrm{d}x$ or $I_n = \sec^{n-2} x \tan x - (n-2)I_n + (n-2)I_{n-2}$ **Depends on all previous M <u>and</u> B marks.** Obtains equation in I_n and I_{n-2} or integrals - not just the final answer		**ddM1**
	$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ *	Fully correct proof. Allow e.g., $(n-2)I_{n-2} + \sec^{n-2} x \tan x = (n-1)I_n$	**A1***
			(6)
ALT **Identity first**	$\int \sec^n x \, \mathrm{d}x = \int \sec^{n-2} x \sec^2 x \mathrm{d}x$	Splits $\sec^n x$ into $\sec^{n-2} x \sec^2 x$ at the start	**M1**
	$\int \sec^{n-2} x \sec^2 x \mathrm{d}x = \int \sec^{n-2} x\left(\tan^2 x + 1\right)\mathrm{d}x$ $\left\{= \int \sec^{n-2} x \mathrm{d}x + \int \tan^2 x \sec^{n-2} x \mathrm{d}x\right\}$	Replaces $\sec^2 x$ with $\tan^2 x + 1$ Requires 1st mark	**B1** **(4th ePen mark)**
	$\int \tan^2 x \sec^{n-2} x \mathrm{d}x = \frac{1}{(n-2)}\tan x \sec^{n-2} x - \frac{1}{(n-2)}\int \sec^n x \, \mathrm{d}x$ dM1: Uses int. by parts on $\int \tan^2 x \sec^{n-2} x \mathrm{d}x$ to obtain $p \tan x \sec^{n-2} x \pm q\int \sec^n x \, \mathrm{d}x$ oe A1: Any correct expression for $\int \tan^2 x \sec^{n-2} x \mathrm{d}x$		**dM1** **A1**
	$\int \sec^n x \, \mathrm{d}x = \int \sec^{n-2} x \, \mathrm{d}x + \frac{1}{(n-2)}\tan x \sec^{n-2} x - \frac{1}{(n-2)}\int \sec^n x \, \mathrm{d}x$ or $I_n = I_{n-2} + \frac{1}{(n-2)}\tan x \sec^{n-2} x - \frac{1}{(n-2)}I_n$ **Depends on all previous M <u>and</u> B marks.** Obtains equation in I_n and I_{n-2} or integrals - not just the final answer.		**ddM1**
	$\Rightarrow$ e.g., $(n-2+1)I_n = (n-2)I_{n-2} + \tan x \sec^{n-2} x$ $\Rightarrow (n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ *	Fully correct proof. An intermediate step is required for this approach. Allow a final answer of e.g., $(n-2)I_{n-2} + \sec^{n-2} x \tan x = (n-1)I_n$	**A1***
	For either method condone missing dx's, the odd missing variable, notational issues e.g., "$\sec x^{n-2}$" and allow recovery of missing brackets if this is before the given answer. Since this question involves trigonometric and not hyperbolic functions we will condone the occasional e.g., tanh for tan but the final answer must be as shown.		

Question Number	Scheme	Notes	Marks
4(b)	"...showing each step of your working...Solutions relying entirely on calculator technology are not acceptable." $(n-1)I_n = \tan x\sec^{n-2}x + (n-2)I_{n-2}$, $I_n = \dfrac{1}{n-1}\tan x\sec^{n-2}x + \dfrac{n-2}{n-1}I_{n-2}$		
I_6 first	$I_2\left\{=\left[\tan x\right]_0^{\frac{\pi}{4}}\right\}=1$	Correct **value** for I_2 seen or implied. Ignore "calculations" of I_0	**B1**
	$I_6 = \frac{1}{5}\left[\tan x\sec^4 x\right]_0^{\frac{\pi}{4}} + \frac{4}{5}I_4$ or e.g. $I_6 = \frac{1}{5}\tan\frac{\pi}{4}\sec^4\frac{\pi}{4} + \frac{4}{5}I_4$ or e.g. $I_6 = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{5}I_4$ Applies the reduction formula once. No need to see or apply limits. Fine as $5I_6 = ...$ and allow slips e.g., using an incorrect formula rearrangement e.g, $I_n = \tan x\sec^{n-2}x + (n-2)I_{n-2}$ or e.g., $I_n = \dfrac{1}{n-1}\tan x\sec^{n-2}x + (n-2)I_{n-2}$		**M1**
	$= \frac{1}{5}\tan x\sec^4 x + \frac{4}{5}\left(\frac{1}{3}\tan x\sec^2 x + \frac{2}{3}I_2\right) = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{15}(1)\left(\sqrt{2}\right)^2 + \frac{8}{15}(1)$ Applies the given reduction formula again (same slips as before) and uses the limits to reach a numerical expression for I_6. If just a value is given it must be $\frac{28}{15}$		**dM1**
	$=\dfrac{28}{15}$	Correct value (or exact equivalent)	**A1**
			(4)
ALT I_4 first	$I_2 = \left\{\left[\tan x\right]_0^{\frac{\pi}{4}} = \right\}1$	Correct **value** for I_2 seen or implied. Ignore "calculations" of I_0	**B1**
	$I_4 = \frac{1}{3}\left[\tan x\sec^2 x\right]_0^{\frac{\pi}{4}} + \frac{2}{3}I_2$ or e.g. $I_4 = \frac{1}{3}\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} + \frac{2}{3}I_2$ or e.g. $I_4 = \frac{1}{3}(1)\left(\sqrt{2}\right)^2 + \frac{2}{3}I_2\left\{=\frac{4}{3}\right\}$ Applies the reduction formula once. No need to see or apply limits. Fine as $3I_4 = ...$ and allow slips as main scheme		**M1**
	$I_6 = \frac{1}{5}\tan x\sec^4 x + \frac{4}{5}\left(\frac{1}{3}\tan x\sec^2 x + \frac{2}{3}I_2\right) = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{15}(1)\left(\sqrt{2}\right)^2 + \frac{8}{15}$ Applies the reduction formula again (same slips as before) and uses the limits to reach a numerical expression for I_6. If just a value is given it must be $\frac{28}{15}$		**dM1**
	$=\dfrac{28}{15}$	Correct value (or exact equivalent)	**A1**
			Total 10

For either method in part (b), working must be shown and the given reduction formula must be used at least once. So do not allow e.g. I_6 to be attempted directly but I_4 can be evaluated directly without using the given reduction formula using an alternative explicit method e.g. by parts or by substitution. To score the dM1 in these cases they will need to have integrated correctly and reached a numerical expression for I_6. The B mark can be awarded for a correct value for I_4 or I_6 $\left(\frac{4}{3}\text{ or }\frac{28}{15}\right)$:

Parts:

$$I_4 = \int\sec^4 x\,\mathrm{d}x = \int\sec^2 x\sec^2 x\,\mathrm{d}x = \sec^2 x\tan x - 2\int\sec^2 x\tan^2 x\,\mathrm{d}x$$

$$= \sec^2 x\tan x - 2\int\sec^2 x\left(\sec^2 x - 1\right)\mathrm{d}x = \sec^2 x\tan x - 2\int\sec^4 x\,\mathrm{d}x + 2\int\sec^2 x\,\mathrm{d}x$$

$$= \sec^2 x\tan x - 2I_4 + 2\int\sec^2 x\,\mathrm{d}x \Rightarrow 3I_4 = \sec^2 x\tan x + 2\tan x \Rightarrow I_4 = \frac{1}{3}\sec^2 x\tan x + \frac{2}{3}\tan x$$

Substitution:

$$I_4 = \int\sec^4 x\,\mathrm{d}x = \int\sec^2 x\sec^2 x\,\mathrm{d}x = \int\sec^2 x\left(1+\tan^2 x\right)\mathrm{d}x$$

$$u = \tan x \Rightarrow \int\sec^2 x\left(1+\tan^2 x\right)\mathrm{d}x = \int\sec^2 x\left(1+u^2\right)\frac{\mathrm{d}u}{\sec^2 x} = \frac{u^3}{3} + u = \frac{\tan^3 x}{3} + \tan x$$

Question Number	Scheme	Notes	Marks
5.	$\mathbf{M}=\begin{pmatrix}4 & -5 & 0\\ k & 2 & 0\\ -3 & -5 & k\end{pmatrix}$ Ignore labelling of matrices in (a) and allow any brackets/no brackets throughout		
(a)	$\lvert\mathbf{M}\rvert=4(2k)+5(k^2)(+0)$	Correct determinant in any form (all 2 x 2 determinants processed). May use Sarrus e.g., $8k-(-5k^2)$	**1st B1**
	Minors : $\begin{pmatrix}2k & k^2 & -5k+6\\ -5k & 4k & -35\\ 0 & 0 & 8+5k\end{pmatrix}$ or Cofactors : $\begin{pmatrix}2k & -k^2 & 6-5k\\ 5k & 4k & 35\\ 0 & 0 & 8+5k\end{pmatrix}$ A correct first step of minors or cofactors. May have transposed. Terms collected.		**2nd B1**
	$\mathbf{M}^{-1}=\dfrac{1}{5k^2+8k}\begin{pmatrix}2k & 5k & 0\\ -k^2 & 4k & 0\\ 6-5k & 35 & 8+5k\end{pmatrix}$	M1: A full credible attempt at inverse: minors, cofactors, transpose and x 1/ their determinant B1: Any 2 correct rows or columns in Adj($\mathbf{M}$) - ignore any determinant A1: Any fully correct inverse with like terms collected	**M1** **3rd B1** **A1**
			(5)
(b)	e.g., $\mathbf{M}^{-1}=-\dfrac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix}$	Any evidence of recognising $k=-1$	**M1**
	$\Pi_2:2x-z=4\Rightarrow x=s, y=t, z=2s-4$	Attempts parametric form. Could be slips but must be 2 parameters	**M1**
	$-\dfrac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix}\begin{pmatrix}s\\ t\\ 2s-4\end{pmatrix}=\begin{pmatrix}...\\ ...\\ ...\end{pmatrix}$ Completes an attempt at $\mathbf{M}^{-1}\times$their parametric form. Allow losing the $-\frac{1}{3}$. May be achieved with separate vectors provided results brought together. Could be seen as $x=...,\ y=...,\ z=...$ Must complete processing e.g., on the $-(2s-4)\Rightarrow...$		**ddM1**
	$-\dfrac{1}{3}\begin{pmatrix}-2s-5t\\ -s-4t\\ 17s+35t-12\end{pmatrix}$ or e.g., $\begin{pmatrix}\frac{2}{3}s+\frac{5}{3}t\\ \frac{1}{3}s+\frac{4}{3}t\\ -\frac{11}{3}s-\frac{35}{3}t-2s+4\end{pmatrix}$	Correct parametric form for Π_1 May be seen as $x=...,\ y=...,\ z=...$ Terms may not be collected	**A1**
	$3x=2s+5t,\quad 3y=s+4t,\quad 3z=-11s-35t-6s+12$ $\Rightarrow...\Rightarrow 11x-5y+z=4$ dddM1: Eliminates *s* and *t* to obtain a cartesian equation of correct form ("processed" with at least 4 terms but they may be uncollected). Requires all M marks. All of *x*, *y* and *z* must have been functions of both *s* and *t* and there must be some (minimal) algebra. May use e.g., *a*, *b* and *c* for *x*, *y* and *z* Allow for equivalent work e.g., $\begin{pmatrix}\frac{2}{3}s+\frac{5}{3}t\\ \frac{1}{3}s+\frac{4}{3}t\\ -\frac{17}{3}s-\frac{35}{3}t+4\end{pmatrix}\Rightarrow\begin{pmatrix}0\\ 0\\ 4\end{pmatrix}+\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\\ -\frac{17}{3}\end{pmatrix}s+\begin{pmatrix}\frac{5}{3}\\ \frac{4}{3}\\ -\frac{35}{3}\end{pmatrix}t\Rightarrow\mathbf{n}=\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\\ -\frac{17}{3}\end{pmatrix}\times\begin{pmatrix}\frac{5}{3}\\ \frac{4}{3}\\ -\frac{35}{3}\end{pmatrix}=\begin{pmatrix}\frac{11}{3}\\ -\frac{5}{3}\\ \frac{1}{3}\end{pmatrix}$ **and** $\begin{pmatrix}0\\ 0\\ 4\end{pmatrix}.\begin{pmatrix}\frac{11}{3}\\ -\frac{5}{3}\\ \frac{1}{3}\end{pmatrix}=\frac{4}{3}$ $\Rightarrow\frac{11}{3}x-\frac{5}{3}y+\frac{1}{3}z=\frac{4}{3}$ A1: Any correct equation (terms collected) and in *x*, *y* and *z* e.g., $2z-10y+22x-8=0$ A0 if given in e.g., $x_1,\ y_1,\ z_1$ or *a*, *b* & *c* etc.		**dddM1** **A1**
			(6)
	Some guidance for alternatives overleaf		**Total 11**

Q5(b) alternatives:

$$\mathbf{M}^{-1}=\frac{1}{5k^2+8k}\begin{pmatrix}2k & 5k & 0\\ -k^2 & 4k & 0\\ 6-5k & 35 & 8+5k\end{pmatrix}=-\frac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix},\quad \mathbf{M}=\begin{pmatrix}4 & -5 & 0\\ k & 2 & 0\\ -3 & -5 & k\end{pmatrix}=\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}$$

There are obviously a lot of possible alternatives here. The mark dependency applies as before. If the learner would score more by the main scheme then apply that but otherwise try and match the overall approach to one of the following examples.

Allow the first M mark for any recognition that $k=-1$ (which may be seen in $\mathbf{M}$ or $\mathbf{M}^{-1}$)

Score the next two M marks (M1dM1) together for a full method where enough work has been done so that a cartesian equation can be produced by a straightforward calculation or algebraic manipulation. If this is not the case, only the first mark will be available (unless using the main scheme scores more marks e.g., an acceptable attempt at the parametric form of Π_2 is also seen).

Then award the first A if everything is correct so far.

Examples:

Using $\mathbf{MP}=\mathbf{Q}$ with $P(x,y,z)$ & $Q(a,b,c)\Rightarrow a=4x-5y,\ c=-3x-5y-z$ (4 marks)

$2a-c=4\Rightarrow 8x-10y+3x+5y+z=4$ (dddM1) $\Rightarrow 11x-5y+z=4$ (A1)

May work with $P(a,b,c)$ & $Q(x,y,z)$ and swap letters at the end

OR $\Pi_2: 2x-z=4$ so 3 points on Π_2 are e.g., $Q_1(2,0,0)$, $Q_2(3,0,2)$ & $Q_3(2,1,0)$

You may have to check to ensure these points are not collinear e.g., the vectors $\mathbf{Q_1Q_2}$ and $\mathbf{Q_1Q_3}$ are not multiples of each other otherwise this method is not viable. This will also be apparent if the three y-coordinates are all zero.

Could apply inverse to these points i.e., $\mathbf{P}=\mathbf{M}^{-1}\mathbf{Q}$ or could solve $\mathbf{MP}=\mathbf{Q}$

$$\Rightarrow P_1\left(\tfrac{4}{3},\tfrac{2}{3},-\tfrac{22}{3}\right),\ P_2(2,1,-13)\ \&\ P_3(3,2,-19)$$

$$P_2-P_1\Rightarrow\frac{1}{3}\begin{pmatrix}2\\1\\-17\end{pmatrix},\quad P_3-P_1\Rightarrow\frac{1}{3}\begin{pmatrix}5\\4\\-35\end{pmatrix}\quad\Rightarrow\text{normal by vector product}:\begin{pmatrix}11\\-5\\1\end{pmatrix}\ \text{(first four marks scored)}$$

$$\Pi_1: 11x-5y+z=11\left(\tfrac{4}{3}\right)-5\left(\tfrac{2}{3}\right)-\tfrac{22}{3}=\ldots 4\ \text{(dddM1A1)}$$

OR A point on Π_2 is $(2,0,0)$ and 2 vectors (must have different directions) in Π_2 are $\begin{pmatrix}1\\0\\2\end{pmatrix}$ and $\begin{pmatrix}0\\1\\0\end{pmatrix}$

Point becomes $\left(\tfrac{4}{3},\tfrac{2}{3},-\tfrac{22}{3}\right)$ and vectors become $\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\\ -\frac{17}{3}\end{pmatrix}$ and $\begin{pmatrix}\frac{5}{3}\\ \frac{4}{3}\\ -\frac{35}{3}\end{pmatrix}\Rightarrow$ normal: $\begin{pmatrix}11\\-5\\1\end{pmatrix}$ as before

OR: The following approach using row vectors for the normal using $\mathbf{M}$ is is also valid:

$\Pi_2: 2x-z=4$ has normal $\begin{pmatrix}2\\0\\-1\end{pmatrix}$ and so Π_1 normal is $\begin{pmatrix}2 & 0 & -1\end{pmatrix}\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}=\begin{pmatrix}11 & -5 & 1\end{pmatrix}$

<u>**and**</u> e.g., $Q_1(2,0,0)\Rightarrow P_1\left(\tfrac{4}{3},\tfrac{2}{3},-\tfrac{22}{3}\right)$ (4 marks)

$$\Pi_1: 11x-5y+z=11\left(\tfrac{4}{3}\right)-5\left(\tfrac{2}{3}\right)-\tfrac{22}{3}=\ldots 4\ \text{(dddM1A1)}$$

Could obtain the normal using the transpose of $\mathbf{M}$: $\begin{pmatrix}4 & -1 & -3\\ -5 & 2 & -5\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}2\\0\\-1\end{pmatrix}=\begin{pmatrix}11\\-5\\1\end{pmatrix}$

But note that this approach needs a point on Π_1 - i.e., there must be work to achieve the "4" (which just happens to be common to both planes).

Question Number	Scheme	Notes	Marks
6.	$x = \cosh t + t, \quad y = \cosh t - t$		
(a)	$\dfrac{dx}{dt} = \sinh t + 1, \quad \dfrac{dy}{dt} = \sinh t - 1$	Correct derivatives. Move to guidance overleaf at the point at which they move to exponentials if applicable and may see a combination of approaches. Could be implied	**B1**
	$(\sinh t + 1)^2 + (\sinh t - 1)^2$ $\{= \sinh^2 t + 2\sinh t + 1 + \sinh^2 t - 2\sinh t + 1\}$ $\{= 2\sinh^2 t + 2\}$	Uses $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$ Correct for their derivatives. No need to expand but if it is only seen expanded the form must be correct (sign/coefficient errors only)	**M1**
	$\left\{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 = \right\} 2(1 + \sinh^2 t)$ or $= 2 + 2\sinh^2 t = 2\cosh^2 t$ * Uses either intermediate step shown to complete the proof with no errors. A0 if e.g., sin accidentally used for sinh, x used for t, but condone $\sinh t^2$ used instead of $\sinh^2 t$ or $(\sinh t)^2$ & condone the occasional missing variable. "Meet in the middle" approaches need a conclusion.		**A1***
			(3)
(b)	$\left\{S = 2\pi\int y\, ds =\right\} 2\pi\int(\cosh t - t)\sqrt{2\cosh^2 t}\,\{dt\}$ Uses $S = 2\pi\int y\, ds$ with the given y (not x) & the result from (a). Allow if x used anywhere for t, condone missing brackets and limits and dt not required		**M1**
	$S = 2\pi\sqrt{2}\int_0^{\ln 3}(\cosh^2 t - t\cosh t)\,dt$ * Reaches given answer with no errors (condone missing brackets) with "$S =$" seen at some point in this part. Limits can appear at end. Allow e.g., $S = 2\sqrt{2}\,\pi\int_0^{\ln 3} -t\cosh t + \cosh^2 t\, dt$ *		**A1***
			(2)
(c)	"Solutions relying entirely on calculator technology are not acceptable."		
	$\displaystyle\int \cosh^2 t\, dt = \int\left(\frac{1}{2} + \frac{1}{2}\cosh 2t\right)dt$	Uses/states $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$ Does not need to be seen as integrand	**M1**
	$\displaystyle\int t\cosh t\, dt = t\sinh t - \int \sinh t\, dt$ M1: Applies integration by parts the right way round on $kt\cosh t$ and obtains the correct form (could have +). If only seen fully integrated must be $kt\cosh t \to k(t\sinh t - \cosh t)$ A1: Correct expression. Could be implied		**M1** **A1**
	$\displaystyle\Rightarrow 2\sqrt{2}\pi\int(\cosh^2 t - t\cosh t)\,dt = 2\sqrt{2}\pi\left[\frac{1}{2}t + \frac{1}{4}\sinh 2t - t\sinh t + \cosh t\right]$ oe A1: Correct form (sign/coeff. slips only inc. e.g. missing/wrong $2\sqrt{2}\pi$) A1: All correct. May use e.g., $\sinh 2t \to 2\sinh t\cosh t$ and may be unsimplified (**Note that both M marks required for these marks**). Must be combined but this could be implied later.		**A1** **A1**
	$\displaystyle = 2\sqrt{2}\pi\left\{\left(\frac{1}{2}\ln 3 + \frac{10}{9} - \frac{4}{3}\ln 3 + \frac{5}{3}\right) - (1)\right\}$	Uses limits 0 and ln3 with an expression of the correct form. Hyperbolics credibly evaluated (and no "e"s). Allow any recognisable attempt at substitution here but look for subtraction of $f(0) \neq 0$	**ddM1**
	$\left\{S = 2\sqrt{2}\,\pi\left(\frac{16}{9} - \frac{5}{6}\ln 3\right)\right\}$ $\Rightarrow \{S =\}\dfrac{\pi\sqrt{2}}{9}(32 - 15\ln 3)$	This answer but allow e.g., $\sqrt{2}\pi\left(\dfrac{32 - 15\ln 3}{9}\right)$ Not just values for the constants unless form seen	**A1**
	Condone confusion with "t" and "x"		**(7)**
	See overleaf for examples using exponential definitions and some general guidance for other approaches to the integration in (c).		**Total 12**

Q6(a): Using exponential definitions:

$$x=\cosh t+t=\frac{e^t+e^{-t}}{2}+t,\quad y=\cosh t-t=\frac{e^t+e^{-t}}{2}-t$$

$$\frac{dx}{dt}=\frac{e^t-e^{-t}}{2}+1,\quad \frac{dy}{dt}=\frac{e^t-e^{-t}}{2}-1 \text{ (B1)}$$

$$\left(\frac{e^t-e^{-t}}{2}+1\right)^2+\left(\frac{e^t-e^{-t}}{2}-1\right)^2 \text{ (M1)}$$

$$=\frac{1}{2}\left(e^t-e^{-t}\right)^2+2 \text{ or e.g., } =\frac{1}{2}\left(e^{2t}+2+e^{-2t}\right)$$

$$=2\left(\frac{e^t+e^{-t}}{2}\right)^2=2\cosh^2 t\,*$$

(A1*: Must see $k\left(e^t+e^{-t}\right)^2$ **and** at least one additional line after substitution)

Q6(c) example using exponential definitions:

$$\int\cosh^2 t\,dt=\int\left(\frac{e^t+e^{-t}}{2}\right)^2 dt=\frac{1}{4}\int\left(e^{2t}+2+e^{-2t}\right)dt \text{ (M1: integrable form - sign errors only)}$$

$$\int t\cosh t\,dt=\frac{1}{2}\int\left(te^t+te^{-t}\right)dt=\frac{1}{2}\left(te^t-\int e^t\,dt-te^{-t}+\int e^{-t}\,dt\right)$$

(M1: applies parts, twice if necessary, correct form A1: All correct for the integral of $t\cosh t$)

$$\Rightarrow\left(2\sqrt{2}\pi\right)\left(\left(\frac{1}{8}e^{2t}+\frac{1}{2}t-\frac{1}{8}e^{-2t}\right)-\left(\frac{1}{2}te^t-\frac{1}{2}e^t-\frac{1}{2}te^{-t}-\frac{1}{2}e^{-t}\right)\right)$$

A1: Correct form (sign/coefficient slips only) A1: All correct. May be unsimplified

$$\Rightarrow 2\sqrt{2}\pi\left[\frac{1}{8}e^{2t}+\frac{1}{2}t-\frac{1}{8}e^{-2t}-\frac{1}{2}te^t+\frac{1}{2}e^t+\frac{1}{2}te^{-t}+\frac{1}{2}e^{-t}\right]_0^{\ln 3}$$

$$=2\sqrt{2}\pi\left(\left(\frac{9}{8}+\frac{1}{2}\ln 3-\frac{1}{72}-\frac{3}{2}\ln 3+\frac{3}{2}+\frac{1}{6}\ln 3+\frac{1}{6}\right)-\left(\frac{1}{8}+0-\frac{1}{8}-0+\frac{1}{2}+0+\frac{1}{2}\right)\right)$$

ddM1: Uses limits 0 and ln3. No exponentials.

Allow any credible attempt at substitution here but look for subtraction of $f(0)\neq 0$

$$\left\{S=2\sqrt{2}\,\pi\left(\frac{16}{9}-\frac{5}{6}\ln 3\right)=\right\}\ \frac{\pi\sqrt{2}}{9}\left(32-15\ln 3\right) \text{ A1 as before}$$

Q6(c): For other potentially creditworthy attempts at the integration that work on the two terms separately, the following guidance applies:

M1: Reaches an integrable form for $\cosh^2 t$ (sign/coefficient errors only)

M1: Correct form for $\int t\cosh t\,dt$ A1: Fully correct integration of $\int t\cosh t\,dt$

A1: Correct form for S A1: Fully correct S

ddM1A1: As main scheme

Using parts on $\cosh t(\cosh t-t)$:

$$=\sinh t(\cosh t-t)-\int\left(\sinh^2 t-\sinh t\right)dt \text{ (2nd M1: correct form, A1: fully correct)}$$

$$=\sinh t(\cosh t-t)-\int\left(-\tfrac{1}{2}+\cosh 2t-\sinh t\right)dt \text{ (1st M1: } \sinh^2 t=\pm\tfrac{1}{2}\pm\tfrac{1}{2}\cosh 2t\text{)}$$

$$\Rightarrow 2\sqrt{2}\,\pi\left[\sinh t\cosh t-t\sinh t+\tfrac{1}{2}t-\tfrac{1}{4}\sinh 2t+\cosh t\right] \text{ (A1: correct form, A1: fully correct)}$$

Then ddM1A1 as main scheme

Question Number	Scheme	Notes	Marks
7.	$A(-1, 5, 1),\ B(1, 0, 3),\ C(2, -1, 2),\ D(3, 6, -1)$		
(a)	e.g., $\mathbf{AB}=\begin{pmatrix}1\\0\\3\end{pmatrix}-\begin{pmatrix}-1\\5\\1\end{pmatrix}=\begin{pmatrix}2\\-5\\2\end{pmatrix},\ \mathbf{AC}=\begin{pmatrix}3\\-6\\1\end{pmatrix},\ \mathbf{AD}=\begin{pmatrix}4\\1\\-2\end{pmatrix}\ \mathbf{BC}=\begin{pmatrix}1\\-1\\-1\end{pmatrix},\ \mathbf{BD}=\begin{pmatrix}2\\6\\-4\end{pmatrix},\ \mathbf{CD}=\begin{pmatrix}-1\\-7\\3\end{pmatrix}$ Attempts any 3 edges (must attempt to subtract) which have a common vertex. Using position vectors **OA** etc is M0		**M1**
	e.g., $\begin{vmatrix}2 & -5 & 2\\3 & -6 & 1\\4 & 1 & -2\end{vmatrix}$ or $\begin{pmatrix}2\\-5\\2\end{pmatrix}\bullet\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\3 & -6 & 1\\4 & 1 & -2\end{vmatrix}$ or $\begin{pmatrix}2\\-5\\2\end{pmatrix}\bullet\left(\begin{pmatrix}3\\-6\\1\end{pmatrix}\times\begin{pmatrix}4\\1\\-2\end{pmatrix}\right)$ Completes an attempt to form an appropriate scalar triple product with their edges with a common vertex. Condone incorrect $\frac{1}{6}$ e.g., $\frac{1}{3}$ or $\frac{1}{2}$ but M0 if triple product is part of a calculation e.g., all square rooted		**dM1**
	$=\dfrac{1}{6}\lvert 2\times11-5\times10+2\times27\rvert=\ldots\ \left\{\dfrac{1}{6}\times26=\right\}\ \dfrac{13}{3}$ ddM1: Finds value appropriately (may be negative if omission of modulus). Must include the $\frac{1}{6}$. M0 if a clear systematic error in processing and must have 2 consistent products (or values) in a sum of 3 if method unclear. If just a value is given it must be consistent, but accept a determinant form followed by a value unless clear evidence of an invalid method (e.g., determinant interpreted as a "modulus") A1: Correct (positive) volume (allow exact equivalents)		**ddM1** **A1**
	Protracted methods may score the 1st mark as above but need to be complete attempts that find a value for the volume to receive credit.		**(4)**
(b)	$\mathbf{AB}\times\mathbf{AC}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\2 & -5 & 2\\3 & -6 & 1\end{vmatrix}=\begin{pmatrix}7\\4\\3\end{pmatrix}$ Result must be seen/used in (b) if done in (a). M1: Completes cross product using attempts at two sides of ABC (Must have subtracted for the vectors & obtain 2 correct components for their product if method unclear). Allow determinant $\to$ vector unless clear evidence of invalid method. No position vectors A1: Correct normal vector (any multiple).		**M1** **A1**
	OR $-a+5b+c=d,\ a+3c=d,\ 2a-b+2c=d\Rightarrow a=\ldots\{7\},\ b=\ldots\{4\},\ c=\ldots\{3\}$ M1: Uses A, B & C to form 3 equations & solves to find all values A1: Correct. **OR** e.g., $(x,\ y,\ z)=(1,\ 5,\ 1)+s(2,\ -5,\ 2)+t(3,-6,\ 1)\to x=-1+2s+3t,\ y=5-5s-6t, z=1+2s+t$ (M1A1) and then eliminates parameters to an unsimplified plane equation of correct form dM1A1		
	$\begin{pmatrix}7\\4\\3\end{pmatrix}\bullet\begin{pmatrix}-1\\5\\1\end{pmatrix}=\ldots\{16\}$	Finds value for scalar product with appropriate vectors (normal & a point). Allow slips if valid method indicated but if value only given it must be consistent. Could sub. point into $7x+4y+3z+d=0\Rightarrow d=\ldots$	**dM1**
	$7x+4y+3z=16$	Correct 4 term equation (any multiple)	**A1**
			(4)
(c)	$\{\mathbf{OT}=\}\begin{pmatrix}3\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}7\\4\\3\end{pmatrix}$ oe	Attempts consistent parametric form of **OT** May see e.g., $\dfrac{x-3}{7}=\dfrac{y-6}{4}=\dfrac{z+1}{3}$ (I)	**M1**
	$7(3+7\lambda)+4(6+4\lambda)+3(-1+3\lambda)=16\Rightarrow\lambda=\ldots$ Substitutes parametric form of **OT** into their plane equation (at least 3 terms, allow slips e.g., $16\to0$) and solves for λ. Could solve (I) with their plane eqn. and score dM1ddM1 together		**dM1**
	$\lambda=-\dfrac{13}{37}\Rightarrow T$ is $\left(\dfrac{20}{37},\dfrac{170}{37},-\dfrac{76}{37}\right)$ Allow $x=\ldots, y=\ldots, z=\ldots$	ddM1: Uses their value of λ in their **OT** equation to obtain coordinates/vector. Give BOD if no working unless a clearly trivial/invented answer is seen A1: Correct exact coordinates (or vector)	**ddM1** **A1**
	Alternative methods need to reach a point/vector via a valid strategy for any marks.		**(4)**
			Total 12