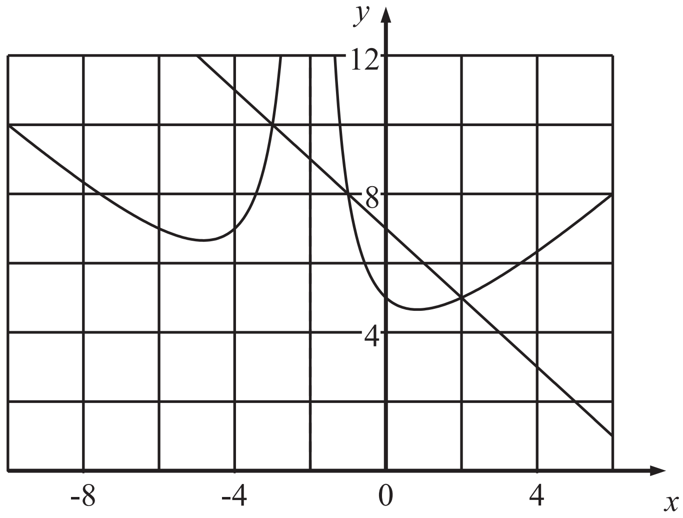
目录

IAL FP2 Mark Scheme

Question Number	Scheme	Marks
1(a)	$\frac{d^3y}{dx^3}+3\frac{dy}{dx}+3x\frac{d^2y}{dx^2}=-2\sin x$	M1M1
	$\frac{d^3y}{dx^3}=-2\sin x-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}$	A1 (3)
(b)	$\frac{d^3y}{dx^3}=-3\times 5=-15$	B1 (1)
(c)	$\frac{d^2y}{dx^2}=-3\times 0\times 5+2=2$	B1
	$y=2+5x+x^2-\frac{5}{2}x^3$	M1A1 (3)
		[7]
(a)	Accept the dashed notation throughout this question.	
M1	Differentiate $3x\frac{dy}{dx}$ with respect to x. The product rule must be used for $x\frac{dy}{dx}$ with at least one term correct	
M1	Differentiate $\frac{d^2y}{dx^2}$ and $2\cos x$. $\frac{d^2y}{dx^2}\to\frac{d^3y}{dx^3}$ $2\cos x\to\pm 2\sin x$	
A1	$\frac{d^3y}{dx^3}=-3\left(x\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)-2\sin x$. Give A0 if not rearranged to have $\frac{d^3y}{dx^3}=\ldots$	
(b)		
B1	$\frac{d^3y}{dx^3}=-15$ provided 3 terms in result in (a)	
(c)		
B1	$\frac{d^2y}{dx^2}=2$ can be implied by a correct x^2 term in the expansion	
M1	Use of a correct Taylor expansion with their values for $\frac{d^3y}{dx^3}$ and $\frac{d^2y}{dx^2}$ 2! or 2, 3! or 6·	
A1	$y=2+5x+x^2-\frac{5}{2}x^3$ Must include $y=\ldots$ or $f(x)=\ldots$ provided $f(x)$ has been defined to be y somewhere in the work.	

Question Number	Scheme	Marks
2 (a)	$\frac{3r+1}{r(r-1)(r+1)}=\frac{A}{r}+\frac{B}{r-1}+\frac{C}{r+1}$ $\frac{3r+1}{r(r-1)(r+1)}=-\frac{1}{r}+\frac{2}{r-1}-\frac{1}{r+1}$	M1A1 (2)
(b)	$\frac{2}{1}-\frac{1}{2}-\frac{1}{3}$ $\quad \frac{2}{n-3}-\frac{1}{n-2}-\frac{1}{n-1}$ $\frac{2}{2}-\frac{1}{3}-\frac{1}{4}$ $\quad \frac{2}{n-2}-\frac{1}{n-1}-\frac{1}{n}$ $\frac{2}{3}-\frac{1}{4}-\frac{1}{5}$ $\quad \frac{2}{n-1}-\frac{1}{n}-\frac{1}{n+1}$ $\frac{2}{4}-\frac{1}{5}-\frac{1}{6}$	M1
	$=2-\frac{1}{2}+\frac{2}{2}-\frac{1}{n}-\frac{1}{n}-\frac{1}{n+1}$	dM1A1
	$\frac{5}{2}-\frac{2}{n}-\frac{1}{n+1}=\frac{5n(n+1)-4(n+1)-2n}{2n(n+1)},=\frac{5n^2-n-4}{2n(n+1)}$	M1, A1 cso (5)
(c)	$\sum_2^{20}-\sum_2^{14}$ $=\frac{5\times20^2-20-4}{2\times20\times21}-\frac{5\times14^2-14-4}{2\times14\times15}$	M1
	$=\frac{13}{210}$	A1 (2)
		[9]

(a)	
M1	Correct method for obtaining the PFs
A1	Correct PFs
(b)	
M1	Show sufficient terms at both ends (eg 3 at start and 2 at end) to demonstrate the cancelling. (This can be implied by correct work at the next line) Must be using PFs of the correct form and start at $r=2$ unless extra terms are ignored at next stage. Can be split into $\sum\left(\frac{1}{r-1}-\frac{1}{r}\right)+\sum\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$
dM1	Extract the non-cancelled terms (min 4 correct terms but 5/2 counts as 3 correct) Depends on first M of (b)
A1	Correct terms extracted
M1	Write terms using the common denominator, numerator need not be simplified. Must start with a min of 3 terms inc terms with denominators n and $(n+1)$
A1cso	Correct answer from correct working
(c)	
M1	Form and use the difference of the 2 summations shown using their result from (b) or an earlier form seen in (b)
A1	Correct **exact** answer, as shown or equivalent

Question Number	Scheme	Marks
3		This sketch on its own scores no marks, but it may be seen in the work
	$\dfrac{x^2+3x+10}{x+2}=7-x$	
	$x^2+3x+10=14+5x-x^2$	M1
	$x^2-x-2=0 \quad (x-2)(x+1)=0$	dM1
	CVs $\quad 2,-1$	A1A1
	$\dfrac{-(x^2+3x+10)}{x+2}=7-x$	
	$-x^2-3x-10=14+5x-x^2$	M1
	$8x=-24 \quad$ CV -3	A1
	$x<-3 \quad -1<x<2$	dddM1A1A1 **[9]**
NB	**No algebra implies no marks**	
M1	Form a quadratic equation or inequality, no simplification needed	
dM1	Solve the 3TQ any valid method Depends on the first M mark.	
A1	Either CV	
A1	Both CVs	
M1	Change the sign of LHS or RHS and obtain an equation (quadratic or linear, no simplification needed)	
A1	Correct CV from solving the linear equation	
dddM1	$x<$ their smallest CV and x between their other 2 CVs All M marks above needed	
A1	Either inequality correct	
A1	Both inequalities correct	
	"and" between the inequalities is acceptable. If $\cap$ used, deduct an A mark.	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4
(a)</td><td>$\left|18\sqrt{3}-18\text{i}\right| = 18\sqrt{(3+1)} = 36$
$\tan\theta = \dfrac{-18}{18\sqrt{3}} \quad \theta = -\dfrac{\pi}{6}, \quad 18\sqrt{3}-18\text{i} = 36\left(\cos\left(-\dfrac{\pi}{6}\right)+\text{i}\sin\left(-\dfrac{\pi}{6}\right)\right)$</td><td>B1
M1,A1cao
(3)</td></tr>
<tr><td>(b)</td><td>$z^4 = 36\left(\cos-\dfrac{\pi}{6}+\text{i}\sin-\dfrac{\pi}{6}\right) = 36\left(\cos\left(2k\pi-\dfrac{\pi}{6}\right)+\text{i}\sin\left(2k\pi-\dfrac{\pi}{6}\right)\right)$
$z = \sqrt{6}\left(\cos\left(\dfrac{12k\pi-\pi}{24}\right)+\text{i}\sin\left(\dfrac{12k\pi-\pi}{24}\right)\right)$
$k=0 \quad z_0 = \sqrt{6}\left(\cos\left(\dfrac{-\pi}{24}\right)+\text{i}\sin\left(\dfrac{-\pi}{24}\right)\right) = \sqrt{6}\,\text{e}^{\text{i}\left(-\frac{\pi}{24}\right)}$
$k=1 \quad z_1 = \sqrt{6}\left(\cos\left(\dfrac{11\pi}{24}\right)+\text{i}\sin\left(\dfrac{11\pi}{24}\right)\right) = \sqrt{6}\text{e}^{\text{i}\frac{11\pi}{24}}$
$k=2 \quad z_2 = \sqrt{6}\left(\cos\left(\dfrac{23\pi}{24}\right)+\text{i}\sin\left(\dfrac{23\pi}{24}\right)\right) = \sqrt{6}\text{e}^{\text{i}\frac{23\pi}{24}}$
$k=-1 \quad z_3 = \sqrt{6}\left(\cos\left(-\dfrac{13\pi}{24}\right)+\text{i}\sin\left(-\dfrac{13\pi}{24}\right)\right) = \sqrt{6}\text{e}^{\text{i}\left(-\frac{13\pi}{24}\right)}$</td><td>M1
M1
B1
A1ft

A1ft (5)
[8]</td></tr>
<tr><td>(a)
B1
M1

A1cao
(b)
M1
M1
B1
A1ft
A1ft

NB</td><td>Correct modulus
Attempt argument using $\tan\theta = \dfrac{\pm 18}{18\sqrt{3}}$ or other valid method. Can be implied by $\theta = \pm\dfrac{\pi}{6}$
Correct answer in the required form.

Valid method for generating at least 2 roots, rotation through $\dfrac{\pi}{2}$ accepted
Apply de Moivre or use the rotation method
Any one correct root
Second root in required form
All 4 roots in the required form
Follow through their $\sqrt[4]{36}$ but 36 not acceptable.
Argument in degrees – M1M1B1A0A0 (ie treat as mis-read)
Incorrect argument: B0A1ftA1ft available
Answers in $r(\cos\theta+\text{i}\sin\theta)$ form – deduct final A marks</td><td></td></tr>
</table>

Question Number	Scheme	Marks
5	$w=\frac{z-3\mathrm{i}}{z+2\mathrm{i}}$	
	$w(z+2\mathrm{i})=z-3\mathrm{i} \quad z=\frac{\mathrm{i}(2w+3)}{1-w}$	M1
	$\lvert z\rvert=1 \quad \left\lvert\frac{\mathrm{i}(2w+3)}{1-w}\right\rvert=1$	dM1
	$\lvert\mathrm{i}(2w+3)\rvert=\lvert 1-w\rvert$	
	$w=u+\mathrm{i}v \quad (2u+3)^2+4v^2=(1-u)^2+v^2$	ddM1
	$4u^2+12u+9+4v^2=1-2u+u^2+v^2$	
	$3u^2+3v^2+14u+8=0$	dddM1
	$u^2+v^2+\frac{14}{3}u+\frac{8}{3}=0$	A1
	$\left(u+\frac{7}{3}\right)^2+v^2=-\frac{8}{3}+\frac{49}{9}=\frac{25}{9}$	
(i)	Centre $\left(-\frac{7}{3},0\right)$	A1
(ii)	Radius $\frac{5}{3}$	A1 (7)
		[7]
(a) M1	re-arrange to $z=$	
dM1	dep (on first M1) using $\lvert z\rvert=1$ with their previous result	
ddM1	dep (on both previous M marks) use $w=u+\mathrm{i}v$ (or any other pair of letters inc (x, y)) and find the moduli (or square of it)	
dddM1	dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms)	
A1	for a correct equation in u and v with coeffs of u^2 and v^2 both 1	
A1	Correct centre, must be in coordinate brackets. Completion of square need not be shown.	
A1	Correct radius **Centre and radius must come from a correct circle equation for the A marks**	

Question Number	Scheme	Marks
6.	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{(x\cot x+2)}{x}y=\frac{4\sin x}{x^2}$	B1
	IF $=\mathrm{e}^{\int\frac{(x\cot x+2)}{x}\mathrm{d}x}$	M1
	$=\mathrm{e}^{(\ln\sin x+2\ln x)}$	A1
	$=x^2\sin x$	A1
	$\frac{\mathrm{d}}{\mathrm{d}x}(\text{their IF}\times y)=\text{their IF}\times\text{"}\frac{4\sin x}{x^2}\text{"}$	M1
	$yx^2\sin x=\int 4\sin^2 x\,\mathrm{d}x=4\int\frac{1-\cos 2x}{2}\mathrm{d}x=4\left(\frac{x}{2}-\frac{1}{4}\sin 2x\right)\quad(+C)$	dM1A1
	$y=\frac{2x-\sin 2x+C}{x^2\sin x}$ oe	A1cao **[8]**

B1	Divide through by x^2
M1	Attempt an IF of the form $\mathrm{e}^{\int\frac{k(x\cot x+2)}{x}\mathrm{d}x}$
A1	$(\ln\sin x+2\ln x)$
A1	Correct IF
M1	Multiply through by their IF and write LHS in form shown – can be implied by next line. Allow if IF is seen instead of their function provided an IF has been attempted. Allow use of their RHS
dM1	Attempt to integrate $\sin^2 x$, including using $\sin^2 x=\frac{1}{2}(1\pm\cos 2x)$ $\cos 2x\to k\sin 2x$ depends on previous M mark
A1	Correct integration, constant not needed
A1	Include the constant and treat it correctly. Must have $y=\ldots$

Question Number	Scheme	Marks
7 **(a)**	$r\sin\theta = 2a\sin\theta + 2a\sin\theta\cos\theta$ OR $r\sin\theta = 2a\sin\theta + a\sin 2\theta$	B1
	$\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 2a\cos\theta + 2a\cos^2\theta - 2a\sin^2\theta$ OR $\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 2a\cos\theta + 2a\cos 2\theta$	M1 A1
	$2\cos^2\theta + \cos\theta - 1 = 0$ terms in any order	
	$(2\cos\theta - 1)(\cos\theta + 1) = 0$	
	$\cos\theta = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ $(\theta = \pi$ need not be seen$)$	dM1A1
	$r = 2a \times \frac{3}{2} = 3a$	A1 (6)
(b)	Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4a^2(1+\cos\theta)^2\, \mathrm{d}\theta$	
	$= 2a^2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 + 2\cos\theta + \cos^2\theta)\mathrm{d}\theta$	M1
	$= 2a^2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right)\mathrm{d}\theta$	M1
	$= 2a^2\left[\theta + 2\sin\theta + \frac{1}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	dM1A1
	$= 2a^2\left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} - \left(\frac{\pi}{6} + 1 + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{12}\right)\right]$	M1 NB: A1 on e-PEN
	$= 2a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right)$	
	Area of $\Delta OAB = \frac{1}{2} \times 3a \times (2+\sqrt{3})a \times \sin\frac{\pi}{6}$ $\left(= \frac{3}{4}a^2(2+\sqrt{3})\right)$	
	Shaded area $= 2a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right) - \frac{3}{4}a^2(2+\sqrt{3}) = \frac{a^2}{4}(2\pi - 14 + 5\sqrt{3})$	M1A1cao (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	Multiply r by $\sin\theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen	
M1	Differentiate $r\sin\theta$ or $r\cos\theta$ (using product rule or using double angle formula first)	
A1	Correct derivative for $r\sin\theta$	
dM1	Use $\sin^2\theta+\cos^2\theta=1$ to form a 3TQ in $\cos\theta$ and attempt its solution by a valid method	
A1	Correct value for θ	
A1	Correct r	
(b)		
M1	Use area $=\frac{1}{2}\int r^2\mathrm{d}\theta$ with $r=2a+2a\cos\theta$, no limits needed,	
M1	Use a double angle formula to obtain a function ready for integrating (Alt method uses integration by parts – may be seen)	
dM1	Attempt the integration $\cos2\theta\to\frac{1}{k}\sin2\theta$ $k=\pm2$ or ±1	
A1	Correct integration,	
M1	Substitute the limits (need not be simplified). Limits $\frac{\pi}{6}$ and their θ from (a) provided this is $>\frac{\pi}{6}$ NB: A1 on e-PEN	
M1	Obtain the area of ΔOAB and subtract from their previous area	
A1	Correct answer	

Question Number	Scheme	Marks
8 (a)	$x = e^u \quad \frac{dx}{du} = e^u$ or $\frac{du}{dx} = e^{-u}$ or $\frac{dx}{du} = x$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -e^{-u}\frac{du}{dx}\frac{dy}{du} + e^{-u}\frac{d^2y}{du^2}\frac{du}{dx} = e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right)$	M1A1
	$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 8y = 4\ln x$	
	$e^{2u} \times e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right) + 3e^u \times e^{-u}\frac{dy}{du} - 8y = 4\ln\left(e^u\right)$	dM1
	$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \quad *$	A1*cso (6)
B1	$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{dy}{dx}$ using chain rule here or seen later	
M1	Obtaining $\frac{d^2y}{dx^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{d^2y}{dx^2}$ any equivalent form	
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark	
A1*cso	Obtaining the **given** result from completely correct work	
	ALTERNATIVE 1	
	$x = e^u \quad \frac{dx}{du} = e^u = x$	B1
	$\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du} = x\frac{dy}{dx}$	M1
	$\frac{d^2y}{du^2} = 1\frac{dx}{du} \times \frac{dy}{dx} + x\frac{d^2y}{dx^2} \times \frac{dx}{du} = x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2}$	M1A1
	$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}$	
	$\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right) + 3x \times \frac{1}{x}\frac{dy}{du} - 8y = 4\ln\left(e^u\right)$	
	$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \quad *$	dM1A1*cso (6)

Question Number	Scheme	Marks
B1	$\frac{dx}{du}=e^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{dy}{du}$ using chain rule here or seen later	
M1	Obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark	
A1*cso	Obtaining the **given** result from completely correct work	
	ALTERNATIVE 2:	
	$u=\ln x \quad \frac{du}{dx}=\frac{1}{x}$	B1
	$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{1}{x}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2}=-\frac{1}{x^2}\frac{dy}{du}+\frac{1}{x}\frac{d^2y}{du^2}\times\frac{du}{dx}=-\frac{1}{x^2}\frac{dy}{du}+\frac{1}{x^2}\frac{d^2y}{du^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{dy}{du}+\frac{1}{x^2}\frac{d^2y}{du^2}\right)+3x\times\frac{1}{x}\frac{dy}{du}-8y=4u$	
	$\frac{d^2y}{du^2}+2\frac{dy}{du}-8y=4u$ *	M1A1*cso
	Notes as for main scheme	

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1 as shown in schemes above
M1 obtaining a first derivative with chain rule
M1 obtaining a second derivative with product rule
A1 correct second derivative with 2 or 3 variables present
dM1 Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2 + 2m - 8 = 0$	
	$(m+4)(m-2)=0, \quad m=-4,2$	M1A1
	CF $= Ae^{-4u} + Be^{2u}$	A1
	PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, $c =$ 0)	
	$\frac{dy}{du} = a \quad \frac{d^2y}{du^2} = 0$	M1
	$0 + 2a - 8(au+b) = 4u$	
	$a = -\frac{1}{2} \quad b = -\frac{1}{8}$	dM1A1
	$\therefore y = Ae^{-4u} + Be^{2u} - \frac{1}{2}u - \frac{1}{8}$	B1ft (7)
(c)	$y = Ax^{-4} + Bx^2 - \frac{1}{2}\ln x - \frac{1}{8}$	B1 (1)
		[14]

(b) M1	Writing down the correct aux equation and solving to $m = \ldots$ (usual rules)
A1	Correct solution $(m=-4,2)$
A1	Correct CF – can use any (single) variable
M1	Using an appropriate PI and finding $\frac{dy}{du}$ **and** $\frac{d^2y}{du^2}$ Use of $y = \lambda u$ scores M0
dM1	Substitute in the equation to obtain values for the unknowns. Depends on the second M1
A1	Correct unknowns two or three (with $c = 0$)
B1ft	A complete solution, follow through their CF and a non-zero PI. Must have $y =$ a function of u Allow recovery of incorrect variables.
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x No ft here x^{-4} or $e^{-4\ln x}$ and x^2 or $e^{2\ln x}$ allowed. Must start $y = \ldots$

Question Number	Scheme	Marks
1.	$\mathrm{i}\left(1+\sqrt{3}\right)=\dfrac{\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}}{\mathrm{i}^2\left(1+\sqrt{3}\right)+3}$	M1
	$-\mathrm{i}\left(1+\sqrt{3}\right)^2+3\mathrm{i}\left(1+\sqrt{3}\right)=\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}$	
	$-1-2\sqrt{3}-3+3+3\sqrt{3}=1+\sqrt{3}+p$	dM1
	$p=-2$	A1
		[3]
M1	Substitute $\mathrm{i}\left(1+\sqrt{3}\right)$ for w and z	
dM1	Solve to $p = ...$	
A1	Correct value for p	
	Some solve for p first:	
M1	Obtain an expression for p in terms of w and/or z	
dM1	Substitute $\mathrm{i}\left(1+\sqrt{3}\right)$ for w and z	
A1	Correct value for p	

Question Number	Scheme	Marks
2 (a)	$\frac{r+2}{r(r+1)}-\frac{r+3}{(r+1)(r+2)}=\frac{(r+2)^2-r(r+3)}{r(r+1)(r+2)}$	M1
	$=\frac{r^2+4r+4-r^2-3r}{r(r+1)(r+2)}=\frac{r+4}{r(r+1)(r+2)}$ *	A1* (2)
(b)	$r=1$ $\quad \frac{3}{1\times2}-\frac{4}{2\times3}$ $\qquad r=n-1$ $\quad \frac{n+1}{(n-1)n}-\frac{n+2}{n(n+1)}$	
	$r=2$ $\quad \frac{4}{2\times3}-\frac{5}{3\times4}$ $\qquad r=n$ $\quad \frac{n+2}{n(n+1)}-\frac{n+3}{(n+1)(n+2)}$	M1
	$r=3$ $\quad \frac{5}{3\times4}-\frac{6}{4\times5}$	
	$\sum_{r=1}^{n}\frac{r+4}{r(r+1)(r+2)}=\frac{3}{2}-\frac{n+3}{(n+1)(n+2)}$	A1
	$\sum_{r=1}^{n}\frac{r+4}{r(r+1)(r+2)}=\frac{3(n+1)(n+2)-2n-6}{2(n+1)(n+2)}=\frac{n(3n+7)}{2(n+1)(n+2)}$	dM1 A1cao (4)
		[6]
(a) **M1**	Attempt a single fraction with the correct denominator (or 2 separate fractions with the correct common denominator)	
A1*	Correct result obtained with no errors in the working. Must include LHS as shown in question or LHS = ...	
(b) **M1**	Show sufficient terms to demonstrate the cancelling, min 3 at start and 1 at end or 2 at start and 2 at end. Award by implication if the correct 2 remaining terms are seen	
A1	Extract the correct 2 remaining terms	
dM1	Attempt common denominator of the form $k(n+1)(n+2)$	
A1cao	Correct result obtained. No need to show a, b and c explicitly.	

Question Number	Scheme	Marks
3	$x^2+x-2<\frac{1}{2}x+\frac{5}{2}$	
	$2x^2+x-9<0$	M1
	CVs $x=\frac{-1\pm\sqrt{73}}{4}$	A1
	$-x^2-x+2<\frac{1}{2}x+\frac{5}{2}$	M1
	$2x^2+3x+1>0 \quad (2x+1)(x+1)>0$	M1
	CVs $x=-\frac{1}{2},\ -1$	A1
	$\frac{-1-\sqrt{73}}{4}<x<-1,\quad -\frac{1}{2}<x<\frac{-1+\sqrt{73}}{4}$	M1A1
		[7]
	No algebra implies no marks	
NB	The first 5 marks can all be awarded if equations rather than inequalities are shown	
M1	Obtain and solve a 3TQ (any valid method including calculator)	
A1	2 correct CVs Allow decimal equivalents (1.886..., -2.386...), min 3 sf, rounded or truncated	
M1	Multiply either side by -1	
M1	Obtain and solve a 3TQ (any valid method including calculator)	
A1	2 correct CVs	
M1	Form 2 double inequalities with their CVs. No overlap between these inequalities. Correct inequality signs required here or for final mark	
A1	Correct inequalities obtained. Values must be exact, but note that 0.5 is exact. Allow “and” but not "$\cap$". May be written in set language with "$\cup$" and round brackets	

Question Number	Scheme	Marks
4 (a)	$y^2 = z^{-1} \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{z^2}\frac{\mathrm{d}z}{\mathrm{d}x}$ oe eg $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{\mathrm{d}z}{\mathrm{d}x}$	B1
	$2y\frac{\mathrm{d}y}{\mathrm{d}x} + 4y^2 = 6xy^4$	
	$-\frac{1}{z^2}\frac{\mathrm{d}z}{\mathrm{d}x} + \frac{4}{z} = \frac{6x}{z^2}$	M1
	$\frac{\mathrm{d}z}{\mathrm{d}x} - 4z = -6x$ *	A1 * (3)
(b)	$\text{IF} = \mathrm{e}^{\int -4\mathrm{d}x} = \mathrm{e}^{-4x}$	B1
	$\mathrm{e}^{-4x}\left(\frac{\mathrm{d}z}{\mathrm{d}x} - 4z\right) = \mathrm{e}^{-4x} \times -6x$	
	$z\mathrm{e}^{-4x} = -6\int x\mathrm{e}^{-4x}\mathrm{d}x$	M1
	$= -6\left[-\frac{1}{4}x\mathrm{e}^{-4x} + \int \frac{1}{4}\mathrm{e}^{-4x}\mathrm{d}x\right]$	M1
	$= -6\left[-\frac{1}{4}x\mathrm{e}^{-4x} - \frac{1}{16}\mathrm{e}^{-4x}\right](+c)$ oe	A1
	$= \frac{3}{2}x\mathrm{e}^{-4x} + \frac{3}{8}\mathrm{e}^{-4x}(+c)$	
	$z = \frac{3}{2}x + \frac{3}{8} + c\mathrm{e}^{4x}$ oe	A1 (5)
ALT	$\frac{\mathrm{d}z}{\mathrm{d}x} - 4z = -6x$	
	$m - 4 = 0 \Rightarrow m = 4 \Rightarrow$ CF is $z = A\mathrm{e}^{4x}$	B1
	PI: $z = \lambda + \mu x$	M1
	$\frac{\mathrm{d}z}{\mathrm{d}x} = \mu \Rightarrow \mu - 4(\lambda + \mu x) = -6x$	
	$4\mu = 6 \quad 4\lambda = \mu, \Rightarrow \mu = \frac{3}{2}, \lambda = \frac{3}{8}$	M1,A1
	$z = \frac{3}{2}x + \frac{3}{8} + A\mathrm{e}^{4x}$	A1
(c)	$y^2 = \frac{1}{\frac{3}{2}x + \frac{3}{8} + c\mathrm{e}^{4x}} = \frac{8}{(12x + 3 + A\mathrm{e}^{4x})}$ oe	B1ft (1)
		[9]

Question Number	Scheme	Marks
(a)		
B1	Correct derivative seen explicitly or used	
M1	Substitutions made. Only award when an equation in x and z only is reached (if working equation I to II) or an equation in x and y is reached (if working II to I)	
A1 *	Correct result obtained with no errors in working	
(b)		
B1	Correct IF seen explicitly or used	
M1	Multiply through by their IF and integrate the LHS. Accept I for e^{-4x} on LHS only	
M1	Apply parts in the correct direction to RHS to obtain $Axe^{-4x} + B\int e^{-4x}dx$ with $A = \pm\frac{3}{2}$ and $B = \pm\frac{3}{2}$	
A1	Correct integration of RHS, constant not needed	
A1	Include the constant and treat it correctly. Answer in form $z = \ldots$	
ALT		
B1	Correct CF May not be seen until GS is formed	
M1	For a PI of the correct form	
M1	Differentiate their PI, substitute in the equation and extract 2 equations for the unknowns	
A1	Solve the two equations to obtain correct values for the unknowns	
A1	Correct GS obtained	
(c)		
B1ft	Any equivalent to that shown. (no need to change letter for constant if rearranged) Must start $y^2 = \ldots$ and must include a constant.	

Question Number	Scheme	Marks
5(a)	$-2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	M1
	$+5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2},=3\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1, B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\left(2-x^2\right)+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(10x\frac{\mathrm{d}y}{\mathrm{d}x}-2x\right)+5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=3\frac{\mathrm{d}y}{\mathrm{d}x}$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)
ALT 1	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{3y-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{\left(2-x^2\right)}$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{\left[3\frac{\mathrm{d}y}{\mathrm{d}x}-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$	M1M1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{\left[3\frac{\mathrm{d}y}{\mathrm{d}x}-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-10x\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\left(2-x^2\right)+2x\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}}{\left(2-x^2\right)^2}$	M1 (NB: B1 on ePEN)
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)

Question Number	Scheme	Marks
ALT 2	$\frac{d^2y}{dx^2}=\frac{3y}{(2-x^2)}-\frac{5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)}$	
	$\frac{d^3y}{dx^3}=\frac{3\frac{dy}{dx}(2-x^2)-3y(-2x)}{(2-x^2)^2}-\frac{\left[5\left(\frac{dy}{dx}\right)^2+5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)-5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	M1M1A1
	$\frac{d^3y}{dx^3}=\frac{3\frac{dy}{dx}(2-x^2)-\left((2-x^2)\frac{d^2y}{dx^2}+5x\frac{dy}{dx}\right)(-2x)}{(2-x^2)^2}-\frac{\left[5\left(\frac{dy}{dx}\right)^2+5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)-5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	M1(B1 on ePEN)
	$\frac{d^3y}{dx^3}=\frac{1}{(2-x^2)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$ *	A1*
(b)	$x=0 \Rightarrow 2\frac{d^2y}{dx^2}=9 \quad \frac{d^2y}{dx^2}=\frac{9}{2}$	B1
	$\frac{d^3y}{dx^3}=\frac{1}{2}\left(-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)=\frac{1}{2}\left(-5\times\frac{1}{16}+\frac{3}{4}\right)=\frac{7}{32}$	M1
	$y=3+\frac{1}{4}x+\frac{9}{2}\frac{x^2}{2!}+\frac{7}{32}\frac{x^3}{3!}$	M1
	$y=3+\frac{1}{4}x+\frac{9}{4}x^2+\frac{7}{192}x^3$	A1 (4)
		[9]

Question Number	Scheme	Marks
(a)		
M1	Differentiate $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule	
M1	Differentiate $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using product and chain rule	
A1	Correct derivative of $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	
B1	Correct derivative of $3y$	
A1*	Correct result obtained from fully correct working	
ALT 1	*Rearrange and use quotient rule*	
M1	Use the quotient rule. Denominator must be $\left(2-x^2\right)^2$ and numerator to be the difference of 2 terms	
M1	Differentiate $\left[3y-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\frac{\mathrm{d}y}{\mathrm{d}x}$	
A1*	Correct result obtained from fully correct working	
ALT 2	*Rearrange, separate into 2 fractions and then use quotient* rule	
M1	Use the quotient rule on both fractions. Denominators must be $\left(2-x^2\right)^2$ and numerator of each to be the difference of 2 terms	
M1	Differentiate $3y$ using the chain rule **and** differentiate $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\frac{\mathrm{d}y}{\mathrm{d}x}$	
A1*	Correct result obtained from fully correct working	
(b)		
B1	Correct value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	
M1	Use the given result from (a) to obtain a value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	
M1	Taylor's series formed using their values for the derivatives (accept 2! or 2 and 3! or 6)	
A1	Correct series, must start (or end) $y = \ldots$ but accept $\mathrm{f}(x)$ provided $y = \mathrm{f}(x)$ defined somewhere	

Question Number	Scheme	Marks
6(a)	$m^2+2m+5=0 \Rightarrow m=-1\pm 2i$	M1
	C F: $y=\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)$ OR $y=\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)$ or $y=P\mathrm{e}^{(-1+2\mathrm{i})x}+Q\mathrm{e}^{(-1-2\mathrm{i})x}$	A1
	PI: $y=a\cos x+b\sin x$	B1
	$y'=-a\sin x+b\cos x \qquad y''=-a\cos x-b\sin x$	
	$-a\cos x-b\sin x-2a\sin x+2b\cos x+5a\cos x+5b\sin x=6\cos x$	M1
	$-b-2a+5b=0 \qquad -a+2b+5a=6$	M1
	$a=\frac{6}{5} \quad b=\frac{3}{5}$	A1
	GS: $y=$ their CF $+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1ft (7)
(b)	$x=0, y=0 \quad 0=A+\frac{6}{5} \Rightarrow A=-\frac{6}{5}$	M1
	$y'=-\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)+\mathrm{e}^{-x}\left(-2A\sin 2x+2B\cos 2x\right)$ $-\frac{6}{5}\sin x+\frac{3}{5}\cos x$	M1A1ft
	$x=0 \quad \frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 0=+\frac{6}{5}+2B+\frac{3}{5} \Rightarrow B=-\frac{9}{10}$	dM1
	PS: $y=\mathrm{e}^{-x}\left(-\frac{6}{5}\cos 2x-\frac{9}{10}\sin 2x\right)+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1 (5)
		[12]
ALT	$y=\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	
	$x=0 \quad y=0 \quad 0=P+Q+\frac{6}{5}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-x}\left(2\mathrm{i}P\mathrm{e}^{\mathrm{i}2x}-2\mathrm{i}Q\mathrm{e}^{-\mathrm{i}2x}\right)-\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)-\frac{6}{5}\sin x+\frac{3}{5}\cos x$	M1A1ft
	$0=2\mathrm{i}P-2\mathrm{i}Q+\frac{9}{5}$	
	$P+Q=-\frac{6}{5} \qquad P-Q=\frac{9}{10}\mathrm{i}$	
	$P=\frac{1}{2}\left(-\frac{6}{5}+\frac{9}{10}\mathrm{i}\right) \qquad Q=\frac{1}{2}\left(-\frac{6}{5}-\frac{9}{10}\mathrm{i}\right)$	dM1
	PS: $y=\frac{1}{2}\mathrm{e}^{-x}\left(-\frac{6}{5}+\frac{9}{10}\mathrm{i}\right)\mathrm{e}^{2\mathrm{i}x}+\frac{1}{2}\mathrm{e}^{-x}\left(-\frac{6}{5}-\frac{9}{10}\mathrm{i}\right)\mathrm{e}^{-2\mathrm{i}x}+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1 (5)

Question Number	Scheme	Marks
(a)		
M1	Form and solve the auxiliary equation	
A1	Correct CF, either form (Often not seen until GS stated)	
B1	Correct form for the PI	
M1	Differentiate twice and sub in the original equation	
M1	Obtain a pair of simultaneous equations and attempt to solve	
A1	Correct values for both unknowns	
A1ft	Form the GS. Must start $y = \ldots$ Follow through their CF (writing CF scores A0) Must have scored a minimum of 2 of the M marks	
(b)		
	For CF $y = \mathrm{e}^{-x}\left(A\cos 2x + B\sin 2x\right)$	
M1	Sub $x = 0,\ y = 0$ in their GS and obtain a value for A	
M1	Differentiate their GS Product rule must be used	
A1ft	Correct differentiation of their GS provided this has 4 terms	
dM1	Sub $x = 0,\ \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and their A and obtain a value for B Depends on both previous M marks	
A1	Fully correct PS. Must start $y = \ldots$	
ALT(b)		
	For CF $y = \mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x} + Q\mathrm{e}^{-\mathrm{i}2x}\right)$ or $y = P\mathrm{e}^{(-1+2\mathrm{i})x} + Q\mathrm{e}^{(-1-2\mathrm{i})x}$	
M1	Sub $x = 0,\ y = 0$ in their GS and obtain an equation in P and Q	
M1	Differentiate their GS Product rule must be used if $y = \mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x} + Q\mathrm{e}^{-\mathrm{i}2x}\right)$ used	
A1ft	Correct differentiation of their GS	
dM1	Sub $x = 0,\ \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to obtain a second equation and solve the pair of equations The solution must allow for P and Q to be complex	
A1	Fully correct PS. Must start $y = \ldots$	

Question Number	Scheme	Marks
7 **(a)**	$x = r\cos\theta = 3\sin 2\theta\cos\theta$	B1
	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 6\cos 2\theta\cos\theta - 3\sin 2\theta\sin\theta = 0$	M1
	$2\cos\theta\left(\cos^2\theta - 2\sin^2\theta\right) = 0$	M1
ALT	For the 2 M marks:	
	$x = 6\sin\theta\cos^2\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 6\cos^3\theta - 12\sin^2\theta\cos\theta = 0$	
	$\tan\phi = \dfrac{1}{\sqrt{2}}$ *	A1* (4)
(b)	$\tan\phi = \dfrac{1}{\sqrt{2}} \Rightarrow \sin\phi = \dfrac{1}{\sqrt{3}},\ \cos\phi = \dfrac{\sqrt{2}}{\sqrt{3}}$	M1
	$R = 3\times 2\times\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{2}}{\sqrt{3}} = 2\sqrt{2}$	A1 (2)
(c)	Area of sector $= \dfrac{1}{2}\int r^2\mathrm{d}\theta = \dfrac{9}{2}\int \sin^2 2\theta\,\mathrm{d}\theta$	M1
	$= \dfrac{9}{2}\int_0^{\arctan\left(\frac{1}{\sqrt{2}}\right)} \dfrac{1}{2}\left(1-\cos 4\theta\right)\mathrm{d}\theta$	M1
	$= \dfrac{9}{2}\left[\dfrac{1}{2}\left(\theta - \dfrac{1}{4}\sin 4\theta\right)\right]_0^{\arctan\frac{1}{\sqrt{2}}}$	M1A1
	$= \dfrac{9}{4}\left[\arctan\dfrac{1}{\sqrt{2}} - \dfrac{1}{4}\sin 4\left(\arctan\dfrac{1}{\sqrt{2}}\right) - 0\right]$	dM1
	$\sin 4\phi = 2\sin 2\phi\cos 2\phi = 4\sin\phi\cos\phi\left(2\cos^2\phi - 1\right)$ $= 4\times\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{2}}{\sqrt{3}}\left(2\times\dfrac{2}{3}-1\right) = \dfrac{4\sqrt{2}}{9}$	M1
	Area of sector $= \dfrac{9}{4}\left(\arctan\dfrac{1}{\sqrt{2}} - \dfrac{1}{4}\times\dfrac{4\sqrt{2}}{9}\right) = \dfrac{9}{4}\arctan\dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{2}}{4}$	A1 (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	State $x = (r\cos\theta) = 3\sin 2\theta\cos 2\theta$ May be given by implication	
M1	Attempt to differentiate $x = r\cos\theta$ or $x = r\sin\theta$ Product rule must be used	
M1	Use a correct double angle formula **and** equate the derivative of $r\cos\theta$ to 0	
ALT	**M1** Attempt the differentiation of $x = r\cos\theta$ or $x = r\sin\theta$ using the product rule (after using a double angle formula) **M1** Use a correct double angle formula **and** equate the derivative of $r\cos\theta$ to 0	
A1*	Complete to the given answer and no extras with no errors in the working. Accept θ or ϕ All values seen must be exact	
(b)		
M1	Attempt exact values for $\sin\theta$ and $\cos\theta$ and use these to obtain a value for *R*. Values for $\sin\theta$ and/or $\cos\theta$ may have been seen in (a)	
A1	A correct, exact value for *R*, as shown or any equivalent. Award M1A1 for a correct exact answer	
(c)		
M1	Use of Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta$ Limits not needed (ignore any shown)	
M1	Use the double angle formula to obtain $k\int \frac{1}{2}(1 \pm \cos 4\theta)\mathrm{d}\theta$ Ignore any limits given This is NOT dependent NB: There are other, lengthy, methods of reaching this point	
M1	Attempt the integration $\cos 4\theta \to \pm\frac{1}{4}\sin 4\theta$ (Not dependent)	
A1	Correct integration of $1 - \cos 4\theta$	
dM1	Correct use of correct limits. Depends on second and third M marks 0 at lower limit need not be shown	
M1	Attempt an exact numerical value for $\sin 4\left(\arctan\frac{1}{\sqrt{2}}\right)$	
A1	Correct final answer. Award M1A1 for a correct exact final answer	

Question Number	Scheme	Marks
8(a)	$z^n = e^{in\theta} = \cos n\theta + i\sin n\theta$	
	$\frac{1}{z^n} = e^{-in\theta} = \cos(-n\theta) + i\sin(-n\theta) = \cos n\theta - i\sin n\theta$	
	$z^n + \frac{1}{z^n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta = 2\cos n\theta$ *	M1A1cso (2)
(b)	$\left(z+\frac{1}{z}\right)^6 = z^6 + 6z^5 \times \frac{1}{z} + \frac{6\times5}{2!}z^4 \times \frac{1}{z^2} + \frac{6\times5\times4}{3!}z^3 \times \frac{1}{z^3}$ $+\frac{6\times5\times4\times3}{4!}z^2 \times \frac{1}{z^4} + \frac{6\times5\times4\times3\times2}{5!}z \times \frac{1}{z^5} + \frac{1}{z^6}$	M1A1
	$(2\cos\theta)^6 = z^6 + 6z^4 + 15z^2 + 20 + 15\times\frac{1}{z^2} + 6\times\frac{1}{z^4} + \frac{1}{z^6}$	
	$64\cos^6\theta = z^6 + \frac{1}{z^6} + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20$	M1
	$64\cos^6\theta = 2\cos 6\theta + 6\times 2\cos 4\theta + 15\times 2\cos 2\theta + 20$	M1
	$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$ *	A1* (5)
(c)	$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 = 10$	
	$32\cos^6\theta = 10$	M1A1
	$\cos\theta = \pm\sqrt[6]{\frac{5}{16}}$	
	$\theta = 0.6027...,\ 2.5388... \qquad \theta = 0.603,\ 2.54$	M1A1 (4)
(d)	$\int_0^{\frac{\pi}{3}} (32\cos^6\theta - 4\cos^2\theta)\,d\theta$	
	$= \int_0^{\frac{\pi}{3}} (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 - 4\cos^2\theta)\,d\theta$	
	$= \int_0^{\frac{\pi}{3}} (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 - 2 - 2\cos 2\theta)\,d\theta$	M1
	$= \left[\frac{1}{6}\sin 6\theta + \frac{3}{2}\sin 4\theta + \frac{13}{2}\sin 2\theta + 8\theta\right]_0^{\frac{\pi}{3}}$	M1A1
	$= (0) + \frac{3}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{13}{2}\times\frac{\sqrt{3}}{2} + \frac{8\pi}{3} \quad (-0)$	dM1
	$= \frac{5\sqrt{3}}{2} + \frac{8\pi}{3}$ oe	A1 (5)
		[16]

Question Number	Scheme	Marks
(a)		
M1	Attempt to obtain $z^n + \frac{1}{z^n}$	
A1cso	Reach the given result with clear working and no errors Must see $\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$ changed to $\cos n\theta - \mathrm{i}\sin n\theta$ (ie both included)	
(b)	*The first 3 marks apply to the binomial expansion only*	
M1	Apply the binomial expansion to $\left(z+\frac{1}{z}\right)^6$ Coefficients must be numerical (ie ${}^nC_r$ is not acceptable). The expansion must have 7 terms with at least 4 correct	
A1	Correct expansion, terms need not be simplified	
M1	Simplify the coefficients and pair the appropriate terms on RHS (At least 2 pairs must be correct)	
M1	Use the result from (a) throughout. Must include 2^6 or 64 now	
A1*	Obtain the given result with no errors in the working	
(c)		
M1	Use the result from (b) to simplify the given equation	
A1	Reach $32\cos^6\theta = 10$ oe	
M1	Solve to obtain at least one correct value for θ, in radians and in the given range, 3 sf or better	
A1	2 correct values, and no extras, in radians and in the given range. Must be 3 sf here Ignore extras outside the range	
(d)		
M1	Use the result in (b) to change $\cos^6\theta$ to a sum of multiple angles ready for integration and use $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$ on $\cos^2\theta$ Limits not needed, ignore any shown	
M1	Integrate their expression to obtain an expression containing terms in $\sin 6\theta, \sin 4\theta, \sin 2\theta$ and θ Limits not needed	
A1	Correct integration Limits not needed	
dM1	Substitute limit pi/3. Depends second M mark	
A1	Correct, exact, answer (any equivalent to that shown). AwardM1A1 for a correct final answer following fully correct working.	
	There are other ways to integrate the function in (d), eg parts on one or both of the powers of $\cos\theta$, using $\cos^6\theta = (\cos^2\theta)^3 = \frac{1}{8}(1+\cos 2\theta)^3 = \ldots$ If in doubt about the marking of alternative methods which are not completely correct, send to review	

Question Number	Scheme	Marks
1		
1(a)	$\dfrac{2}{r(r+1)(r-1)}=\dfrac{1}{r-1}-\dfrac{2}{r}+\dfrac{1}{r+1}$	M1A1A1 (3)
1(b)	$r=2 \quad 1-\dfrac{2}{2}+\dfrac{1}{3}$	
	$r=3 \quad \dfrac{1}{2}-\dfrac{2}{3}+\dfrac{1}{4}$	
	$r=4 \quad \dfrac{1}{3}-\dfrac{2}{4}+\dfrac{1}{5}$	M1
	$r=n-1 \quad \dfrac{1}{n-2}-\dfrac{2}{n-1}+\dfrac{1}{n}$	
	$r=n \quad \dfrac{1}{n-1}-\dfrac{2}{n}+\dfrac{1}{n+1}$	M1
	$\sum_{r=2}^{n}\left(\dfrac{1}{r-1}-\dfrac{2}{r}+\dfrac{1}{r+1}\right)=\left(1-\dfrac{2}{2}+\dfrac{1}{2}+\dfrac{1}{n}-\dfrac{2}{n}+\dfrac{1}{n+1}\right)$	A1
	$\dfrac{1}{2}\sum_{r=1}^{n}\dfrac{2}{r(r+1)(r-1)}=\dfrac{1}{2}\times\left(\dfrac{1}{2}-\dfrac{1}{n}+\dfrac{1}{n+1}\right)=\dfrac{n^2+n-2}{4n(n+1)}$	dM1A1 (5) **[8]**
(a) **M1** **A1A1**	Attempt PFs by any valid method (by implication if 3 correct fractions seen) A1 any 2 fractions correct; A1 third fraction correct	
(b) **M1** **M1**	Method of differences with at least 3 terms at start and 2 at end OR 2 at start and 3 at end. Must start at 2 and end at n One M mark for the initial terms and a second for the final. Last lines may be missing $k/(n-1)$ and $c/(n-2)$ These 2 M marks may be implied by a correct extraction of terms. If starting from 1, M0M1 can be awarded.	
A1	Extract the remaining terms. $1-2/2$ may be missing and $1/n-2/n$ may be combined	
dM1	Include the 1/2 and attempt a common denominator of the required form. Depends on both previous M marks	
A1	$\dfrac{n^2+n-2}{4n(n+1)}$	

1

Question Number	Scheme	Marks
1(a)	**Special Case:** $\dfrac{2}{r\left(r^2-1\right)}=\dfrac{2r}{r^2-1}-\dfrac{2}{r}$ seen, award M1A1A0 Award M1A0A0 provided of the form $\dfrac{2}{r\left(r^2-1\right)}=\dfrac{Ar}{r^2-1}-\dfrac{B}{r}$	
1(b)	Terms listed as described above – award M1M1. Further progress unlikely as too many terms needed to establish the cancellation.	

Question Number	Scheme	Marks
2	$w = \dfrac{z+2}{z-\mathrm{i}} \quad z \neq \mathrm{i}$	
	$z = \dfrac{2+\mathrm{i}w}{w-1}$	M1
	$\lvert z\rvert = 2 \Rightarrow \left\lvert\dfrac{2+\mathrm{i}w}{w-1}\right\rvert = 2 \Rightarrow \lvert 2+\mathrm{i}w\rvert = 2\lvert w-1\rvert$	
	$\lvert 2+\mathrm{i}u-v\rvert = 2\lvert u+\mathrm{i}v-1\rvert$	M1 A1
	$(2-v)^2+u^2 = 4\left((u-1)^2+v^2\right)$	M1 A1
	$3u^2+3v^2-8u+4v=0$ oe	
	$\left(u-\dfrac{4}{3}\right)^2+\left(v+\dfrac{2}{3}\right)^2=\dfrac{20}{9}$ or $u^2+v^2-\dfrac{8}{3}u+\dfrac{4}{3}v=0$	dM1
	(i) centre is $\left(\dfrac{4}{3},-\dfrac{2}{3}\right)$	A1
	(ii) radius is $\dfrac{2\sqrt{5}}{3}$ oe	A1 [8]
M1 **M1** **A1** **M1** **A1** **dM1** **(i)A1** **(ii)A1** **ALT 1** **M1** **M1** **A1**	Rearrange equation to $z = \ldots$ Change w to $u+\mathrm{i}v$ and use $\lvert z\rvert = 2$ Allow if a different pair of letters used. Correct equation Correct use of Pythagoras on either side. Allow with 2 or 4 (RHS) Correct unsimplified equation Attempt the circle form. Coefficients for u^2 and v^2 must be 1. Depends on all 3 previous M marks Correct centre given (no decimals) (Use of rounded decimals changes the values) Correct radius given, any equivalent form (but no decimals) **NB:** These 2 A marks can only be awarded if the results have been deduced from a correct circle equation. Change w to $u+\mathrm{i}v$ Allow a different pair of letters. Rearrange equation to $z = \ldots$ and use $\lvert z\rvert = 2$ Correct equation Then as above.	
ALT 2	Very rare but may be seen: i maps to $\infty \Rightarrow \pm 2\mathrm{i}$ map to a diameter of C So $\dfrac{2\mathrm{i}+2}{\mathrm{i}}$ and $\dfrac{-2\mathrm{i}+2}{-3\mathrm{i}}$ are ends of a diameter Calculate centre and radius	 M1A1 M2A1 M1A1A1

1

Question Number	Scheme	Marks
3(a)	$y = r\sin\theta = \sin\theta + \sin\theta\cos\theta$ OR $r\sin\theta = \sin\theta + \frac{1}{2}\sin 2\theta$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta - \sin^2\theta + \cos^2\theta$ OR $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta + \cos 2\theta$ $0 = \cos\theta + 2\cos^2\theta - 1 \quad = (2\cos\theta - 1)(\cos\theta + 1)$	M1
	$\cos\theta = \frac{1}{2}$ $(\cos\theta = -1$ outside range for $\theta)$ $\theta = \frac{\pi}{3}$	M1
	A is $\left(1\frac{1}{2}, \frac{\pi}{3}\right)$	A1 (4)
3(b)	Area $= \frac{1}{2}\int_0^{\frac{\pi}{3}} (1+\cos\theta)^2 \mathrm{d}\theta$	B1
	$= \frac{1}{2}\int \left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right)\mathrm{d}\theta$	M1A1
	$= \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}$	dM1A1
	$= \frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16} = \frac{\pi}{4} + \frac{9\sqrt{3}}{16}$	A1 (6) [10]
(a) **B1**	Use of $r\sin\theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen.	
M1	Differentiate $r\sin\theta$ or $r\cos\theta$	
M1	Set $\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 0$ and solve the resulting equation. Only the solution used need be shown.	
A1	Correct coordinates of A	
(b)B1	Use of Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta$ with $r = 1 + \cos\theta$, limits not needed.	
M1	Attempt $(1+\cos\theta)^2$ (minimum accepted is $(1 + k\cos\theta + \cos^2\theta)$) and change $\cos^2\theta$ to an expression in $\cos 2\theta$ using $\cos^2\theta = \frac{1}{2}(\pm\cos 2\theta \pm 1)$	
A1	Correct integrand; limits not needed. $\frac{1}{2}$ may be missing.	
dM1	Attempt to integrate all terms. $\cos 2\theta \to \pm\frac{1}{k}\sin 2\theta$ $k = \pm 1$ or ± 2 Limits not needed. Depends on the previous M mark	
A1	Correct integration and correct limits seen	
A1	Substitute correct limits and obtain the correct answer in the required form.	

Question Number	Scheme	Marks
	Alternative for (b) *using integration by parts* (Very rare but may be seen)	
	Area $=\frac{1}{2}\int_0^{\frac{\pi}{3}}(1+\cos\theta)^2\mathrm{d}\theta$	B1
	$=\frac{1}{2}\left[\int (1+2\cos\theta)\mathrm{d}\theta+ \quad \int \cos^2\theta\,\mathrm{d}\theta\right]$	
	$=\frac{1}{2}\left[\int (1+2\cos\theta)\mathrm{d}\theta+ \quad \cos\theta\sin\theta+\int \sin^2\theta\,\mathrm{d}\theta\right]$	M1A1
	$=\frac{1}{2}\left[\theta+2\sin\theta+ \quad \sin\theta\cos\theta+\int (1-\cos^2\theta)\mathrm{d}\theta\right]_0^{\frac{\pi}{3}}$	
	$=\frac{1}{2}\left[\theta+2\sin\theta+ \quad \frac{1}{2}(\sin\theta\cos\theta+\theta)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$=\frac{\pi}{4}+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{16}=\frac{\pi}{4}+\frac{9\sqrt{3}}{16}$	A1
B1	Use of Area $=\frac{1}{2}\int r^2\mathrm{d}\theta$ with $r=1+\cos\theta$, limits not needed.	
M1	Attempt $(1+\cos\theta)^2$ (minimum accepted is $(1+k\cos\theta+\cos^2\theta)$) and attempt first stage of $\int \cos^2\theta\,\mathrm{d}\theta$ by parts. Reach $\int \cos^2\theta\,\mathrm{d}\theta=\cos\theta\sin\theta\pm\int \sin^2\theta\,\mathrm{d}\theta$ Limits not needed	
A1	Correct so far. Limits not needed.	
dM1	Attempt to integrate all terms. $\int (1+2\cos\theta)\mathrm{d}\theta$ **and** attempt to complete $\int \cos^2\theta\,\mathrm{d}\theta$ using Pythagoras identity. Limits not needed. Depends on the previous M mark	
A1A1	As main scheme	

Question Number	Scheme	Marks
4 (a)	$\frac{d^2y}{dx^2}=\frac{4}{y}\left(\frac{dy}{dx}\right)^2-3$	M1
	$\frac{d^3y}{dx^3}=-\frac{4}{y^2}\left(\frac{dy}{dx}\right)^3+\frac{8}{y}\times\frac{d^2y}{dx^2}\times\frac{dy}{dx}$	M1A1A1
	$\frac{d^3y}{dx^3}=-\frac{4}{y^2}\left(\frac{dy}{dx}\right)^3+\frac{8}{y}\left(\frac{4}{y}\left(\frac{dy}{dx}\right)^2-3\right)\left(\frac{dy}{dx}\right)$	
	$\frac{d^3y}{dx^3}=\frac{28}{y^2}\left(\frac{dy}{dx}\right)^3-\frac{24}{y}\left(\frac{dy}{dx}\right)$ *	A1* (5)
ALT	$\frac{dy}{dx}\frac{d^2y}{dx^2}+y\frac{d^3y}{dx^3}-8\frac{dy}{dx}\times\frac{d^2y}{dx^2}+3\frac{dy}{dx}=0$	M1A1A1
	$\frac{d^3y}{dx^3}=\frac{1}{y}\left(7\frac{dy}{dx}\right)\left(\frac{4}{y}\left(\frac{dy}{dx}\right)^2-3\right)-\frac{3}{y}\frac{dy}{dx}$	M1
	$\frac{d^3y}{dx^3}=\frac{28}{y^2}\left(\frac{dy}{dx}\right)^3-\frac{24}{y}\left(\frac{dy}{dx}\right)$ *	A1* (5)
4(b)	At $x=0$ $\frac{d^2y}{dx^2}=\frac{4}{8}(1)^2-3=-\frac{5}{2}$ oe	B1
	$\frac{d^3y}{dx^3}=\frac{28}{64}\times1^3-\frac{24}{8}\times1=-\frac{41}{16}$	M1
	$y=8+x-\frac{5}{2}\times\frac{x^2}{2!}-\frac{41}{16}\times\frac{x^3}{3!}+\ldots$	M1
	$y=8+x-\frac{5}{4}x^2-\frac{41}{96}x^3+\ldots$	A1 (4) [9]

Question Number	Scheme	Marks
5(a)		
M1	Divide through by y No need to re-arrange the equation until later	
M1	Attempt the differentiation using product rule and chain rule and obtain $\frac{d^3y}{dx^3} = \ldots$	
A1A1	A1 Either RHS term correct A1 Second RHS term correct and no extras	
A1*	Eliminate $\frac{d^2y}{dx^2}$ and obtain the **given** result	
ALT		
M1	Re-arrange the equation (Will probably be seen later in work)	
M1	Attempt the differentiation using product rule and chain rule	
A1A1	A1 Two terms correct A1 All correct and no extras	
A1*	Eliminate $\frac{d^2y}{dx^2}$ and obtain the correct result	
5(b)B1	Correct value for $\frac{d^2y}{dx^2}$	
M1	Use the *given* expression from (a) to obtain a value for $\frac{d^3y}{dx^3}$ Award if correct value seen.	
M1	Taylor's series formed using their values for the derivatives (2! or 2, 3! or 6)	
A1	Correct series, must start (or end) $y = \ldots$ Correct terms must be seen, order may be different. Can have f(x) = ... provided f(x) = y is defined somewhere.	

Question Number	Scheme	Marks
5		
NB	Question states "Use algebra..." so purely graphical solutions (using calculator?) score 0/7. A sketch and some algebra to find intersection points can score.	
	$2x^2+x-3\geq 0$	
	$2x^2+x-3=3(1-x)\Rightarrow 2x^2+4x-6=0$	M1
	$2x^2+4x-6\Rightarrow x^2+2x-3=(x+3)(x-1)=0$	
	$x=-3,\ 1$	A1
	$2x^2+x-3\leq 0$	
	$-2x^2-x+3=3(1-x)\Rightarrow 2x^2-2x=0$	M1
	$2x(x-1)=0,\ \ x=0,\ 1$	A1
	$x<-3\quad 0<x<1\quad x>1$	dM1A1A1 [7]
	The first 4 marks can be awarded with any inequality sign or =	
M1	Assume $2x^2+x-3\geq 0$ and obtain a 3TQ	
A1	Correct CVs obtained from a correct equation.	
M1	Assume $2x^2+x-3\leq 0$ and obtain a 2 or 3TQ	
A1	Correct CVs obtained from a correct equation.	
dM1	Form 3 distinct inequalities with their 3 CVs. Can have $<$ or $\leq$, $>$ or $\geq$. Must have scored both previous M marks. Accept $x<-3\quad 0<x\quad x\neq 1$	
A1	All 3 correct CVs used correctly	
A1	Inequalities fully correct. "and" between the inequalities is acceptable. If $\cap$ is used, award A0 here. Fully correct set language accepted.	
ALT	*Squaring both sides*	
	$(2x^2+x-3)^2>9(1-x)^2$	
	$4x^4+4x^3-20x^2+12x>0$	M1A1
	$x(x+3)(x-1)(x-1)>0$	M1
	CVs: $x=0,-3,1$	A1
	Then as main scheme	
	These 4 marks can be awarded with any inequality sign or =	
M1	Square both sides and collect terms to obtain a quartic with 4 or 5 terms	
A1	Correct quartic	
M1	Factorise their quartic	
A1	3 correct CVs	

Question Number	Scheme	Marks
6(a)	$m^2 - 6m + 8 = 0$ $(m-2)(m-4) = 0,\ \ m = 2, 4$ $(\text{CF} =)\quad Ae^{2x} + Be^{4x}$ $\text{PI}:\ y = \lambda x^2 + \mu x + \nu$ $y' = 2\lambda x + \mu \quad y'' = 2\lambda$ $2\lambda - 6(2\lambda x + \mu) + 8(\lambda x^2 + \mu x + \nu) = 2x^2 + x$ $\lambda = \frac{1}{4},\ -12\lambda + 8\mu = 1,\ \ 2\lambda - 6\mu + 8\nu = 0$ $\lambda = \frac{1}{4},\ \mu = \frac{1}{2},\ \nu = \frac{5}{16}$ $y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$	 M1 A1 B1 M1 M1 A1A1 A1ft (8)
(a)M1 **A1** **B1** **M1** **M1** **A1** **A1** **A1ft**	Form aux equation and attempt to solve (any valid method). Equation need not be shown if CF is correct or complete solution $(m = 2, 4)$ is shown Correct CF $\ y = ..$ not needed. Correct form for PI Their PI (minimum 2 terms) differentiated twice and substituted in the equation Coefficients equated Any 2 values correct All 3 values correct A complete solution, follow through their CF and PI. All 3 M marks must have been earned. Must start $y = ...$	
6(b)	$y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$ $1 = A + B + \frac{5}{16}$ $\frac{dy}{dx} = 2Ae^{2x} + 4Be^{4x} + \frac{1}{2}x + \frac{1}{2} \qquad 0 = 2A + 4B + \frac{1}{2}$ $A = \frac{13}{8} \quad B = -\frac{15}{16} \quad$ oe $y = \frac{13}{8}e^{2x} - \frac{15}{16}e^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16} \quad$ oe	 M1 M1 dM1A1 A1ft (5) [13]
(b) **M1** **M1** **dM1** **A1** **A1ft**	 Substitute $y = 1$ and $x = 0$ in their complete solution from (a) Differentiate and substitute $\frac{dy}{dx} = 0$, $x = 0$ Solve the 2 equations to $A = ...$ or $B = ...$ Depends on the two previous M marks Both values correct Particular solution, follow through their general solution and A and B. Must start $y = ...$	

1

Question Number	Scheme	Marks
7(a)	$(\cos\theta + \mathrm{i}\sin\theta)^4 = \cos 4\theta + \mathrm{i}\sin 4\theta$	
	$\cos^4\theta + 4\cos^3\theta(\mathrm{i}\sin\theta) + \dfrac{4\times3}{2!}\cos^2\theta(\mathrm{i}\sin\theta)^2$ $+\dfrac{4\times3\times2}{3!}\cos\theta(\mathrm{i}\sin\theta)^3 + (\mathrm{i}\sin\theta)^4$	M1
	$= \cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta + \mathrm{i}^2 6\cos^2\theta\sin^2\theta + 4\mathrm{i}^3\cos\theta\sin^3\theta + \mathrm{i}^4\sin^4\theta$	A1
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$	M1
	$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	A1
	$\tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta} = \dfrac{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}$	
	$\tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta} = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ *	M1A1* (6)
7(b)	$x = \tan\theta$ $\quad \dfrac{2\tan\theta - 2\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} = \dfrac{1}{2}\tan 4\theta = 1$	
	$\tan 4\theta = 2$	M1
	$x = \tan\theta = 0.284,\ 1.79$	A1A1 (3) [9]
(a)		
M1	Correct use of de Moivre and attempt the complete expansion	
A1	Correct expansion. Coefficients to be single numbers but powers of i may still be present.	
M1	Equate the real and imaginary parts	
A1	Correct expressions for $\cos 4\theta$ and $\sin 4\theta$	
M1	Use $\tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta}$ and divide numerator and denominator by $\cos^4\theta$ Only tangents now.	
A1*	Correct **given** answer, no errors seen.	
(b)		
M1	Substitute $x = \tan\theta$ and re-arrange to $\tan 4\theta = \pm 2$ or $\pm\dfrac{1}{2}$	
A1A1	A1 for either solution; A2 for both. Deduct one mark only for failing to round either or both to 3 sf (One correct answer but not rounded scores A0A0; two correct answers neither rounded scores A1A0; two correct answers, only one rounded, scores A1A0)	

Question Number	Scheme	Marks
	Alternative for first 4 marks of 7(a): $\sin 4\theta = \frac{1}{2\mathrm{i}}\left(z^4 - z^{-4}\right) = \frac{1}{2\mathrm{i}}\left(\left(\cos\theta - \mathrm{i}\sin\theta\right)^4 - \left(\cos\theta + \mathrm{i}\sin\theta\right)^{-4}\right)$ $= \frac{1}{2\mathrm{i}}\left(\cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4\mathrm{i}\cos\theta\sin^3\theta + \sin^4\theta\right)$ $-\frac{1}{2\mathrm{i}}\left(-\cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta + 6\cos^2\theta\sin^2\theta - 4\mathrm{i}\cos\theta\sin^3\theta - \sin^4\theta\right)$ $= 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$ Similar work leads to $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ Remaining 2 marks as main scheme	 M1 M1 A1 A1
M1 **A1** **M1** **A1**	For the expression derived from de Moivre for **either** $\sin 4\theta$ or $\cos 4\theta$ Both shown and correct Attempt the binomial expansion for either, reaching a simplified expression Both simplified expressions correct	

Question Number	Scheme	Marks
8(a)	$v = y^{-2} \quad \frac{dv}{dy} = -2y^{-3}$	B1
	$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = -\frac{y^3}{2}\frac{dv}{dx}$	M1A1
	$-\frac{y^3}{2}\frac{dv}{dx} + 6xy = 3xe^{x^2}y^3$	
	$\frac{1}{2}\frac{dv}{dx} - \frac{6xy}{y^3} = -3xe^{x^2}$	
	$\frac{dv}{dx} - 12vx = -6xe^{x^2}$ *	dM1A1* (5)
ALT 1	$y = v^{-\frac{1}{2}} \quad \frac{dy}{dv} = -\frac{1}{2}v^{-\frac{3}{2}}$	B1
	$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dx}$	M1A1
	$-\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dx} + 6xv^{-\frac{1}{2}} = 3xe^{x^2}v^{-\frac{3}{2}}$	dM1
	$-\frac{1}{2}\frac{dv}{dx} + 6xv = 3xe^{x^2}$	
	$\frac{dv}{dx} - 12vx = -6xe^{x^2}$ *	A1* (5)
ALT 2	$v = y^{-2} \quad \frac{dv}{dy} = -2y^{-3}$	B1
	$\frac{dv}{dx} = \frac{dv}{dy} \times \frac{dy}{dx} = -2y^{-3}\frac{dy}{dx}$	M1A1
	$-2y^{-3}\frac{dy}{dx} - 12y^{-2}x = -6xe^{x^2}$	dM1
	$\frac{dy}{dx} + 6xy = 3xe^{x^2}y^3 \qquad x > 0$	A1* (5)
8(a)	**All Methods:**	
B1	Correct derivative	
M1	Attempt $\frac{dy}{dx}$ or $\frac{dv}{dx}$ using the chain rule	
A1	Correct derivative	
dM1	Substitute in equation (I) to obtain an equation in v and x only OR in equation (II) to obtain an equation in x and y only (ALT 2)	
A1*	Correct completion with no errors seen	

Question Number	Scheme	Marks
8(b)	IF: $\mathrm{e}^{\int -12x\mathrm{d}x} = \mathrm{e}^{-6x^2}$	M1A1
	$v\mathrm{e}^{-6x^2} = \int -6x\mathrm{e}^{x^2} \times \left(\mathrm{e}^{-6x^2}\right)\mathrm{d}x = \int -6x\mathrm{e}^{-5x^2}\,\mathrm{d}x$	dM1
	$v\mathrm{e}^{-6x^2} = \frac{6}{10}\mathrm{e}^{-5x^2}\ (+c)$	A1
	$v\left(= y^{-2}\right) = \frac{6}{10}\mathrm{e}^{x^2} + c\mathrm{e}^{6x^2}$	ddM1
	$y^2 = \dfrac{1}{\frac{6}{10}\mathrm{e}^{x^2} + c\mathrm{e}^{6x^2}}$ oe eg $y^2 = \dfrac{10}{6\mathrm{e}^{x^2} + k\mathrm{e}^{6x^2}}$	A1 (6) **[11]**
(b)		
M1	IF of form $\mathrm{e}^{\int \pm 12x\mathrm{d}x}$ and attempt the integration.	
A1	Correct IF	
dM1	Multiply through by their IF and integrate the LHS. Depends on first M mark of (b)	
A1	Correct integration of the complete equation with or without constant	
ddM1	Include the constant and multiply through by e^{6x^2} Depends on both previous M marks of (b)	
A1	Any equivalent to that shown. (No need to change letter used for constant when rearranging)	

WFM02 Further Pure Mathematics F2 Mark Scheme

Question Number	Scheme	Notes	Marks
1	$z^5 - 32\text{i} = 0 \Rightarrow r^5 = 32 \Rightarrow r = 2$	Correct value for r. May be shown explicitly or used correctly.	B1
	$5\theta = \frac{\pi}{2} + 2n\pi \Rightarrow \theta = \frac{\pi}{10} + \frac{2n\pi}{5}$	Applies a correct strategy for establishing at least 2 values of θ. This can be awarded if if the initial angle $\left(\frac{\pi}{2} \text{ or } \frac{\pi}{10}\right)$ is incorrect but otherwise their strategy is correct.	M1
	$z = 2\text{e}^{\text{i}\frac{\pi}{10}},\ 2\text{e}^{\text{i}\frac{\pi}{2}},\ 2\text{e}^{\text{i}\frac{9\pi}{10}},\ 2\text{e}^{\text{i}\frac{13\pi}{10}},\ 2\text{e}^{\text{i}\frac{17\pi}{10}}$ or $z = 2\text{e}^{\left(\frac{\pi}{10}+\frac{2n\pi}{5}\right)\text{i}},\quad n = 0,1,2,3,4$	At least 2 correct, follow through their r	A1ft
		All correct. Must have $r = 2$	A1
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
2	$\dfrac{x}{2-x} \leqslant \dfrac{x+3}{x}$		
Way 1	$\dfrac{x}{2-x} \leqslant \dfrac{x+3}{x} \Rightarrow \dfrac{x}{2-x} - \dfrac{x+3}{x} \leqslant 0$	Collects to one side	M1
	$\dfrac{x}{2-x} - \dfrac{x+3}{x} \leqslant 0 \Rightarrow \dfrac{x^2-(2-x)(x+3)}{x(2-x)} \leqslant 0$ M1: Attempt common denominator A1: Correct fraction		M1 A1
	$x = 0,\ 2$	These critical values	B1
	$x^2-(2-x)(x+3)=0$ $\Rightarrow 2x^2+x-6=0 \Rightarrow x = \ldots$	Solves the 3TQ in the numerator	M1
	$x = \dfrac{3}{2},\ -2$	These critical values	A1
	$x \leqslant -2,\ 0 < x \leqslant \dfrac{3}{2},\ x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$		A1A1
			(8)
			Total 8
	Alternative 1: $\times x^2(2-x)^2$		
	$x^3(2-x) \leqslant x(x+3)(2-x)^2$	Multiplies by a positive expression	M1
	$x^3(2-x) - x(x+3)(2-x)^2 \leqslant 0$	Collects to one side Correct inequality	M1 A1
	$x = 0,\ 2$	These critical values	B1
	$x(2-x)\left[x^2 - (x+3)(2-x)\right] = 0$ $x^2 - (x+3)(2-x) = 0$ $\Rightarrow 2x^2+x-6=0 \Rightarrow x = \ldots$	Attempts to factorise by taking out a factor of $x(2-x)$ and solves resulting 3TQ. May have quartic and apply the factor theorem.	M1
	$x = \dfrac{3}{2},\ -2$	These critical values	A1
	$x \leqslant -2,\ 0 < x \leqslant \dfrac{3}{2},\ x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$		A1A1

Question Number	Scheme	Notes	Marks
3	$w=\frac{(2+\mathrm{i})z+4}{z-\mathrm{i}} \Rightarrow wz-w\mathrm{i}=(2+\mathrm{i})z+4$ $\Rightarrow z=\ldots$	Attempts to make z the subject	M1
	$z=\frac{w\mathrm{i}+4}{w-2-\mathrm{i}}$	Correct equation in any form	A1
	$z=\frac{(u+\mathrm{i}v)\mathrm{i}+4}{u+\mathrm{i}v-2-\mathrm{i}}$ $z=\frac{((u+\mathrm{i}v)\mathrm{i}+4)(u-2-(v-1)\mathrm{i})}{(u-2+(v-1)\mathrm{i})(u-2-(v-1)\mathrm{i})}$	Introduces $w=u+\mathrm{i}v$ and multiplies numerator and denominator by the conjugate of the denominator	M1
	$u(v-1)+(4-v)(u-2)=0$	Sets real part = 0 (with or without denominator) Depends on both M marks above	dM1
		Any correct equation	A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 2	$w=\frac{(2+\mathrm{i})z+4}{z-\mathrm{i}}, z=y\mathrm{i} \Rightarrow w=\frac{(2+\mathrm{i})y\mathrm{i}+4}{y\mathrm{i}-\mathrm{i}}$ $w=\frac{(2+\mathrm{i})y\mathrm{i}+4}{y\mathrm{i}-\mathrm{i}}\times\frac{\mathrm{i}}{\mathrm{i}}$	Solves simultaneously and multiplies numerator and denominator by i	M1
	$u=\frac{2y}{y-1}, v=\frac{y-4}{y-1}$	Correct real and imaginary parts	A1
	$u=\frac{2y}{y-1} \Rightarrow y=\frac{u}{u-2}$	Attempts y in terms of u or v	M1
	$y=\frac{u}{u-2} \Rightarrow v=\frac{\frac{u}{u-2}-4}{\frac{u}{u-2}-1}$	Obtains an equation connecting u and v	M1
		Any correct equation	A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 3	Apply the transformation to any point on the imaginary axis	Eg $(0,0)\to(0,4)$ $(0,1)\to(4,-2)$	M1
	Apply the transformation to a second point on the imaginary axis	This is the second M mark on e-PEN	M1
	Both transformations correct	This is the first A mark on e-PEN	A1
	Complete method to obtain an equation for the line thro' their 2 points in the w-plane		M1
	Correct equation in any form		A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$(x+1)\frac{dy}{dx} - xy = e^{3x} \qquad x > -1$		
	$\frac{dy}{dx} - \frac{xy}{(x+1)} = \frac{e^{3x}}{(x+1)}$	Correctly rearranged equation	B1
	$I = e^{\int \frac{-x}{x+1} dx} = e^{\int \left(-1+\frac{1}{x+1}\right) dx}$	Correct strategy for the integrating factor including an attempt at the integration	M1
	$= e^{-x+\ln(x+1)}$	For $-x + \ln(x + 1)$	A1
	$= (x+1)e^{-x}$	Correct integrating factor	A1
	$y(x+1)e^{-x} = \int \frac{e^{3x}}{x+1} \times (x+1)e^{-x}\, dx$	Uses their integrating factor to reach the form $yI = \int Q\,I\, dx$	M1
	$y(x+1)e^{-x} = \frac{1}{2}e^{2x} + c$	Correct equation (with or without $+ c$)	A1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{ce^{x}}{(x+1)}$	Correct answer (allow equivalent forms). Must have $y =$...	A1
			(7)
(b)	$x = 0, y = 5 \Rightarrow 5 = \frac{1}{2} + c \Rightarrow c = \frac{9}{2}$	Substitutes $x = 0$ and $y = 5$ and attempts to find a value for c.	M1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{9e^{x}}{2(x+1)}$	Cao (oe) Must have $y =$...	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$y=\tan^2 x \Rightarrow \frac{dy}{dx}=2\tan x\sec^2 x$	Correct first derivative any correct form	B1
	$\frac{dy}{dx}=2\tan x\sec^2 x \Rightarrow \frac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x$ M1: Correct application of the product rule and chain rule A1: Correct expression		M1A1
	$\frac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x \Rightarrow \frac{d^3y}{dx^3}=8\sec^4 x\tan x+8\sec^2 x\tan^3 x+8\sec^4 x\tan x$ Or $\frac{d^2y}{dx^2}=6\sec^4 x-4\sec^2 x \Rightarrow \frac{d^3y}{dx^3}=24\sec^4 x\tan x-8\sec^2 x\tan x$ M1: Attempt to differentiate using product and chain rule. At least one term to be correct		M1
	$=8\sec^4 x\tan x+8\sec^2 x\tan x\left(\sec^2 x-1\right)+8\sec^4 x\tan x$ $=24\sec^4 x\tan x-8\sec^2 x\tan x=8\sec^2 x\tan x\left(3\sec^2 x-1\right)$ Fully correct expression		A1
			(5)
(b)	$\left(y\right)_{\frac{\pi}{3}}=3,\ \left(y'\right)_{\frac{\pi}{3}}=8\sqrt{3},\ \left(y''\right)_{\frac{\pi}{3}}=80,\ \left(y'''\right)_{\frac{\pi}{3}}=352\sqrt{3}$	Attempts the values up to the third derivative when $x=\frac{\pi}{3}$	M1
	$y=3+8\sqrt{3}\left(x-\frac{\pi}{3}\right)+\frac{80}{2!}\left(x-\frac{\pi}{3}\right)^2+\frac{352\sqrt{3}}{3!}\left(x-\frac{\pi}{3}\right)^3+\ldots$ Correct application of the Taylor series 2! or 2, 3! or 6		M1
	$y=3+8\sqrt{3}\left(x-\frac{\pi}{3}\right)+40\left(x-\frac{\pi}{3}\right)^2+\frac{176\sqrt{3}}{3}\left(x-\frac{\pi}{3}\right)^3+\ldots$ Correct expansion Must start $y=$ ... or $\tan^2 x=$ .... f(x) only accepted if f(x) has been defined to be $\tan^2 x$		A1
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$\lvert z+1-13\text{i}\rvert = 3\lvert z-7-5\text{i}\rvert \Rightarrow (x+1)^2+(y-13)^2 = 9\left\{(x-7)^2+(y-5)^2\right\}$ Correct application of Pythagoras Accept 3 or 9 on RHS		M1
	$\Rightarrow x^2+y^2-16x-8y+62=0$	Correct equation in any form with terms collected	A1
	Centre (8, 4)	Correct centre. i included scores A0	A1
	$r^2 = 64+16-62=...$	Correct method for r or r^2	M1
	$r=\sqrt{18}$ or $3\sqrt{2}$	Correct radius. Must be exact.	A1
			(5)
(b)	$\arg(z-8-6\text{i}) = -\frac{3\pi}{4} \Rightarrow y-6=x-8$	Converts the given locus to the correct Cartesian form	B1
	$\Rightarrow x^2+y^2-16x-8y+62=0$ $\Rightarrow x^2+(x-2)^2-16x-8(x-2)+62=0 \Rightarrow x=...$ or $\Rightarrow (y+2)^2+y^2-16x-8(y+2)+62=0 \Rightarrow y=...$	Uses both Cartesian equations to obtain an equation in one variable and attempts to solve	M1
	$x=7-2\sqrt{2}$ or $y=5-2\sqrt{2}$	One correct "coordinate"	A1
	R is $7-2\sqrt{2}+(5-2\sqrt{2})\text{i}$ **or** $x=7-2\sqrt{2}$ and $y=5-2\sqrt{2}$	Correct complex number or coordinates and no others. Must be exact	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	$x=t^2 \Rightarrow \frac{dx}{dy}=2t\frac{dt}{dy}$ oe	Correct application of the chain rule	M1
	$\Rightarrow \frac{dy}{dx}=\frac{1}{2t}\frac{dy}{dt}\left(\text{or e.g. } \frac{1}{2\sqrt{x}}\frac{dy}{dt}\right)$	Any correct expression for $\frac{dy}{dx}$ or equivalent equation	A1
	$2\sqrt{x}\frac{dy}{dx}=\frac{dy}{dt} \Rightarrow x^{-\frac{1}{2}}\frac{dy}{dx}+2\sqrt{x}\frac{d^2y}{dx^2}=\frac{d^2y}{dt^2}\frac{dt}{dx}$ (NB $\frac{d^2y}{dt^2}=2\frac{dy}{dx}+4x\frac{d^2y}{dx^2}$)	Fully correct strategy to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$ Chain rule used on at least one term. Depends on the first M mark	dM1
	$4x\frac{d^2y}{dx^2}+2\left(1+2\sqrt{x}\right)\frac{dy}{dx}-15y=15x \Rightarrow 4x\frac{d^2y}{dx^2}+4\sqrt{x}\frac{dy}{dx}+2\frac{dy}{dx}-15y=15x$ $\Rightarrow \frac{d^2y}{dt^2}+2\frac{dy}{dt}-15y=15t^2$ * ddM1: Substitutes into the given differential equation. The full substitution must be seen. Depends on both M marks. A1*: Cso		ddM1 A1*
			(5)
(b)	$m^2+2m-15=0 \Rightarrow m=3,-5$	Attempts to solve $m^2+2m-15=0$	M1
	$y=Ae^{-5t}+Be^{3t}$	Correct CF	A1
	$y=at^2+bt+c \Rightarrow \frac{dy}{dt}=2at+b \Rightarrow \frac{d^2y}{dt^2}=2a$ $\Rightarrow 2a+4at+2b-15at^2-15bt-15c=15t^2$ Starts with the correct PI form and differentiates twice and substitutes		M1
	$-15a=15 \Rightarrow a=\ldots$ $4a-15b=0 \Rightarrow b=\ldots$ $2a+2b-15c=0 \Rightarrow c=\ldots$	Complete method to find a, b and c by comparing coefficients. Values for all 3 needed. Depends on the second M mark.	dM1
	$y=Ae^{-5t}+Be^{3t}-t^2-\frac{4}{15}t-\frac{38}{225}$	Correct GS. Must start $y=\ldots$	A1
			(5)
(c)	$y=Ae^{-5\sqrt{x}}+Be^{3\sqrt{x}}-x-\frac{4}{15}\sqrt{x}-\frac{38}{225}$	Correct equation (follow through their answer to (b)) Must start $y=\ldots$	B1ft
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$x = r\cos\theta = (1+\sin\theta)\cos\theta$ $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = \cos^2\theta - (1+\sin\theta)\sin\theta$ or $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = -\sin\theta + \cos 2\theta$	Differentiates $r\cos\theta$ using product rule or double angle formula	M1
		Correct derivative in any form	A1
	$\cos^2\theta - (1+\sin\theta)\sin\theta = 0 \Rightarrow 1-\sin^2\theta - \sin\theta - \sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ or $-\sin\theta + \cos 2\theta = 0 \Rightarrow -\sin\theta + 1 - 2\sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ Sets $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$ and proceeds to a 3TQ in sinθ Depends on the first M mark		dM1
	$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ $\Rightarrow \sin\theta = \frac{1}{2}, (-1) \Rightarrow \theta = ...$	Solves for θ. Depends o both M marks above.	ddM1
	$\left(\frac{3}{2}, \frac{\pi}{6}\right)$	Correct coordinates and no others. Need not be in coordinate brackets.	A1
			(5)
(b)	$\int (1+\sin\theta)^2\,\mathrm{d}\theta = \int (1+2\sin\theta+\sin^2\theta)\,\mathrm{d}\theta$ $= \int \left(1+2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$	Attempts $\left(\frac{1}{2}\right)\int r^2\,\mathrm{d}\theta$ and applies $\sin^2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ Ignore any limits shown	M1
	$\int (1+\sin\theta)^2\,\mathrm{d}\theta = \frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$	Correct integration (Ignore limits)	A1
	$\frac{1}{2}\left[\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2}\left[\frac{3\pi}{4} - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8}\right)\right]\left(= \frac{\pi}{4} + \frac{9\sqrt{3}}{16}\right)$	Applies the limits of $\frac{\pi}{2}$ and their $\frac{\pi}{6}$ Substitution must be shown but no simplification needed	M1
	Trapezium: $\frac{1}{2}\left(2+\left(2-\frac{3}{2}\sin\frac{\pi}{6}\right)\right)\times\frac{3}{2}\cos\frac{\pi}{6}$ $\left(=\frac{39\sqrt{3}}{32}\right)$	Uses a correct strategy for the area of trapezium *OQSP*	M1
	Area of $R = \frac{39\sqrt{3}}{32} - \frac{\pi}{4} - \frac{9\sqrt{3}}{16}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32}\left(21\sqrt{3} - 8\pi\right)$	Cao	A1
			(6)

Question Number	Scheme	Notes	Marks
9(a)	$n^5-(n-1)^5=n^5-\left(n^5-5n^4+10n^3-10n^2+5n-1\right)=...$ Starts the proof by expanding the bracket		M1
	$5n^4-10n^3+10n^2-5n+1$*	Correct proof with no errors. Full expansion of $(n-1)^5$ must be shown.	A1*
			(2)
(b)	$1^5-0^5=5(1)^4-10(1)^3+10(1)^2-5(1)+1$ $2^5-1^5=5(2)^4-10(2)^3+10(2)^2-5(2)+1$ .... $(n-1)^5-(n-2)^5=5(n-1)^4-10(n-1)^3+10(n-1)^2-5(n-1)+1$ $(n)^5-(n-1)^5=5(n)^4-10(n)^3+10(n)^2-5(n)+1$ $n^5=5\sum_{r=1}^{n}r^4-10\sum_{r=1}^{n}r^3+10\sum_{r=1}^{n}r^2-5\sum_{r=1}^{n}r+n$ M1: Applies the result from part (a) between 1 and n and sums **both** sides Min 3 lines shown A1: Correct equation If only the last line is seen, award M1A1 These marks can be implied by a correct following stage.		M1A1
	$n^5=5\sum_{r=1}^{n}r^4-10\times\frac{1}{4}n^2(n+1)^2+10\times\frac{1}{6}n(n+1)(2n+1)-5\times\frac{1}{2}n(n+1)+n$ M1: Introduces at least 2 correct summation formulae A1: Correct equation		M1A1
	$5\sum_{r=1}^{n}r^4=\frac{5}{2}n^2(n+1)^2-\frac{5}{3}n(n+1)(2n+1)+\frac{5}{2}n(n+1)+n^5-n=...$ $5\sum_{r=1}^{n}r^4=n(n+1)\left[\frac{5}{2}n(n+1)-\frac{5}{3}(2n+1)+\frac{5}{2}+n^3-n^2+n-1\right]$ Makes $5\sum_{r=1}^{n}r^4$ or $\sum_{r=1}^{n}r^4$ the subject and takes out a factor of $n(n+1)$		M1
	$\sum_{r=1}^{n}r^4=\frac{1}{30}n(n+1)\left[15n(n+1)-10(2n+1)+15+6\left(n^3-n^2+n-1\right)\right]$ $=\frac{1}{30}n(n+1)\left[6n^3+9n^2+n-1\right]=\frac{1}{30}n(n+1)(2n+1)(...)$ Takes out a factor of $n(n+1)(2n+1)$ Depends on all previous method marks		dM1
	$=\frac{1}{30}n(n+1)(2n+1)\left(3n^2+3n-1\right)$	cao	A1
			(7)
			Total 9

Question	Scheme	Marks
1(a)	$r=\sqrt{(-4)^2+\left(-4\sqrt{3}\right)^2}=\ldots$	M1
	$\tan\theta=\frac{-4\sqrt{3}}{-4}\Rightarrow\theta=\tan^{-1}\left(\sqrt{3}\right)\pm\pi$	M1
	$8\left(\cos\left(-\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)$	A1
		(3)
(b)	$z=re^{\mathrm{i}\theta}\Rightarrow\left(re^{\mathrm{i}\theta}\right)^3=-4-4\sqrt{3}\Rightarrow r^3\left(e^{3\mathrm{i}\theta}\right)=8e^{-\mathrm{i}\frac{2\pi}{3}}$	
	$\Rightarrow r=\sqrt[3]{8}=2$	M1
	$3\theta=-\frac{2\pi}{3}(+2k\pi)\Rightarrow\theta=-\frac{2\pi}{9}+\left(\frac{2k\pi}{3}\right)$	M1
	So $z=2e^{-\frac{8\pi}{9}\mathrm{i}},2e^{-\frac{2\pi}{9}\mathrm{i}},2e^{\frac{4\pi}{9}\mathrm{i}}$	A1ft A1
		(4)
		(7 marks)

Notes:

(a)
M1: For a correct attempt at the modulus, implied by a correct modulus if no method seen and allow recovery if correct answer follows a minor slip in notation.

M1: For an attempt to find a value of θ in the correct quadrant. Accept $\tan^{-1}\left(\sqrt{3}\right)\pm\pi$ or $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\pm\pi$

May be implied by sight of an of $-\frac{2}{3}\pi,\frac{4}{3}\pi,-\frac{5}{6}\pi,\frac{7}{6}\pi$.

A1: cao as in scheme, no other solution.

(b)

M1: Applies De Moivre's Theorem and proceeds to find a value for r ie $\left(\text{their }8\right)^{\frac{1}{3}}$

M1: Proceeds to find at least one value for θ – ie their argument/3.

A1ft: At least two roots correct for their r and θ. (Must come from correct method, watch for correct roots coming from an incorrect angle due to errors.)

A1: All three correct roots and no others. Accept e.g $2e^{\mathrm{i}-\frac{8\pi}{9}}$ as a slip in notation, so allow marks.

Question	Scheme	Marks
2	$2m^2-5m-3=0\Rightarrow(2m+1)(m-3)=0\Rightarrow m=\ldots$	M1
	So C.F. is $(y_{CF}=)\,A\mathrm{e}^{-\frac{1}{2}x}+B\mathrm{e}^{3x}$	A1
	P.I. is $y_{PI}=ax\mathrm{e}^{3x}$	B1
	$\frac{\mathrm{d}y_{PI}}{\mathrm{d}x}=3ax\mathrm{e}^{3x}+a\mathrm{e}^{3x},\ \frac{\mathrm{d}^2y_{PI}}{\mathrm{d}x^2}=9ax\mathrm{e}^{3x}+3a\mathrm{e}^{3x}+3a\mathrm{e}^{3x}$ $\Rightarrow 2(9ax+6a)\mathrm{e}^{3x}-5(3ax+a)\mathrm{e}^{3x}-3ax\mathrm{e}^{3x}=2\mathrm{e}^{3x}\Rightarrow a=\ldots$	M1
	$a=\frac{2}{7}$	A1
	General solution is $y=A\mathrm{e}^{-\frac{1}{2}x}+B\mathrm{e}^{3x}+\frac{2}{7}x\mathrm{e}^{3x}$	B1ft
		(6)
		(6 marks)

Notes:

M1: Forms and solves the auxiliary equation.
A1: Correct complementary function (no need for $y=\ldots$)

B1: Correct form for the particular integral. Accept any PI that includes $ax\mathrm{e}^{3x}$, so e.g. $(ax+b)\mathrm{e}^{3x}$ is fine.

M1: Attempts to differentiate their PI twice and substitutes into the left hand side of the equation. The derivatives must be changed functions. There is no need to reach a value for the unknown(s) but their PI must contain an unknown constant.

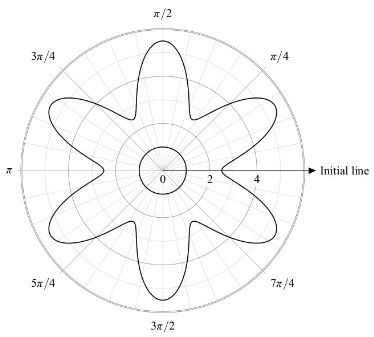
A1: Correct value of a (and any other coefficients as zero). Must have had a suitable PI

B1ft: For $y=$ their CF + their PI. Must include the $y=$. The PI must be a function of x that matches their initial choice of PI, with their constants substituted.

Question	Scheme	Marks
3(a)	Meet when $x^2-8x=\frac{4x}{4-x}\Rightarrow\left(x^2-8x\right)\left(4-x\right)=4x\Rightarrow x\left(4x-32-x^2+8x-4\right)=0$	**M1**
	(so $x=0$ or) $x^2-12x+36=0$	**A1**
	$\Rightarrow x\left(x-6\right)^2=0\Rightarrow x=...$	**M1**
	Meet at (6,–12)	**A1**
	e.g. touch at (6,–12) as repeated root.	**B1**
		(5)
Alt	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2-8x\right)=2x-8$ and $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4x}{4-x}\right)=\frac{4(4-x)-4x(-1)}{(4-x)^2}=\frac{16}{(4-x)^2}$	**M1A1**
	$2x-8=\frac{16}{(4-x)^2}\Rightarrow(x-4)^3=8\Rightarrow x=...$	**M1**
	Meet at (6,–12)	**A1**
	e.g. $6^2-6\times9=-12$ and $\frac{4\times6}{4-6}=-12$, so curves meet at tangent at (6,–12)	**B1**
		(5)
(b)	$x^2-8x=\frac{4x}{4+x}\Rightarrow x\left(x-8\right)\left(4+x\right)-4x=0\Rightarrow x\left(x^2-4x-36\right)=0\Rightarrow x=..$	**M1**
	$x=(0),2\pm2\sqrt{10}\Rightarrow$ critical value is (0 and) $2-2\sqrt{10}$	**A1**
	Other C.V.'s are 0, ±4	**B1**
	E.g. extremes are $x<2-2\sqrt{10}$ and $x>6$ or any two suitable ranges.	**M1**
	Solution is $x<2-2\sqrt{10},-4<x<0,4<x<6,x>6$	**A1A1**
		(6)
		(11 marks)

Notes:

(a)
M1: Attempts to find intersection by setting equations equal and cross multiplies and factorises the x out or cancels.
A1: Correct quadratic reached. May be implied by solutions of 0,6 seen from the cubic (by calculator)
M1: Solves the quadratic to find roots.
A1: Obtains the correct point where the curves meet.
B1: Correct reason given for why the curves touch. Accepted "repeated root" as reason. As a minimum, accept "$(x-6)^2 = 0$ therefore touches". Alternatively, accept discriminant = 0 shown with conclusion, or may find gradient at both points and show equal, with conclusion.
Alt:
M1: Attempts derivatives of both curves
A1: Both derivatives correct.
M1: Sets derivatives equal and solves to find x value where gradients agree.
A1: Obtains the correct point where the curves meet.
B1: Correct value checked in both curves with conclusion that they meet at a tangent or equivalent working as per main scheme.
(b)
M1: Attempts to find the intersection of the other branch of $\frac{4x}{4-|x|}$ with x^2-8x. Allow for any attempt at solving $\frac{4x}{4+x} = x^2 - 8x$ that reaches a value for x
A1: Correct value of $2-2\sqrt{10}$ identified. (No need to see the second root rejected for this mark.)
B1: Both 0 and ± 4 identified as critical values for the ranges needed at some stage in working.
M1: Forms at least two suitable ranges from their critical values (allow if e.g. $\leqslant$ is used instead of $<$). Likely to be the extreme ranges, so look for $x <$ their $2-2\sqrt{10}$ and $x >$ their 6. However, allow if this latter is included as part of the range $x > 4$ for this mark.
A1: At least two correct ranges.
A1: Fully correct answer as in scheme.

Question	Scheme		Marks
4(a)		Completes to a closed loop with "petals" containing circle of radius 1 (whether the circle is drawn or not)	M1
		Fully correct – 6 petals in roughly the right places, but allow if curvature is not quite smooth.	A1
		Circle centre O radius 1.	B1
			(3)
(b)	$\left(\frac{1}{2}\right)\int r^2 \, d\theta = \left(\frac{1}{2}\right)\int\left(\underline{16-12\cos 6\theta+\frac{9}{4}\cos^2 6\theta}\right) d\theta$		M1
	$=\frac{1}{2}\int_0^{2\pi}\left(16-12\cos 6\theta+\underline{\frac{9}{8}(1+\cos 12\theta)}\right) d\theta$		M1
	$=\frac{1}{2}\left[16\theta-2\sin 6\theta+\frac{9}{8}\left(\theta+\frac{1}{12}\sin 12\theta\right)\right]_0^{2\pi}$		M1 A1
	$A_{outer}=\frac{1}{2}\int_0^{2\pi} r^2 \, d\theta=\frac{1}{2}\int_0^{2\pi}\left(16-12\cos 6\theta+\frac{9}{4}\cos^2 6\theta\right) d\theta$ $=\frac{1}{2}\left(32\pi-0+\frac{9}{8}(2\pi+0)-(0)\right)$		dM1
	So Area required is $\frac{1}{2}\left(32\pi+\frac{9\pi}{4}\right)-\pi(1^2)=\ldots$		B1
	$=\frac{129}{8}\pi$		A1
			(7)
			(10 marks)

Notes:

(a)
M1: Allow for any closed loop that oscillates, though may not have the correct number of "petals" but require at least 4 . Need not have correct places of maximum radius.
A1: Fully correct sketch, 6 "petals" in the right places, with maximum radius between the 5 and 6 radius lines, minimum between the 2 and 3 radius lines.
B1: For a circle of radius 1 and centre O drawn.
(b)
M1: Attempts to square r as part of an integral for the outer curve, achieving a 3 term quadratic in $\cos 6\theta$
M1: Applies the double angle formula to the $\cos^2$ term from their expansion (not dependent on the first M, but must have a $\cos^2$ term). Accept $\cos^2 6\theta \to \frac{1}{2}(\pm 1 \pm \cos 12\theta)$
M1: Attempts to integrate, achieving the form $\alpha\theta+\beta\sin 6\theta+\gamma\sin 12\theta$ where $\alpha,\beta,\gamma \neq 0$

A1: Correct integration – limits and the $\frac{1}{2}$ not needed. Look for $16\theta - 2\sin 6\theta + \frac{9}{8}\left(\theta + \frac{1}{12}\sin 12\theta\right)$ oe.

dM1: Depends on at least two of the previous M's being scored. For a correct overall strategy for the area contained in the outer loop, with an attempt at the r^2 (should be 3 term expansion). Correct appropriate limits and the $\frac{1}{2}$ should be present or implied by working, but note variations on the scheme are possible, e.g. $2\times\frac{1}{2}\int_0^{\pi} r^2 \, \mathrm{d}\theta$, in which the $2\times\frac{1}{2}$ may be implied rather than seen.

B1: Subtracts correct area of π for inner circle

A1: cso. Check carefully the integration was correct as the sin terms disappear with the limits.

Question	Scheme	Marks
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}(4+\ln x)^{-\frac{1}{2}}\times\frac{1}{x}$	M1 A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}\frac{0-\left(\sqrt{4+\ln x}+x\times\frac{1}{2}(4+\ln x)^{-\frac{1}{2}}\times\frac{1}{x}\right)}{x^2(4+\ln x)}$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{4x}(4+\ln x)^{-\frac{3}{2}}\times\frac{1}{x}-\frac{1}{x^2}\times\frac{1}{2}(4+\ln x)^{-\frac{1}{2}}$ oe	M1
	$=\frac{\dots}{4x^2(4+\ln x)^{\frac{3}{2}}}=-\frac{9+2\ln x}{4x^2(4+\ln x)^{\frac{3}{2}}}$ *	M1 A1*
		(5)
Alt(a)	$y^2=4+\ln x\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}$	M1 A1
	$\Rightarrow 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-\frac{1}{x^2}$	M1
	$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{2yx^2}-\frac{2}{8x^2y^3}=\frac{-2y^2-1}{4x^2y^3}$	M1
	$=-\frac{9+2\ln x}{4x^2(4+\ln x)^{\frac{3}{2}}}$ *	A1*
		(5)
(b)	$y_{x=1}=2,\ \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=1}=\frac{1}{4},\ \left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right\|_{x=1}=-\frac{9}{32}$	M1
	So $y=2+\frac{1}{4}(x-1)-\frac{1}{2!}\times\frac{9}{32}(x-1)^2+\dots$	M1
	$=2+\frac{1}{4}(x-1)-\frac{9}{64}(x-1)^2+\dots$	A1
		(3)
		(8 marks)

Notes:

(a)

M1: Attempts the derivative of y using the chain rule, look for $\frac{K}{x}(4+\ln x)^{-\frac{1}{2}}$ oe

A1: Correct derivative.

M1: Attempts the second derivative of y using the product or quotient rule and chain rule. Look for the correct form for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the answer up to slips in coefficients.

M1: Attempts to simplify to get correct denominator. Must be correct work for their second derivative, but may have been errors in differentiating.

A1*: For a correct unsimplified second derivative, with no errors before reaching the given answer.

Note it is a given answer so needs a suitable intermediate line with at least the formation of the correct common denominator between two fractions before reaching the answer.

Alt:

M1: Squares and uses implicit differentiation to achieve $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\beta}{x}$

A1: Correct derivative.

M1: Differentiates again using implicit differentiation and product rule. Look for $\gamma y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\delta\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{\nu}{x^2}$

M1: Makes $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ the subject and forms single fraction with denominator kx^2y^3

A1*: Obtains the correct second derivative, with no errors seen in working.

(b)

M1: Evaluates y, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at $x = 1$, if substitution is not seen, accept stated values for all three following attempts at the first and second derivatives as an attempt to find these.

M1: Applies Taylor's theorem with their values.

A1: Correct expression (don't be concerned if the $y =$ is missing.)

5(b) Alt	$y=\sqrt{4+\ln\left(1+(x-1)\right)}=\sqrt{4+\left((x-1)-\frac{(x-1)^2}{2}+\ldots\right)}$	**M1**
	$=4^{\frac{1}{2}}+\frac{1}{2}\times4^{-\frac{1}{2}}\times\left((x-1)-\frac{(x-1)^2}{2}\right)+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times4^{-\frac{3}{2}}\times\left((x-1)-\ldots\right)^2+\ldots$	**M1**
	$=2+\frac{1}{4}(x-1)-\frac{1}{8}(x-1)^2-\frac{1}{64}(x-1)^2+\ldots=2+\frac{1}{4}(x-1)-\frac{9}{64}(x-1)^2+\ldots$	**A1**
		(3)

Notes:

M1: Writes the x as $1+(x-1)$ and attempts to expand using the Maclaurin series for $\ln(1+x)$ with correct expansion of $\ln(1+(x-1))$.

M1: Attempts a binomial expansion using their ln expansion. Alternatively, may gain this before the first M if they expand using ln's, e.g. $4^{\frac{1}{2}}+\frac{1}{2}4^{-\frac{1}{2}}\ln x+\frac{\frac{1}{2}\times\frac{-1}{2}}{2!}(\ln x)^2$

A1: Fully correct expression (don't be concerned if the $y =$ is missing.)

Question	Scheme	Marks
6(a)	Let $x = \arctan A$ and $y = \arctan B$ then $\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$ Or $\tan(\arctan A - \arctan B) = \dfrac{\tan\arctan A - \tan\arctan B}{1+\tan\arctan A\tan\arctan B}$	B1
	$\tan(x-y) = \dfrac{A-B}{1+AB} \Rightarrow x-y = \arctan\left(\dfrac{A-B}{1+AB}\right)$	M1
	So $\arctan A - \arctan B = x-y = \arctan\left(\dfrac{A-B}{1+AB}\right)$ *	A1*
		(3)
(b)	$A = r+2, B = r \Rightarrow \left(\dfrac{A-B}{1+AB} = \right)\dfrac{r+2-r}{1+(r+2)r} = \dfrac{2}{\dots}$	M1
	$= \dfrac{2}{r^2+2r+1} = \dfrac{2}{(1+r)^2}$ *	A1*
		(2)
(c)	$\sum\limits_{r=1}^{n} \arctan\left(\dfrac{2}{(1+r)^2}\right) = \sum\limits_{r=1}^{n}\left(\arctan(r+2) - \arctan(r)\right) = \dots$	M1
	$= \left(\cancel{\arctan 3} - \arctan 1\right) + \left(\cancel{\arctan 4} - \arctan 2\right) + \left(\cancel{\arctan 5} - \cancel{\arctan 3}\right) + \dots$ $+ \left(\arctan(n+1) - \cancel{\arctan(n-1)}\right) + \left(\arctan(n+2) - \cancel{\arctan n}\right)$	A1
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \arctan 1$	M1
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \dfrac{\pi}{4}$	A1
		(4)
(d)	As $n \to \infty$, $\arctan n \to \dfrac{\pi}{2}$	M1
	So $\sum\limits_{r=1}^{\infty} \arctan\left(\dfrac{2}{(1+r)^2}\right) = \dfrac{\pi}{2} + \dfrac{\pi}{2} - \arctan 2 - \dfrac{\pi}{4} = \dfrac{3\pi}{4} - \arctan 2$	A1
		(2)
		(11 marks)

Notes:
(a) **B1:** For any correct statement or use of the compound angle formula with **consistent variables** of x and y or arctan A and arctan B. Can be either way round (may be working in reverse). **M1:** Attempts to apply tan or arctan on an appropriate identity with either x and y or arctan A and arctan B. Should have $\frac{\tan x \pm \tan y}{1 \pm \tan x \tan y}$ (oe with arctans or A's and B's) as part of the identity, and allow if they change between x,y and arctan's during the step. **A1*:** Must have scored the B and M marks. Replaces tan x and tan y by A and B respectively if appropriate with fully correct work leading to the given result and conclusion made and no erroneous statements. **Note: for working in reverse e.g.** Let $x = \arctan A$ and $y = \arctan B$ then $\arctan A - \arctan B = \arctan\left(\frac{A-B}{1+AB}\right) \Leftrightarrow x - y = \arctan\left(\frac{A-B}{1+AB}\right) \Leftrightarrow \tan(x-y) = \frac{A-B}{1+AB}$ Scores M1 $\Leftrightarrow \tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$ Scores B1 – but enter as the first mark. Which is the correct identity for tan($x - y$) hence the result is true. Score A1 The conclusion here must include reference to the identity being true, e.g. with a tick, or statement, before deducing the final result. **(b)** **M1:** Substitutes in $A = r + 2$ and $B = r$ and simplifies the numerator to 2 (may be implied) **A1*:** Expands the denominator (must be seen) and then factorises to the given result, no errors seen. **(c)** **M1:** Applies the result of (a) to the series – allow if they have a different A and B due to error. **A1:** At least first three and final two brackets of terms correctly written out – must be clear enough to show cancelling. **M1:** Extracts the non-cancelling terms. **A1:** Correct result with no errors seen – must see the arctan 1 before reaching $\frac{\pi}{4}$. **Note:** Insufficient terms to gain the first A is not an error, so M1A0M1A1 is possible if e.g. only the first two terms are shown. Condone missing brackets on arctan n + 1 etc. **(d)** **M1:** Identifies the value arctan tends towards as n increase. Need not see limits, as long as the value is identified. **A1:** Correct answer.

Question	Scheme	Marks
7(a)	$z=(0+)\mathrm{i}y \Rightarrow w=\dfrac{(1+\mathrm{i})\mathrm{i}y+2(1-\mathrm{i})}{\mathrm{i}y-\mathrm{i}}=\dfrac{-y+2+\mathrm{i}(y-2)}{\mathrm{i}(y-1)}=\dfrac{y-2+\mathrm{i}(y-2)}{y-1}$	M1
	$\Rightarrow u=v$ or $\operatorname{Im} w=\operatorname{Re} w$	A1
		(2)
(b)	$w=\dfrac{(1+\mathrm{i})z+2(1-\mathrm{i})}{z-\mathrm{i}} \Rightarrow z=\dfrac{2(1-\mathrm{i})+\mathrm{i}w}{w-1-\mathrm{i}}=\dfrac{2-v+\mathrm{i}(u-2)}{u-1+\mathrm{i}(v-1)}$	M1
	$\dfrac{2-v+\mathrm{i}(u-2)}{u-1+\mathrm{i}(v-1)}\times\dfrac{u-1-\mathrm{i}(v-1)}{u-1-\mathrm{i}(v-1)}$ $=\dfrac{(2-v)(u-1)+(u-2)(v-1)+\mathrm{i}\left((u-1)(u-2)-(2-v)(v-1)\right)}{\ldots}$ $\operatorname{Im} z=0 \Rightarrow (u-1)(u-2)-(2-v)(v-1)=0$	M1
	$\Rightarrow (u-1)(u-2)-(2-v)(v-1)=0 \Rightarrow u^2-3u+2+v^2-3v+2=0$	A1
	$\Rightarrow \left(u-\dfrac{3}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2=\dfrac{1}{2}$	M1
	Centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ and radius is $\dfrac{\sqrt{2}}{2}$	A1A1
		(6)
		(8 marks)

Notes:

(a)

M1: Correct method to find the equation of the image line – e.g. substitutes in $z=\mathrm{i}y$ and rearranges to Cartesian form. May use $x+\mathrm{i}y$ and later set $x=0$. Alternatively, may start as in (b) and then set $(2-v)(u-1)+(u-2)(v-1)=0 \Rightarrow 2u-v-uv-2+uv+2-2v-u=0$ etc.

Another alternative is to find the image points of two points on the imaginary axis and to find the line from these.

A1: For $u=v$ oe equation. Accept Im w = Re w, or $x=y$ if they have set $w=x+\mathrm{i}y$.

(b)

Note: Accept work done in part (a) that is relevant to part (b) for credit if appropriate.

M1: Makes z the subject, substitutes $w=u+\mathrm{i}v$ into the equation.

M1: Multiplies the numerator by the complex conjugate of denominator **and** extracts the imaginary part and sets it equal to zero to form an equation in u and v. Do not be concerned about the denominator.

A1: Correct equation in u and v for the circle.

M1: Completes the square on their equation to extract centre and radius. Not dependent, so allow as long as a suitable equation in u and v has been reached.

A1: Correct centre or correct radius. Accept either $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ or $\left(\dfrac{3}{2},\dfrac{3}{2}\right)$ for the centre.

A1: Correct centre and correct radius. As above. Accept equivalent forms (need not be simplified)

Allow the final two A marks if all that is wrong is an error in the denominator. (M1M0A0M1A1A1 is possible.)

7(b) Alt1	Real axis is $z = x(+0\mathrm{i})$, so $u+\mathrm{i}v = \dfrac{(1+\mathrm{i})x+2(1-\mathrm{i})}{x-\mathrm{i}} = \dfrac{(1+\mathrm{i})x+2(1-\mathrm{i})}{x-\mathrm{i}} \times \dfrac{x+\mathrm{i}}{x+\mathrm{i}} =$ $\dfrac{(1+\mathrm{i})x^2+2x(1-\mathrm{i})+(\mathrm{i}-1)x+2(\mathrm{i}+1)}{x^2+1} = \dfrac{x^2+x+2+\mathrm{i}\left(x^2-x+2\right)}{x^2+1}$	**M1**
	$u = \dfrac{x^2+x+2}{x^2+1} = 1+\dfrac{x+1}{x^2+1}; v = \dfrac{x^2-x+2}{x^2+1} = 1-\dfrac{x-1}{x^2+1} \Rightarrow u+v = 2+\dfrac{2}{x^2+1}$ $\Rightarrow (u-1)^2+(v-1)^2 = \dfrac{(x+1)^2+(x-1)^2}{\left(x^2+1\right)^2} = \dfrac{2x^2+2}{\left(x^2+1\right)^2} = \dfrac{2}{x^2+1} = u+v-2$	**M1** **A1**
	$\Rightarrow \left(u-\dfrac{3}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2 = \dfrac{1}{2}$	**M1**
	Centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ and radius is $\dfrac{\sqrt{2}}{2}$	**A1A1**
		(6)

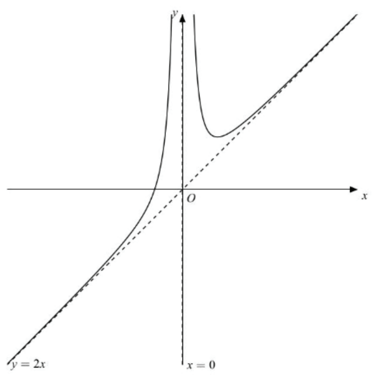
Notes

M1: Sets $z = x$ in the equation (or uses $x + \mathrm{i}y$ and later sets $y = 0$) and multiplies by complex conjugate.
M1: Eliminates x from the equations (one suitable method is shown, others are possible).
A1: Correct equation in u and v for the circle.
M1: Completes the square on their equation to extract centre and radius

A1: Correct centre or correct radius. Accept either $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ or $\left(\dfrac{3}{2},\dfrac{3}{2}\right)$ for the centre.

A1: Correct centre and correct radius. As above.

7(b) Alt 2	Unlikely to be seen As i and −i are inverse points of the line, so their images are inverse points of the circle. $\mathrm{i} \to \infty, -\mathrm{i} \to \dfrac{-\mathrm{i}+1+2-2\mathrm{i}}{-2\mathrm{i}} = \dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ Hence (as ∞ is the other point) the centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ $0 \to \dfrac{2-2\mathrm{i}}{-\mathrm{i}} = 2+2\mathrm{i}$ So radius is $\left\lvert\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}-2-2\mathrm{i}\right\rvert = \ldots$ $= \dfrac{\sqrt{2}}{2}$	**M1** **M1** **A1** **M1** **A1** **A1**
(b) Alt 3	M1: Attempt to find images of three different points on the real axis. M1: Correct method to find centre from three points – e.g. intersection of two perpendicular bisectors. A1: Correct equation for the centre. M1: Uses centre and one point to find radius. A1: Correct centre A1: Correct radius	

Question	Scheme	Marks
8(a)	$\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}x}-2$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}+2yx(y-4x)=2-8x^3 \to \frac{\mathrm{d}v}{\mathrm{d}x}+2+2(v+2x)x(v+2x-4x)=2-8x^3$	
	$\to \frac{\mathrm{d}v}{\mathrm{d}x}+2+2x(v^2-4x^2)=2-8x^3$	M1
	$\to \frac{\mathrm{d}v}{\mathrm{d}x}=-2xv^2$ *	A1*
		(4)
(b)	$\frac{1}{v^2}\frac{\mathrm{d}v}{\mathrm{d}x}=-2x \Rightarrow \int v^{-2}\,\mathrm{d}v=-2\int x\,\mathrm{d}x$	B1
	$\Rightarrow \frac{v^{-1}}{-1}=-2\frac{x^2}{2}(+c)$	M1
	$\Rightarrow \frac{1}{v}=x^2+c$	A1
	$\Rightarrow v=\frac{1}{x^2+c}$	A1
		(4)
(c)	$y=2x+\frac{1}{x^2+c}$	B1ft
		(1)
(d)	$-1=2\times-1+\frac{1}{1+c}\Rightarrow c=\ldots$	M1
	$y=2x+\frac{1}{x^2}$	A1
	 Attempts the sketch for their equation, with at least one of - One branch correct - Vertical asymptote for their equation - Long term behaviour tends to infinity - Minimum in quadrant 1	M1
	Fully correct shape, two branches tending to infinity as x tends to infinity both directions, with minimum in first quadrant No need for oblique asymptote marked.	A1
	y-axis a vertical asymptote labelled	B1ft
		(5)
		(14 marks)

Notes:

(a)

B1: Correct differentiation of the given transformation. Allow any correct connecting derivative, e.g. $\frac{\mathrm{d}y}{\mathrm{d}v}=1+2\frac{\mathrm{d}x}{\mathrm{d}v}$ or $\frac{\mathrm{d}v}{\mathrm{d}y}=1-2\frac{\mathrm{d}x}{\mathrm{d}y}$
M1: For a complete substitution into the equation (*I*).
M1: Applies difference of squares, or completely expands brackets of the left hand side. Alternatively, may rearrange and factorise to give $8x^2y-2xy^2-8x^3=-2x\left(y^2-4xy+4x^2\right)=-2x(y-2x)^2$ before substituting.
A1*: Reaches the given answer with no errors seen.
(b)
B1: Correct separation of the variables.
M1: Attempts the integration, usual rule, power increased by 1 on at least one term. No need for + *c* for the method.
A1: Correct integration including the + *c*
A1: Correct expression for *v*.
(c)
B1: Follow through their answer to (b), so $y=2x+$ their v from (b)
(d)
M1: Uses the point $(-1,-1)$ to find a value for the constant in their equation. Must have had a constant of integration in their equation to score this mark.
A1: Correct equation for *y* following a correct general solution. Withhold this mark for $y=2x+\frac{1}{x^2}+c$ leading to the correct equation.
Note: the following three marks may be scored from a correct equation that arose from having no constant in (b) or from $y=2x+\frac{1}{x^2}+c$ (which gives the same equation).
M1: Attempts a sketch for their curve. See scheme. Look for at least one of the key features for their equation shown.
A1: Correct shape, two branches tending to infinity as *x* tends to infinity both directions with a minimum in first quadrant. Not a follow through mark, so must be the correct curve.
B1ft: Correct vertical asymptote at $x = 0$. Need not be labelled if it is clearly the *y*-axis. Follow through their equation as long as there is at least one vertical asymptote (ie for a negative *c* they need a pair of asymptotes symmetric about the *y*-axis).

Question Number	Scheme	Marks
1 **(a)**	$2n+1=A(n+1)^2+Bn^2 \Rightarrow 2n+1=An^2+2An+1+Bn^2$	
	$A=1\ \ B=-1$ or $\dfrac{1}{n^2}-\dfrac{1}{(n+1)^2}$	B1 (1)
(b)	$\displaystyle\sum_{r=5}^{n}\frac{2r+1}{r^2(r+1)^2}=\sum_{r=5}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right)$ $=\left(\dfrac{1}{5^2}-\cancel{\dfrac{1}{6^2}}\right)+\left(\cancel{\dfrac{1}{6^2}}-\cancel{\dfrac{1}{7^2}}\right)+\ldots\left(\cancel{\dfrac{1}{n^2}}-\dfrac{1}{(n+1)^2}\right)$	M1
	$\displaystyle\sum_{r=5}^{n}\frac{2r+1}{r^2(r+1)^2}=\frac{1}{5^2}-\frac{1}{(n+1)^2}$	A1
	$=\dfrac{n^2+2n+1-25}{25(n+1)^2}=\dfrac{n^2+2n-24}{25(n+1)^2}$	M1A1 (4)
		[5]
	Notes	
(a) **B1**	Both values correct with or without working seen, may be in the expression. Ignore incorrect working.	
(b) **M1**	Show sufficient terms to demonstrate the cancelling. Require at least one cancelling term seen. Must start at $r=5$ - M0 if starting at e.g $r=1$ unless there is a full process to complete the difference method (same condition) **and** apply f(n) − f(4)	
A1	Extract the two correct terms, or in the Alt obtains a correct overall expression.	
M1	Write the terms with a (non-zero) common denominator with at least numerator correct for their terms. Not dependent - may be scored following M0 if no cancelling terms were shown, but must have had exactly two terms to combine from differences.	
A1	Correct answer in the required form or accept correct values stated following an unsimplified form. (Allow as long as correct terms were extracted, even if no cancelling terms were shown.) Note: this means M0A0M1A1 can be scored for answers which show only the last two lines of the scheme with no cancelling process shown. Note: if e.g. r is used in place of n allow full marks if recovered, but A0 if left in terms of r.	
Alt (b) for first two marks	$\displaystyle\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}=\sum_{r=1}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right)$ $=\left(\dfrac{1}{1^2}-\cancel{\dfrac{1}{2^2}}\right)+\left(\cancel{\dfrac{1}{2^2}}-\cancel{\dfrac{1}{3^2}}\right)+\ldots\left(\cancel{\dfrac{1}{n^2}}-\dfrac{1}{(n+1)^2}\right)=1-\dfrac{1}{(n+1)^2}$ $\displaystyle\rightarrow\sum_{r=5}^{n}\frac{2r+1}{r^2(r+1)^2}=1-\frac{1}{(n+1)^2}-\left(1-\frac{1}{25}\right)\left(=\frac{n^2+2n}{(n+1)^2}-\frac{24}{25}\right)$	M1A1

Question Number	Scheme	Marks
2 (a)	E.g. $(x+3)(x-5)=9 \Rightarrow x^2-2x-24=0 \Rightarrow x=...$ OR $(x-5)(x+3)^2-9(x+3)=0 \;\; \Rightarrow (x+3)(x-6)(x+4)=0 \Rightarrow x=...$ OR $\dfrac{(x+3)(x-5)-9}{x+3}<0 \Rightarrow x^2-2x-24=0 \Rightarrow x=...$	M1
	CVs: $6,\ -4\ ;\ -3$	A1;B1
	$x<-4, \quad -3<x<6$	dM1A1A1
	OR: $x \in (-3,6)\cup(-\infty,-4)$ or any equivalent notation.	(6)
(b)	$x<6, \quad x \neq -3$ or any equivalent notation.	B1ftB1 (2)
		[8]
	Notes	
(a) **M1**	For a correct **algebraic** method to find the intersection points of $y=x-5$ and $y=\dfrac{9}{x+3}$. May set these equal and form a quadratic and solve. May multiply through by $(x+3)^2$ and collect on one side or use any other valid method Eg work from $\dfrac{(x+3)(x+2)-12}{x+3}>0$ Answers only from a calculator score M0. Must reach at least a quadratic or cubic before answers given. Do not be concerned with the equality or inequality for this mark.	
A1	For 6, −4 obtained via a valid algebraic method.	
B1	for the CV -3 seen anywhere	
dM1	Obtaining (any) inequalities using all of their critical values and no other numbers.	
A1	For at least one correct interval allowing for … or „ used instead of < and >	
A1cso	Both correct ranges and no extras. Use of … or „ scores A0. May be written in set notation, and all work should have been correct so penalise if incorrect inequalities method was used at the start. Accept $x<-4$ and/or $-3<x<6$ with "and" or "or" For candidates who draw a sketch graph and follow with the cvs without any algebra shown only the **B** mark is available. **Those who use some algebra** after their graph may gain marks as earned (possibly all)	
(b) **B1ft**	For the "$x<6$" in some form with the possible exception of the CVs from (a). Allow x „ 6 if already penalised in (a). It is essentially for realising all the extra (valid) values less than −3 are solutions while retaining all their given solutions. If only the CVs themselves are excluded allow B1. Follow through their answer to (a).	
B1	Fully correct answer. May give as intervals $x<-3, -3<x<6$	

Question Number	Scheme	Marks
3	$w=\dfrac{z}{z+4\text{i}}$	
	$w\left(z+4\text{i}\right)=z\Rightarrow z(1-w)=4\text{i}w$ or $z=\dfrac{4\text{i}w}{1-w}$ oe	M1A1
	$\lvert z\rvert=3$ $\left\lvert\dfrac{4\text{i}w}{1-w}\right\rvert=3$	dM1
	$\lvert 4\text{i}w\rvert=3\lvert 1-w\rvert$	
	$w=u+\text{i}v$ $16\left(u^2+v^2\right)=9\left(\left(1-u\right)^2+v^2\right)$	ddM1A1
	$16u^2+16v^2=9\left(1-2u+u^2+v^2\right)$	
	$7u^2+7v^2+18u-9=0$	
	$\left(u+\dfrac{9}{7}\right)^2+v^2=\dfrac{144}{49}$	dddM1
	Centre $\left(-\dfrac{9}{7},0\right)$ Radius $\dfrac{12}{7}$	A1A1 (8)
	Notes	
(a) **M1**	re-arrange to $z=\ldots$. or an expression $z(\alpha w+\beta)=\gamma w+\delta$	
A1	correct result	
dM1	dep (on first M1) using $\lvert z\rvert=3$ with their previous result	
ddM1	dep (on both previous M marks) use $w=u+\text{i}v$ (or $w=x+\text{i}y$ or any other pair of letters) and attempts the squares of the moduli. The i's must be dealt with correctly, but allow e.g. $3^2\rightarrow 3$	
A1	for a correct equation quadratic in u and v after squaring (including squaring coefficients).	
dddM1	dep (on all previous M marks) re-arrange to the completed square form of the equation of a circle (same coeffs for the squared terms) or implied by a correct centre or radius following a correct equation with terms gathered.	
A1	either correct and exact.	
A1	both correct and exact.	
	Note: Allow recovery for the last three A's if all that is incorrect in is the wrong sign in their expression for z, ie $z=\dfrac{-4\text{i}w}{1-w}$	
	If you see alternative methods, e.g. via Apollonian approaches or attempts to use $z=x+\text{i}y$ in the original equation, that you feel are worthy of credit please use Review to consult your team leader.	

Question Number	Scheme	Marks
4	$\dfrac{dy}{dx} - 3y\tan x = e^{4x}\sec^3 x$	
(a)	$e^{-3\int \tan x\,dx} = e^{-3\ln\sec x} = \sec^{-3} x$ or $\cos^3 x$	M1A1
	$\cos^3 x\dfrac{dy}{dx} - 3y\sin x\cos^2 x = e^{4x}\cos^3 x\sec^3 x$	
	$\dfrac{d}{dx}\left(y\cos^3 x\right) = e^{4x} \Rightarrow y\cos^3 x = \int e^{4x}\,dx$	M1
	$y\cos^3 x = \dfrac{1}{4}e^{4x} \quad (+c)$	M1
	$y = \left(\dfrac{1}{4}e^{4x} + c\right)\sec^3 x$ or $y = \left(\dfrac{1}{4}e^{4x} + c\right)\cos^{-3} x$ oe	A1
		(5)
(b)	$y = 4,\quad x = 0 \quad 4 = \left(\dfrac{1}{4} + c\right)$	
	$c = \dfrac{15}{4}$	M1
	$y = \dfrac{1}{4}\left(e^{4x} + 15\right)\sec^3 x$ or $\dfrac{1}{4}\left(e^{4x} + 15\right)\cos^{-3} x$ oe	A1
		(2)
		[7]

	Notes
(a)	
M1	Attempt the integrating factor, including integration of (-3)tan x; $\ln\cos$ or $\ln\sec$ seen
A1	Correct simplified integrating factor $\sec^{-3} x$ or $\cos^3 x$
M1	Multiply the equation by the integrating factor and integrate the LHS. Look for $y \times \text{their IF} = \int\left(e^{4x}\sec^3 x \times \text{their IF}\right)dx$ (condone missing dx)
M1	Integrate RHS, constant not needed. Must be a function they can integrate and a valid attempt (e.g. allowing coefficient slips).
A1	Correct result in the demanded form, including y =.., constant included
(b)	
M1	Use the given initial conditions to obtain a value for c
A1	Fully correct final answer. Must include y = .. but allow A1 if missing and penalised in (a). May be in the form $y\cos^3 x = \ldots$ or $4y\cos^3 x = \ldots$

Question Number	Scheme	Marks
5.	$\dfrac{d^2y}{dx^2} = -\dfrac{2}{y}\left(\dfrac{dy}{dx}\right)^2 + 2$	
(a)	$\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ seen	B1
	$\dfrac{d^3y}{dx^3} = -\dfrac{4}{y}\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2} + \dfrac{2}{y^2}\left(\dfrac{dy}{dx}\right)^3$	M1A1A1
		(4)
ALT:	$\left(\dfrac{dy}{dx}\right)^2 \to 2\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ seen	B1
	$\underline{\dfrac{dy}{dx}\left(\dfrac{d^2y}{dx^2}\right) + y\dfrac{d^3y}{dx^3}} + 4\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^2y}{dx^2}\right) \underline{-2\dfrac{dy}{dx}} = 0$	M1<u>A1</u>
	$\dfrac{d^3y}{dx^3} = \dfrac{1}{y}\left(-5\dfrac{d^2y}{dx^2} + 2\right)\dfrac{dy}{dx}$	A1 (4)
(b)	At $x = 0$ $\dfrac{d^2y}{dx^2} = \dfrac{1}{2}\left(-2\times(1)^2 + 4\right) = 1$	B1
	$\dfrac{d^3y}{dx^3} = \dfrac{1}{2}(-5\times1+2)\times1 = \dfrac{-3}{2}$	M1
	$(y =)2 + x + (1)\dfrac{x^2}{2!} + \left(\dfrac{-3}{2}\right)\dfrac{x^3}{3!} + \ldots$	M1
	$y = 2 + x + \dfrac{1}{2}x^2 - \dfrac{1}{4}x^3 + \ldots$	A1 (4)
		[8]

	Notes
(a)	
B1	$\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ seen in the differentiation
M1	Divide equation by y and differentiate wrt x chain and product rules needed. LHS correct
A1	Either RHS term correct. Need not be simplified.
A1	Both RHS terms correct. Need not be simplified.
ALT	
B1	$\left(\dfrac{dy}{dx}\right)^2 \to 2\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ correct differentiation of middle term.
M1	Differentiate before dividing. Product rule must be used.
A1	Correct differentiation of $y\dfrac{d^2y}{dx^2}$ and $-2y$
A1	Rearrange to a correct expression for $\dfrac{d^3y}{dx^3}$ (need not be simplified)

	Notes
(b)	
B1	Correct value for $\frac{d^2y}{dx^2}$. May be implied by the term in their expansion.
M1	Use their expression from (a) to obtain a value for $\frac{d^3y}{dx^3}$ (May be implied - you may need to check if their value follows from their expression in (a).)
M1	Taylor's series formed using their values for the derivatives, accept 2! or 2 and 3! or 6
A1	Correct series, must start $y = \ldots$, or allow $f(x) = \ldots$ as longs $y = f(x)$ has been defined in the question. Must come from a correct expression for $\frac{d^3y}{dx^3}$

Question Number	Scheme	Marks
6 (a)	$\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta}=4a\cos\theta+4a\cos^2\theta-4a\sin^2\theta$ or $4a\cos\theta+4a\cos2\theta$ oe (Or allow $\frac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta}=-4a\sin\theta-8a\cos\theta\sin\theta$ or $-4a\sin\theta-4a\sin2\theta$)	M1
	E.g. $4a\cos\theta+4a\cos^2\theta-4a\sin^2\theta=0\Rightarrow\cos\theta+\cos^2\theta-(1-\cos^2\theta)=0$	M1
	$2\cos^2\theta+\cos\theta-1=0$ terms in any order	A1
	$(2\cos\theta-1)(\cos\theta+1)=0\Rightarrow\cos\theta=...$	ddM1
	$\left(\cos\theta=\frac{1}{2}\Rightarrow\right)\theta=\frac{\pi}{3}$ $(\theta=\pi$ need not be seen$)$	A1
	$r=4a\times\frac{3}{2}=6a$	A1 (6)
(b)	Area $=\frac{1}{2}\int r^2\mathrm{d}\theta=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}16a^2(1+\cos\theta)^2\,\mathrm{d}\theta$	
	$=\frac{16a^2}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(1+2\cos\theta+\cos^2\theta\right)\mathrm{d}\theta$	M1
	$=8a^2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(1+2\cos\theta+\frac{1}{2}(\cos2\theta+1)\right)\mathrm{d}\theta$	M1
	$=8a^2\left[\theta+2\sin\theta+\frac{1}{2}\left(\frac{1}{2}\sin2\theta+\theta\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	dM1A1
	$8a^2\left[\frac{\pi}{3}+\sqrt{3}+\frac{1}{4}\times\frac{\sqrt{3}}{2}+\frac{\pi}{6}-\left(\frac{\pi}{6}+1+\frac{1}{4}\times\frac{\sqrt{3}}{2}+\frac{\pi}{12}\right)\right]$	A1
	$8a^2\left[\frac{\pi}{4}+\sqrt{3}-1\right]$	
	Area $R=8a^2\left[\frac{\pi}{4}+\sqrt{3}-1\right]-6a^2\left(1+\frac{\sqrt{3}}{2}\right)=a^2\left(2\pi+5\sqrt{3}-14\right)$	M1A1 (7)
		[13]
	Notes	
(a) **M1**	Attempt the differentiation of $r\sin\theta$ using product rule or $\sin2\theta=2\sin\theta\cos\theta$ **OR** for this mark only allow differentiation of r cos θ, inc use of product rule, chain rule or $\cos^2\theta=\frac{1}{2}(1\pm\cos2\theta)$ Allow errors in coefficients as long as the form is correct.	
M1	Sets their derivative of $r\sin\theta$ equal to zero and achieves a quadratic expression in $\cos\theta$	
A1	Correct 3 term quadratic in $\cos\theta$ (any multiple, including a)	
ddM1	Dep on both M marks. Solve their quadratic (usual rules) giving one or two roots	
A1	Correct quadratic solved to give $\theta=\frac{\pi}{3}$	

	Notes
A1*	Correct r obtained from an intermediate step. Accept as shown in scheme, or $r = 4a\left(1+\cos\frac{\pi}{3}\right) = 6a$ or equivalent in stages. No need to see coordinates together in brackets
(b) **M1** **M1** **dM1** **A1** **A1** **M1** **A1**	Note : first 4 marks of (b) do not require limits. Use of correct area formula, $\frac{1}{2}$ may be seen later, inc squaring the bracket to obtain 3 terms - limits need not be shown. Use double angle formula (formula to be of form $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$) to obtain an integrable function - limits need not be shown, $\frac{1}{2}$ from area formula may be missing, Attempt the integration $\cos\theta \to \pm k\sin\theta$ and $\cos 2\theta \to \pm m\sin 2\theta$ - limits not needed – dep on 2nd M mark but not the first. Note if only two terms arise from squaring allow for $\cos 2\theta \to \pm m\sin 2\theta$ Correct integration – substitution of limits not required (NB Not follow through) Include the $\frac{1}{2}$ and substitute the correct limits in a correct integral. Note may be attempted via integral from 0 to $\frac{\pi}{3}$ minus integral from 0 to $\frac{\pi}{6}$ - but attempts at sector formula for the latter is A0. Attempt the area of the triangle - accept valid attempt even if not subtracted from area. E.g. attempts $\frac{1}{2}OA.OB\sin\frac{\pi}{6}$ Correct final answer in the demanded the form.

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$ or $\frac{dv}{dx} = x^{-1}\frac{dy}{dx} - x^{-2}y$ (oe)	M1A1
	$\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x\frac{d^2v}{dx^2}$ or $\frac{d^2v}{dx^2} = -x^{-2}\frac{dy}{dx} + x^{-1}\frac{d^2y}{dx^2} + 2x^{-3}y - x^{-2}\frac{dy}{dx}$ (oe)	dM1A1
	$3\left(2\frac{dv}{dx} + x\frac{d^2v}{dx^2}\right) - \frac{6}{x}\left(v + x\frac{dv}{dx}\right) + \frac{6xv}{x^2} + 3xv = x^2$ (oe in reverse)	ddM1
	$3x\frac{d^2v}{dx^2} + 6\frac{dv}{dx} - 6\frac{dv}{dx} - \frac{6}{x}v + \frac{6v}{x} + 3xv = x^2$	
	$3\frac{d^2v}{dx^2} + 3v = x$ *	A1 * (6)
(b)	$3\lambda^2 + 3 = 0$ so $\lambda = \pm i$	M1
	$(v =)\, Ae^{ix} + Be^{-ix}$ or $(v =)\, C\cos x + D\sin x$	A1
	P.I: Try $(v =)\, kx\ (+l)$	B1
	$\frac{dv}{dx} = k$ $\frac{d^2v}{dx^2} = 0$	
	$3 \times 0 + 3(kx\ (+l)) = x$	M1
	$k = \frac{1}{3}$ $(l = 0)$	
	$v = Ae^{ix} + Be^{-ix} + \frac{1}{3}x$ or $v = C\cos x + D\sin x + \frac{1}{3}x$	A1
	$y = x\left(Ae^{ix} + Be^{-ix} + \frac{1}{3}x\right)$ or $y = x\left(C\cos x + D\sin x + \frac{1}{3}x\right)$	B1ft (6)
		[12]
	Notes	
(a) **M1**	Attempt to find a relevant first derivative from $y = xv$ e.g to get $\frac{dy}{dx}$ or $\frac{dv}{dx}$ - product or quotient rule must be used. Methods via $\frac{d..}{dv}$ would require a chain rule to reach a relevant derivative.	
A1	Correct derivative	
dM1	Attempt to differentiate **their** $\frac{dy}{dx}$ or $\frac{dv}{dx}$ to obtain an expression for $\frac{d^2y}{dx^2}$ or $\frac{d^2v}{dx^2}$ - product rule must be used. Depends on the previous M mark	
A1	Correct expression for $\frac{d^2y}{dx^2}$ or $\frac{d^2v}{dx^2}$	

	Notes
ddM1 **A1***	Depends on both previous M marks. Substitute **their** $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $y = xv$ in the original equation to obtain a differential equation in v and x. Alternatively substitute **their** $\frac{\mathrm{d}v}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$ and $v = \frac{y}{x}$ into equation (II) to obtain a differential equation in y and x Obtain the **given** equation/original equation with no errors in the working. There must be at least one step shown between the initial substitution and the result
(b) **M1** **A1** **B1** **M1** **A1** **B1ft**	 Forms correct AE and attempts to solve (accept $3m^2 + 3$ (=0) leading to any value(s)). Correct CF. Suitable form for PI (ie one that include kx) Differentiate their PI twice and substitute their derivatives in the equation $3\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 3v = x$ Obtain the correct result (either form). Must be $v =$… Reverse the substitution. Follow through their previous line. Must be $y =$..

Question Number	Scheme	Marks
8 (a)	$(\cos\theta+\mathrm{i}\sin\theta)^5=\cos5\theta+\mathrm{i}\sin5\theta$	B1
	$=\cos^5\theta+5\cos^4(\mathrm{i}\sin\theta)+\frac{5\times4}{2!}\cos^3\theta(\mathrm{i}\sin\theta)^2$ $+\frac{5\times4\times3}{3!}\cos^2\theta(\mathrm{i}\sin\theta)^3+\frac{5\times4\times3\times2}{4!}\cos\theta(\mathrm{i}\sin\theta)^4+(\mathrm{i}\sin\theta)^5$	M1
	$=\cos^5\theta+\underline{5\mathrm{i}\cos^4\theta\sin\theta}-10\cos^3\theta\sin^2\theta\underline{-10\mathrm{i}\cos^2\theta\sin^3\theta}+5\cos\theta\sin^4\theta\underline{+\mathrm{i}\sin^5\theta}$	A1
	$\sin5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$ $=5(1-\sin^2\theta)^2\sin\theta-10(1-\sin^2\theta)\sin^3\theta+\sin^5\theta\frac{\mathrm{d}y}{\mathrm{d}x}$ $=5(1-2\sin^2\theta+\sin^4\theta)\sin\theta-10(1-\sin^2\theta)\sin^3\theta+\sin^5\theta$	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1* (5)
	Alternative: Using "$z-\frac{1}{z}$" $z^5-\frac{1}{z^5}=2\mathrm{i}\sin5\theta$ oe	B1
	Binomial expansion of $\left(z-\frac{1}{z}\right)^5$	M1
	$32\sin^5\theta=2\sin5\theta-10\sin3\theta+20\sin\theta$	A1
	Uses double angle formulae etc to obtain $\sin3\theta=3\sin\theta-4\sin^3\theta$ and then use it in their expansion	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1* (5)
(b)	Let $x=\sin\theta$ $16x^5-20x^3+5x=-\frac{1}{5}\Rightarrow\sin5\theta=...$	M1A1
	$\Rightarrow\theta=\frac{1}{5}\sin^{-1}\left(\pm\frac{1}{5}\right)=38.306$ (or $-2.307, 69.692. 110.306, 141.693, 182.306$)	dM1
	(or in radians $-0.0402…0.6685..., 1.216…,1.925…, 2.473…$)	
	Two of (awrt) $x=\sin\theta=-0.963,\ -0.555,\ -0.040,\ 0.620,\ 0.938$	A1
	All of (awrt) $x=\sin\theta=-0.963,\ -0.555,\ -0.040,\ 0.620,\ 0.938$	A1 (5)
(c)	$\int_0^{\frac{\pi}{4}}(4\sin^5\theta-5\sin^3\theta-6\sin\theta)\mathrm{d}\theta=\left(\int_0^{\frac{\pi}{4}}\frac{1}{4}(\sin5\theta-5\sin\theta)-6\sin\theta\right)\mathrm{d}\theta$	M1
	$=\left[\frac{1}{4}\left(-\frac{1}{5}\cos5\theta+5\cos\theta\right)+6\cos\theta\right]_0^{\frac{\pi}{4}}\left(=\left[-\frac{1}{20}\cos5\theta+\frac{29}{4}\cos\theta\right]_0^{\frac{\pi}{4}}\right)$	A1
	$\frac{1}{4}\left[-\frac{1}{5}\cos\frac{5\pi}{4}+5\cos\frac{\pi}{4}-\left(-\frac{1}{5}+5\right)\right]+6\cos\frac{\pi}{4}-6$	
	$=\frac{1}{4}\left[\frac{1}{5}\times\frac{1}{Ö2}+\frac{5}{Ö2}-4\frac{4}{5}\right]+\frac{6}{\sqrt{2}}-6$	dM1
	$=\frac{73\sqrt{2}}{20}-\frac{36}{5}$ oe	A1 (4)
		[14]
	Notes	

(a)	
B1	Applies de Moivre correctly. Need not see full statement, but must be correctly applied.
M1	Use binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$ May only show imaginary parts - ignore errors in real parts. Binomial coefficients must be evaluated.
A1	Simplify coefficients to obtain a simplified result with all imaginary terms correct
M1	Equate imaginary parts and obtain an expression for $\sin 5\theta$ in terms of powers of $\sin\theta$ No $\cos\theta$ now
A1*	Correct **given** result obtained from fully correct working with at least one intermediate line wit the $(1-\sin^2\theta)^2$ expanded. Must see both sides of answer (may be split across lines). A0 if equating of imaginary terms is not clearly implied.
(b)	Note Answers only with no working score no marks as the "hence" has not been used. But if the first M1A1 gained then dM1 may be implied by a correct answer.
M1	Use substitution $x = \sin\theta$ and attempts to use the result from (a) to obtain a value for $\sin 5\theta$
A1	Correct value for $\sin 5\theta$
dM1	Proceeds to apply arcsin and divide by 5 to obtain at least one value for θ. Note for $\sin 5\theta = \frac{1}{5}$ the values you may see are the negatives of the true answers. FYI: $(5\theta = -11.53...,191.53..., 348.46..., 551.53..., 708.46..., 911.53...)$ (Or in radians $-0.201\ldots$ 3.3428..., 6.0819..., 9.6260..., 12.365..., 15.909…)
A1	Proceeds to take sin and achieve at least 2 different correct values for x or $\sin\theta$
A1	For all 5 values of x or $\sin\theta$ awrt 3 d.p. (allow 0.62 and – 0.04)
(c)	
M1	Use previous work to change the integrand into a function that can be integrated
A1	Correct result after integrating. Any limits shown can be ignored
dM1	Substitute given limits, subtracts and uses exact numerical values for trig functions
A1	Final answer correct (oe provided in the given form)

Question Number	Scheme	Notes	Marks
1(a)	$y=\ln(5+3x)\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{5+3x}$	Correct first derivative	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{5+3x}\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{9}{(5+3x)^2}\Rightarrow\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{54}{(5+3x)^3}$ **M1**: Continues the process of differentiating and reaches $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{k}{(5+3x)^3}$ oe Note this may be achieved via the quotient rule e.g. $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{-9\times-2\times3(5+3x)}{(5+3x)^4}$ **A1**: Correct **simplified** third derivative. Allow e.g. $\dfrac{54}{(5+3x)^3}$ or $54(5+3x)^{-3}$.		M1 A1
			(3)
(b)	$y_0=\ln 5,\ y_0'=\dfrac{3}{5},\ y_0''=-\dfrac{9}{25},\ y_0'''=\dfrac{54}{125}$ $\Rightarrow\ln(5+3x)\approx\ln 5+\dfrac{3}{5}x-\dfrac{9}{25}\dfrac{x^2}{2!}+\dfrac{54}{125}\dfrac{x^3}{3!}+\ldots$ Attempts all values at $x=0$ and applies Maclaurin's theorem. Evidence for attempting the values can be taken from at least 2 terms. The form of the expansion must be correct including the factorials or their values. Note that this is "Hence" and so do not allow other methods e.g. Formula Book.		M1
	$\ln(5+3x)\approx\ln 5+\dfrac{3}{5}x-\dfrac{9}{50}x^2+\dfrac{9}{125}x^3+\ldots$ Correct expansion. The "$\ln(5+3x)=$" is not required.		A1
			(2)
(c)	$\ln(5-3x)\approx\ln 5-\dfrac{3}{5}x-\dfrac{9}{50}x^2-\dfrac{9}{125}x^3+\ldots$ Correct expansion even if obtained "from scratch" **OR** for a correct follow through with signs changed on the coefficients of the odd powers of x only in an expansion of the correct form e.g. a polynomial in ascending powers of x.		B1ft
			(1)
(d)	$\ln\dfrac{(5+3x)}{(5-3x)}=\ln(5+3x)-\ln(5-3x)$ $\ln 5+\dfrac{3}{5}x-\dfrac{9}{50}x^2+\dfrac{9}{125}x^3+\ldots-\left(\ln 5-\dfrac{3}{5}x-\dfrac{9}{50}x^2-\dfrac{9}{125}x^3+\ldots\right)$ Subtracts **their** 2 **different** series to obtain at least 2 non-zero terms in ascending powers of x.		M1
	$=\dfrac{6}{5}x+\dfrac{18}{125}x^3+\ldots$ Correct terms. Allow e.g. $0+\dfrac{6}{5}x+0x^2+\dfrac{18}{125}x^3+\ldots$		A1

	Allow both marks to score in (d) provided the **correct** series have been obtained in (b) and (c) by **any** means.		
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
2(a)	$\dfrac{1}{(2n-1)(2n+1)(2n+3)} \equiv \dfrac{A}{2n-1}+\dfrac{B}{2n+1}+\dfrac{C}{2n+3}$ $\Rightarrow A=...,B=...,C=...$ Correct partial fraction attempt to obtain values for A, B and C		M1
	$\dfrac{1}{8(2n-1)}-\dfrac{1}{4(2n+1)}+\dfrac{1}{8(2n+3)}$ or e.g $\dfrac{1}{16n-8}-\dfrac{1}{8n+4}+\dfrac{1}{16n+24}$ or e.g. $\dfrac{\frac{1}{8}}{(2n-1)}-\dfrac{\frac{1}{4}}{(2n+1)}+\dfrac{\frac{1}{8}}{(2n+3)}$ Correct **partial fractions**. (May be seen in (b)) This mark is **not** for the correct values of A, B and C, it is for the correct fractions.		A1
			(2)
(b)	$\dfrac{1}{8}\sum_{r=1}^{n}\left(\dfrac{1}{2r-1}-\dfrac{2}{2r+1}+\dfrac{1}{2r+3}\right)=$ $\dfrac{1}{8}\left(\dfrac{1}{1}-\dfrac{2}{3}+\cancel{\dfrac{1}{5}}\right.$ $+\dfrac{1}{3}\cancel{-\dfrac{2}{5}}+\cancel{\dfrac{1}{7}}$ $\cancel{+\dfrac{1}{5}}\cancel{-\dfrac{2}{7}}+\cancel{\dfrac{1}{9}}$ . . $+\cancel{\dfrac{1}{2n-3}}\cancel{-\dfrac{2}{2n-1}}+\dfrac{1}{2n+1}$ $\left.+\cancel{\dfrac{1}{2n-1}}-\dfrac{2}{2n+1}+\dfrac{1}{2n+3}\right)$ Uses the method of differences to find sufficient terms to establish cancelling. E.g. 3 rows at the start and 2 rows at the end or vice versa This may be implied if they extract the correct non-cancelling terms.		M1
	$=\dfrac{1}{8}\left(1-\dfrac{1}{3}-\dfrac{1}{2n+1}+\dfrac{1}{2n+3}\right)$ or e.g. $=\dfrac{1}{8}\left(1-\dfrac{2}{3}+\dfrac{1}{3}+\dfrac{1}{2n+1}-\dfrac{2}{2n+1}+\dfrac{1}{2n+3}\right)$ Identifies the correct non-cancelling terms. May be unsimplified.		A1
	$=\dfrac{1}{8}\left(\dfrac{2(2n+1)(2n+3)-3(2n+3)+3(2n+1)}{3(2n+1)(2n+3)}\right)=...$ Attempts to combine terms into one fraction. There must have been at least one constant term and at least 2 different algebraic terms with at least 3 terms in the numerator when combining the fractions.		dM1

	$= \dfrac{n(n+2)}{3(2n+1)(2n+3)}$	Cao	A1
			(4)
			Total 6

Question Number	Scheme	Notes	Marks
3(a)	$x^2\frac{dy}{dx}+xy=2y^2 \qquad y=\frac{1}{z}$		
	$\frac{dy}{dx}=-\frac{1}{z^2}\frac{dz}{dx}$	Correct differentiation	B1
	$-\frac{x^2}{z^2}\frac{dz}{dx}+\frac{x}{z}=\frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{dz}{dx}-\frac{z}{x}=-\frac{2}{x^2}$ *	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
			(3)
(a) Way 2	$y=\frac{1}{z}\Rightarrow zy=1\Rightarrow y\frac{dz}{dx}+z\frac{dy}{dx}=0$	Correct differentiation	B1
	$-\frac{y}{z}x^2\frac{dz}{dx}+\frac{x}{z}=\frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{dz}{dx}-\frac{z}{x}=-\frac{2}{x^2}$ *	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(a) Way 3	$y=\frac{1}{z}\Rightarrow z=\frac{1}{y}\Rightarrow\frac{dz}{dx}=-\frac{1}{y^2}\frac{dy}{dx}$	Correct differentiation	B1
	$-\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{xy}=-\frac{2}{x^2}$	Substitutes into differential equation (II)	M1
	$x^2\frac{dy}{dx}+xy=2y^2$	Obtains differential equation (I) with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(b)	$I=e^{-\int\frac{1}{x}dx}=e^{-\ln x}=\frac{1}{x}$	Correct integrating factor of $\frac{1}{x}$	B1
	$\frac{z}{x}=-\int\frac{2}{x^3}dx$	For $Iz=-\int\frac{2I}{x^2}dx$. Condone the "dx" missing.	M1
	$\frac{z}{x}=\frac{1}{x^2}+c$	Correct equation including constant	A1
	$z=\frac{1}{x}+cx$	Correct equation in the required form	A1
			(4)
(c)	$\frac{1}{y}=\frac{1}{x}+cx\Rightarrow-\frac{8}{3}=\frac{1}{3}+3c\Rightarrow c=-1$	Reverses the substitution and uses the given conditions to find their constant	M1

	$\frac{1}{y}=\frac{1}{x}-x \Rightarrow y=\frac{x}{1-x^2}$	Correct equation for y in terms of x. Allow any correct equivalents e.g. $y=\frac{1}{x^{-1}-x}$, $y=\frac{1}{\frac{1}{x}-x}$	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
4(a)	$\frac{dy}{dx} = y^2 - x \Rightarrow \frac{d^2y}{dx^2} = 2y\frac{dy}{dx} - 1$	Correct expression for $\frac{d^2y}{dx^2}$	B1
	$\frac{d^3y}{dx^3} = 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2$ **M1**: Applies the product rule to obtain $\frac{d^3y}{dx^3} = Ay\frac{d^2y}{dx^2} + \ldots$ or $\frac{d^3y}{dx^3} = \ldots + B\left(\frac{dy}{dx}\right)^2$ where … is non-zero **A1**: Correct expression. Apply isw if necessary.		M1 A1
	$\frac{d^3y}{dx^3} = 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 \Rightarrow \frac{d^4y}{dx^4} = 2y\frac{d^3y}{dx^3} + 2\frac{dy}{dx}\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\frac{d^2y}{dx^2}$ $\frac{d^4y}{dx^4} = 2y\frac{d^3y}{dx^3} + 6\frac{dy}{dx}\frac{d^2y}{dx^2}$ Correct expression for $\frac{d^4y}{dx^4}$ or correct values for A and B.		A1
	Note: If e.g. $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$ is obtained, allow recovery in (a) so B0M1A1A1 is possible.		
			(4)
(b)	$(y)_{-1} = 1, (y')_{-1} = 2, (y'')_{-1} = 3, (y''')_{-1} = 14, (y'''')_{-1} = 64$ Attempts the values up to at least the 3rd derivative using $x = -1$ and $y = 1$ Condone slips provided the intention is clear. May be implied by their values.		M1
	$(y =)1 + 2(x+1) + \frac{3(x+1)^2}{2} + \frac{14(x+1)^3}{3!} + \frac{64(x+1)^4}{4!} + \ldots$ Correct application of the Taylor series in powers of $(x + 1)$ If the expansion is just written down with no formula quoted then it must be correct for their values. E.g. $y = -1 + \ldots$ with no evidence $y = f(-1)$ was meant scores M0		M1
	$(y =)1 + 2(x+1) + \frac{3(x+1)^2}{2} + \frac{7(x+1)^3}{3} + \frac{8(x+1)^4}{3} + \ldots$ Correct simplified expansion. The "$y =$" is not required.		A1
			(3)
			Total 7

Question 5
General Guidance

B1: This mark is for sight of -8 seen as part of their working. It may be seen as e.g. embedded in an inequality, as part of their solution if they consider for example $x > -8$, $x < -8$ or -8 is seen in a sketch etc.
Do not allow for just e.g. $x + 8 > 0$,

M1: Any valid attempt to find at least one critical value other than $x = -8$ (see below).
Condone use of e.g. "=", ">", "<" etc as part of their working.
Note these usually come in pairs as $3, -\frac{19}{3}$ or $3, -13$

M1: A valid attempt to find all critical values.
Condone use of e.g. "=", ">", "<" etc as part of their working.

A1: Any 2 critical values other than $x = -8$. May be seen embedded in an inequality or on a sketch.

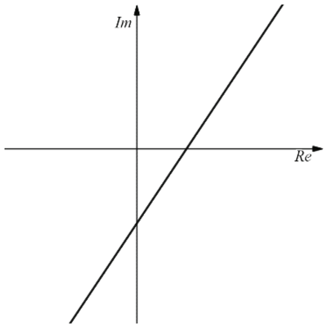
A1: 2 correct regions

A1: All correct with no extra regions

Question Number	Scheme	Notes	Marks
5	$(x=)-8$	This cv stated or used	B1
	For cv's $3,\ -\frac{19}{3}$	**OR** For cv's $3,\ -13$	
	Examples: $x^2-9=(x+8)(6-2x)\Rightarrow x=...$ **or** $(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $3x^2+10x-57=0$	Examples: $x^2-9=-(x+8)(6-2x)\Rightarrow x=...$ **or** $-(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{-(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $x^2+10x-39=0$	M1
	For cv's $3,\ -\frac{19}{3}$	**AND** For cv's $3,\ -13$	
	Examples: $x^2-9=(x+8)(6-2x)\Rightarrow x=...$ **or** $(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $3x^2+10x-57=0$	Examples: $x^2-9=-(x+8)(6-2x)\Rightarrow x=...$ **or** $-(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{-(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $x^2+10x-39=0$	M1
	Any two of: $x=-13,\ -\frac{19}{3},\ 3$	For any two of these cv's. May be seen embedded in their inequalities. **Depends on at least one previous M mark.**	A1
	$-13<x<-8,\ -8<x<-\frac{19}{3},\ x>3$ **A1**: Any 2 of these inequalities. Note that $-13<x<-\frac{19}{3},\ x\neq-8$ would count as 2 correct inequalities. Also condone $-13<x<-\frac{19}{3},\ x>3$ as 2 correct inequalities. **Depends on at least one previous M mark.** **A1**: All correct and no other regions. **Depends on all previous marks.**		A1 A1
	Allow equivalent notation for the inequalities e.g. for $-13<x<-8$ allow $x>-13$ and $x<-8$, $x>-13,\ x<-8,\ -8>x>-13, (-13,-8), \{x:x>-13\cap x<-8\}$ But not $x>-13$ or $x<-8$ Note that $-13<x<-\frac{19}{3},\ x\neq-8,\ x>3$ is fully correct.		
			(6)
			Total 6

Note that it is possible to find all the cv's by squaring both sides of the equation:

	$(x=)-8$	This cv stated or used	B1
	$\frac{(x^2-9)^2}{(x+8)^2}=(6-2x)^2 \Rightarrow x^4-18x^2+81=(36-24x+4x^2)(x^2+16x+64)$ $\Rightarrow 3x^4+40x^3-74x^2-960x+2223=0 \Rightarrow x=...$ M2 Requires a complete attempt to square both sides, multiply up to obtain a quartic equation and an attempt to solve to find at least 1 critical value other than $x=-8$		M1M1
	Any two of: $x=-13,\ -\frac{19}{3},\ 3$	For any two of these cv's. May be seen embedded in their inequalities. **Depends on both previous M marks.**	A1
	$-13<x<-8,\ -8<x<-\frac{19}{3},\ x>3$ **A1**: Any 2 of these inequalities. Note that $-13<x<-\frac{19}{3},\ x\neq-8$ would count as 2 correct inequalities. Also condone $-13<x<-\frac{19}{3},\ x>3$ as 2 correct inequalities. **Depends on at least one previous M mark.** **A1**: All correct and no other regions. **Depends on all previous marks.**		A1 A1
	Allow equivalent notation for the inequalities e.g. for $-13<x<-8$ allow $x>-13$ and $x<-8$, $x>-13,\ x<-8,\ -8>x>-13, (-13,-8),\ \{x: x>-13 \cap x<-8\}$ But not $x>-13$ or $x<-8$ Note that $-13<x<-\frac{19}{3},\ x\neq-8,\ x>3$ is fully correct.		

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="3">6(a)</td><td rowspan="2">
</td><td>A straight line anywhere that is not vertical or horizontal which does not pass through the origin. It may be solid or dotted. Clear “V” shapes score M0.</td><td>M1</td></tr>
<tr><td>A straight line in the correct position. Must have a positive gradient and lie in quadrants 1, 3 and 4. Ignore any intercepts correct or incorrect. If there are other lines that are clearly “construction” lines e.g. a line from 2i to 3 they can be ignored. The line may be solid or dotted. However, if there are clearly several lines then score A0.</td><td>A1</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
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Part (b)

The approaches below are the ones that have been seen most often.
Apply the mark scheme to the overall method the candidate has chosen.
There may be several attempts:

- **If none are crossed out, mark all attempts and score the best single complete attempt**
- **If some attempts are crossed out, mark the uncrossed out work**
- **If everything is crossed out, mark all the work and score the best single complete attempt**

Note that the question does not specify the variables the candidates should work in so they may use: e.g. $z = x + \mathrm{i}y$ and $w = u + \mathrm{i}v$ or $w = x + \mathrm{i}y$ and $z = u + \mathrm{i}v$ or any other letters so please check the work carefully.

Note that the M marks are all dependent on each other.

(b) **Way 1**	$w=\dfrac{\mathrm{i}z}{z-2\mathrm{i}}\Rightarrow z=\dfrac{2w\mathrm{i}}{w-\mathrm{i}}$	Attempts to make z the subject. Must obtain the form $\dfrac{aw\mathrm{i}}{bw+c\mathrm{i}}$, a, b, c real and non-zero.	M1
	$z=\dfrac{2(u+\mathrm{i}v)\mathrm{i}}{u+\mathrm{i}v-\mathrm{i}}$ or e.g. $z=\dfrac{2(x+\mathrm{i}y)\mathrm{i}}{x+\mathrm{i}y-\mathrm{i}}$ $z=\dfrac{2(u+\mathrm{i}v)\mathrm{i}}{u+(v-1)\mathrm{i}}\times\dfrac{u-(v-1)\mathrm{i}}{u-(v-1)\mathrm{i}}$ or equivalent Introduces $w=u+\mathrm{i}v$ or e.g. $w=x+\mathrm{i}y$ and attempts to multiply numerator and denominator by the complex conjugate of the denominator. The above statement would be sufficient e.g. no expansion is needed for this mark.		**d**M1
	$z=\dfrac{-2u}{u^2+(v-1)^2}+\dfrac{2u^2+2v(v-1)}{u^2+(v-1)^2}\mathrm{i}$ or e.g. $z=\dfrac{-2x}{x^2+(y-1)^2}+\dfrac{2x^2+2y(y-1)}{x^2+(y-1)^2}\mathrm{i}$ or $z=\dfrac{-2uv+2u(v-1)+(2u^2+2v(v-1))\mathrm{i}}{u^2+(v-1)^2}$ or e.g. $z=\dfrac{-2xy+2x(y-1)+(2x^2+2y(y-1))\mathrm{i}}{x^2+(y-1)^2}$ Correct expression for z in terms of their variables with real and imaginary parts identified. May be embedded as above or stated explicitly.		A1
	$\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert\Rightarrow y-1=\dfrac{3}{2}\left(x-\dfrac{3}{2}\right)\left(y=\dfrac{3}{2}x-\dfrac{5}{4},\ 6x-4y=5\right)$ $\Rightarrow\dfrac{2u^2+2v(v-1)}{u^2+(v-1)^2}-1=\dfrac{3}{2}\left(\dfrac{-2u}{u^2+(v-1)^2}-\dfrac{3}{2}\right)$ Attempts the Cartesian equation of the locus of z and substitutes for x and y or equivalent using their variables to obtain an equation in u and v (or their variables). Condone slips with the locus of z but must be a linear equation in any form but with a non-zero constant term. **or** $\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert\Rightarrow\left\lvert\dfrac{-2u}{u^2+(v-1)^2}+\dfrac{2u^2+2v(v-1)}{u^2+(v-1)^2}\mathrm{i}-2\mathrm{i}\right\rvert=\left\lvert\dfrac{-2u}{u^2+(v-1)^2}+\dfrac{2u^2+2v(v-1)}{u^2+(v-1)^2}\mathrm{i}-3\right\rvert$ $\Rightarrow\left(\dfrac{-2u}{u^2+(v-1)^2}\right)^2+\left(\dfrac{2u^2+2v(v-1)}{u^2+(v-1)^2}-2\right)^2=\left(\dfrac{-2u}{u^2+(v-1)^2}-3\right)^2+\left(\dfrac{2u^2+2v(v-1)}{u^2+(v-1)^2}\right)^2$ Substitutes their z into the locus of z and applies Pythagoras correctly to obtain an equation in u and v (or their variables). Note that here, further progress is unlikely.		**dd**M1
	$13u^2+13v^2+12u-18v+5=0\Rightarrow u^2+v^2+\dfrac{12}{13}u-\dfrac{18}{13}v+\dfrac{9}{13}=\dfrac{4}{13}$ $\Rightarrow\left(u+\dfrac{6}{13}\right)^2+\left(v-\dfrac{9}{13}\right)^2=\dfrac{4}{13}$ Attempts to complete the square on their equation in u and v where u^2 and v^2 have the same coefficient. Award for e.g. $u^2+v^2+\alpha u+\beta v+...=\left(u+\dfrac{\alpha}{2}\right)^2+\left(v+\dfrac{\beta}{2}\right)^2+...=...$		**ddd**M1

	Attempts using the form $u^2+v^2+2gu+2fv+c=0$ send to review.		
	$\left\lvert w-\left(-\frac{6}{13}+\frac{9}{13}\mathrm{i}\right)\right\rvert=\frac{2}{\sqrt{13}}$	Correct equation in the required form	A1
			Total 8

(b) Way 2	$w=\frac{\mathrm{i}z}{z-2\mathrm{i}}\Rightarrow z=\frac{2w\mathrm{i}}{w-\mathrm{i}}$	Attempts to make z the subject. Must obtain the form $\frac{aw\mathrm{i}}{bw+c\mathrm{i}}$, a, b, c real and non-zero.	M1
	$\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert\Rightarrow\left\lvert\frac{2w\mathrm{i}}{w-\mathrm{i}}-2\mathrm{i}\right\rvert=\left\lvert\frac{2w\mathrm{i}}{w-\mathrm{i}}-3\right\rvert$ $\Rightarrow\left\lvert\frac{2w\mathrm{i}-2w\mathrm{i}-2}{w-\mathrm{i}}\right\rvert=\left\lvert\frac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\rvert$ Introduces z in terms of w into the given locus and attempts to combine terms		**d**M1
	$\left\lvert\frac{-2}{w-\mathrm{i}}\right\rvert=\left\lvert\frac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\rvert\Rightarrow\lvert-2\rvert=\lvert 2w\mathrm{i}-3w+3\mathrm{i}\rvert$ Correct equation with fractions removed		A1
	$\lvert 2(u+\mathrm{i}v)\mathrm{i}-3(u+\mathrm{i}v)+3\mathrm{i}\rvert=2\Rightarrow(3u+2v)^2+(3v-2u-3)^2=4$ Introduces e.g. $w=u+\mathrm{i}v$ and applies Pythagoras correctly		**dd**M1
	$13u^2+13v^2+12u-18v+9=4\Rightarrow u^2+v^2+\frac{12}{13}u-\frac{18}{13}v+\frac{9}{13}=\frac{4}{13}$ $\Rightarrow\left(u+\frac{6}{13}\right)^2+\left(v-\frac{9}{13}\right)^2=\frac{4}{13}$ Attempts to complete the square on their equation in u and v where u^2 and v^2 have the same coefficient. Award for e.g. $u^2+v^2+\alpha u+\beta v+\ldots=\left(u+\frac{\alpha}{2}\right)^2+\left(v+\frac{\beta}{2}\right)^2+\ldots=\ldots$ Attempts using the form $u^2+v^2+2gu+2fv+c=0$ send to review.		**ddd**M1
	$\left\lvert w-\left(-\frac{6}{13}+\frac{9}{13}\mathrm{i}\right)\right\rvert=\frac{2}{\sqrt{13}}$	Correct equation in the required form	A1
			Total 8

(b) Way 3	$w=\dfrac{\mathrm{i}z}{z-2\mathrm{i}} \Rightarrow z=\dfrac{2w\mathrm{i}}{w-\mathrm{i}}$	Attempts to make z the subject. Must obtain the form $\dfrac{aw\mathrm{i}}{bw+c\mathrm{i}}$, a, b, c real and non-zero.	M1
	$\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert \Rightarrow \left\lvert\dfrac{2w\mathrm{i}}{w-\mathrm{i}}-2\mathrm{i}\right\rvert=\left\lvert\dfrac{2w\mathrm{i}}{w-\mathrm{i}}-3\right\rvert$ $\Rightarrow \left\lvert\dfrac{2w\mathrm{i}-2w\mathrm{i}-2}{w-\mathrm{i}}\right\rvert=\left\lvert\dfrac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\rvert$ Introduces z and attempts to combine terms		**d**M1
	$\left\lvert\dfrac{-2}{w-\mathrm{i}}\right\rvert=\left\lvert\dfrac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\rvert \Rightarrow \lvert -2\rvert=\lvert 2w\mathrm{i}-3w+3\mathrm{i}\rvert$ Correct equation with fractions removed		A1
	$\lvert w(2\mathrm{i}-3)+3\mathrm{i}\rvert=\left\lvert(2\mathrm{i}-3)\left(w+\dfrac{3\mathrm{i}}{2\mathrm{i}-3}\right)\right\rvert=\lvert 2\mathrm{i}-3\rvert\left\lvert w+\dfrac{6-9\mathrm{i}}{13}\right\rvert=2$ Attempts to isolate w and rationalise denominator of other term		**dd**M1
	$\sqrt{13}\left\lvert w-\left(-\dfrac{6}{13}+\dfrac{9}{13}\mathrm{i}\right)\right\rvert=2 \Rightarrow \left\lvert w-\left(-\dfrac{6}{13}+\dfrac{9}{13}\mathrm{i}\right)\right\rvert=\dfrac{2}{\sqrt{13}}$ M1: Completes the process by dividing by their $\lvert 2\mathrm{i}-3\rvert$ A1: Correct equation in the required form		**ddd**M1A1
			(6)

Question Number	Scheme	Notes	Marks
7(a)	Condone use of e.g. $C + \mathrm{i}S$ for $\cos x + \mathrm{i}\sin x$ if the intention is clear.		
	$(\cos 5x \equiv)\mathrm{Re}(\cos x + \mathrm{i}\sin x)^5 \equiv \cos^5 x + \begin{pmatrix}5\\2\end{pmatrix}\cos^3 x(\mathrm{i}\sin x)^2 + \begin{pmatrix}5\\4\end{pmatrix}\cos x(\mathrm{i}\sin x)^4$ Identifies the correct terms of the binomial expansion of $(\cos x + \mathrm{i}\sin x)^5$ They may expand $(\cos x + \mathrm{i}\sin x)^5$ completely but there must be an attempt to extract the real terms which must have the correct binomial coefficients combined with the correct powers of sin x and cos x. Condone use of a different variable e.g. θ.		M1
	$(\cos 5x \equiv)\cos^5 x - 10\cos^3 x\sin^2 x + 5\cos x\sin^4 x$ Correct simplified expression. Condone use of a different variable e.g. θ.		A1
	$\equiv \cos x(\cos^4 x - 10\cos^2 x\sin^2 x + 5\sin^4 x)$ $\equiv \cos x\left((1-\sin^2 x)^2 - 10(1-\sin^2 x)\sin^2 x + 5\sin^4 x\right)$ Applies $\cos^2 x = 1 - \sin^2 x$ to obtain an expression in terms of sin x inside the bracket. Condone use of a different variable e.g. θ.		M1
	$\equiv \cos x(16\sin^4 x - 12\sin^2 x + 1)$	Correct expression. Must be in terms of x now. The "cos5x =" is not required.	A1
			(4)
(b)	Allow use of a different variable in (b) e.g. x for <u>all</u> marks.		
	$\cos 5\theta = \sin 2\theta\sin\theta - \cos\theta$ $\Rightarrow \cos\theta(16\sin^4\theta - 12\sin^2\theta + 1) = 2\sin^2\theta\cos\theta - \cos\theta$ $\Rightarrow \cos\theta(16\sin^4\theta - 14\sin^2\theta + 2) = 0$ Uses the result from part (a) with $\sin 2\theta = 2\sin\theta\cos\theta$ and collects terms		M1
	$16\sin^4\theta - 14\sin^2\theta + 2 = 0$ $\Rightarrow \sin^2\theta = \dfrac{7 \pm \sqrt{17}}{16} \Rightarrow \sin\theta = ...$ Solves for $\sin^2\theta$ by any method including calculator and takes square root to obtain at least one value for $\sin\theta$. Depends on the first mark. May be implied by their values of $\sin\theta$ or θ. NB $\dfrac{7 \pm \sqrt{17}}{16} = 0.69519..., 0.17980...$		dM1
	$\sin\theta = \sqrt{\dfrac{7 \pm \sqrt{17}}{16}} \Rightarrow \theta = ...$ NB $\sqrt{\dfrac{7 \pm \sqrt{17}}{16}} = 0.833783...,\ 0.424035...$ A full method to reach at least one value for θ. Depends on the previous mark. May be implied by their values of θ		ddM1
	$(\theta =)0.986,\ 0.438$	Correct values and no others in range. Allow awrt these values.	A1
			(4)
			Total 8

Note that it is possible to do 7(b) by changing to cos θ e.g.

$$\cos\theta\left(16\sin^4\theta-12\sin^2\theta+1\right)=\cos\theta\left(16\left(1-\cos^2\theta\right)^2-12\left(1-\cos^2\theta\right)+1\right)$$

$$\cos\theta\left(16\left(1-\cos^2\theta\right)^2-12\left(1-\cos^2\theta\right)+1\right)=2\sin^2\theta\cos\theta-\cos\theta$$

$$16\cos^4\theta-18\cos^2\theta+4=0$$

$$\cos^2\theta=\frac{9\pm\sqrt{17}}{16}\Rightarrow\cos\theta=\sqrt{\frac{9\pm\sqrt{17}}{16}}$$

$$(\theta=)0.986,\ 0.438$$

This is acceptable as they used part (a) and can be scored as:

M1: Uses part (a) with $\sin^2\theta=1-\cos^2\theta$ and $\sin 2\theta=2\sin\theta\cos\theta$ and collects terms.

dM1: Solves for $\cos^2\theta$ by any method including calculator and takes square root to obtain at least one value for $\cos\theta$. Depends on the first mark. May be implied by their values of $\cos\theta$ or θ.

NB $\frac{9\pm\sqrt{17}}{16}=0.82019...,\ 0.30480...$

dM1: A full method to reach at least one value for θ.
Depends on the previous mark. May be implied by their values of θ

NB $\sqrt{\frac{9\pm\sqrt{17}}{16}}=0.905645...,\ 0.552092...$

A1: $(\theta=)0.986,\ 0.438$

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="2">8(a)</td><td rowspan="2">$y = r\sin\theta = (1-\sin\theta)\sin\theta = \sin\theta - \sin^2\theta$
$\Rightarrow \frac{dy}{d\theta} = \cos\theta - 2\sin\theta\cos\theta$
or e.g.
$\Rightarrow \frac{dy}{d\theta} = \cos\theta - \sin 2\theta$</td><td>Differentiates $(1-\sin\theta)\sin\theta$ to achieve $\pm\cos\theta \pm k\sin\theta\cos\theta$ or equivalent. Use of $y = r\cos\theta$ or $x = r\cos\theta$ scores M0</td><td>M1</td></tr>
<tr><td>Correct derivative in any form.</td><td>A1</td></tr>
<tr><td></td><td colspan="2">$\cos\theta - 2\sin\theta\cos\theta = 0 \Rightarrow \cos\theta(1-2\sin\theta) = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = ...$
Solves to find a value for θ. Depends on the first M.</td><td>dM1</td></tr>
<tr><td></td><td colspan="2">$\left(\frac{1}{2}, \frac{\pi}{6}\right)$
Correct coordinates and no others. Isw if necessary e.g. if written as $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ after correct values seen or implied award A1. Allow e.g. $\theta = \frac{\pi}{6}$, $r = \frac{1}{2}$.
The value of r must be seen in (a) – i.e. do not allow recovery in (b).</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>(4)</td></tr>
<tr><td rowspan="6">(b)
Way 1</td><td colspan="2">Note that the $\frac{1}{2}$ in $\frac{1}{2}\int r^2\,d\theta$ is not required for the first 4 marks</td><td></td></tr>
<tr><td>$\int(1-\sin\theta)^2\,d\theta = \int(1-2\sin\theta+\sin^2\theta)\,d\theta$
$= \int\left(1-2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)d\theta$</td><td>Attempts $\left(\frac{1}{2}\right)\int r^2\,d\theta$ and applies $\sin^2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$</td><td>M1</td></tr>
<tr><td colspan="2">$\int(1-\sin\theta)^2\,d\theta = \frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta\,(+c)$
Correct integration. Condone mixed variables e.g.
$\int(1-\sin\theta)^2\,d\theta = \frac{3}{2}x + 2\cos\theta - \frac{1}{4}\sin 2\theta\,(+c)$</td><td>A1</td></tr>
<tr><td colspan="2">$\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{2}\right)\left[\left(\frac{\pi}{4} + \sqrt{3} - \frac{\sqrt{3}}{8}\right) - (2)\right]\left(= \frac{\pi}{8} + \frac{7\sqrt{3}}{16} - 1\right)$
Applies the limits of 0 and their $\frac{\pi}{6}$ to their integration. The $\frac{1}{2}$ is not required.
For the integration look for at least $\pm\int\sin\theta\,d\theta \to \pm\cos\theta$</td><td>M1</td></tr>
<tr><td colspan="2">Triangle: $\frac{1}{2}\times\frac{1}{2}\sin\frac{\pi}{6}\times\frac{1}{2}\cos\frac{\pi}{6}\left(=\frac{\sqrt{3}}{32}\right)$
Uses a correct strategy for the area of the triangle</td><td>M1</td></tr>
<tr><td>Area of $R = \frac{\pi}{8} + \frac{7\sqrt{3}}{16} - 1 + \frac{\sqrt{3}}{32}$</td><td>Fully correct method for the required area. Depends on all previous method marks.</td><td>dM1</td></tr>
</table>

	$\frac{1}{32}\left(4\pi+15\sqrt{3}-32\right)$	Cao	A1
			(6)
			Total 10

	Note that the $\frac{1}{2}$ in $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ is not required for the first 3 marks		
(b) Way 2	$\int(1-\sin\theta)^2\,\mathrm{d}\theta=\int\left(1-2\sin\theta+\sin^2\theta\right)\mathrm{d}\theta$ $=\int\left(1-2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$	Attempts $\left(\frac{1}{2}\right)\int r^2\,\mathrm{d}\theta$ and applies $\sin^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$.	M1
	$\int(1-\sin\theta)^2\,\mathrm{d}\theta=\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta(+c)$ Correct integration. Condone mixed variables e.g. $\int(1-\sin\theta)^2\,\mathrm{d}\theta=\frac{3}{2}x+2\cos\theta-\frac{1}{4}\sin 2\theta(+c)$		A1
	$\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}}=\left(\frac{1}{2}\right)\left[\left(\frac{3\pi}{4}+0-0\right)-(2)\right]\left(=\frac{3\pi}{8}-1\right)$ Evidence of use of **both** limits 0 and $\frac{\pi}{2}$ to their integration. The $\frac{1}{2}$ is not required. For the integration look for at least $\pm\int\sin\theta\,\mathrm{d}\theta\to\pm\cos\theta$		M1
	Triangle – "Segment": $\frac{1}{2}\times\frac{1}{2}\sin\frac{\pi}{6}\times\frac{1}{2}\cos\frac{\pi}{6}-\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(1-\sin\theta)^2\,\mathrm{d}\theta$ $\frac{\sqrt{3}}{32}-\frac{1}{2}\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(=\frac{15\sqrt{3}}{32}-\frac{\pi}{4}\right)$ Uses a fully correct strategy for the area above the curve between O and P. Requires a correct method for the triangle as in Way 1 and a correct method for the "segment" using **both** their $\frac{\pi}{6}$ and $\frac{\pi}{2}$.		M1
	Area of $R=\frac{3\pi}{8}-1+\frac{15\sqrt{3}}{32}-\frac{\pi}{4}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32}\left(4\pi+15\sqrt{3}-32\right)$	cao	A1
			(6)

Question Number	Scheme	Notes	Marks
9(a)(i)	$x=t^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{2}t^{-\frac{1}{2}}\frac{\mathrm{d}t}{\mathrm{d}y} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ or $t=x^2 \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}x}=2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}=\ldots$ Applies the chain rule and proceeds to an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$		M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}$	Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y and t	A1
(a)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}t}t^{-\frac{1}{2}}\frac{\mathrm{d}t}{\mathrm{d}x}+2t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ **dM1**: Uses the product rule to differentiate an equation of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=kt^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}$ or equivalent e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=kx\frac{\mathrm{d}y}{\mathrm{d}t}$ to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\alpha t^{-\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}+\ldots$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\ldots+\beta t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ **<u>or equivalent expressions</u>** where … is non-zero **A1**: <u>Any</u> correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$		**d**M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}t}t^{-\frac{1}{2}}\frac{\mathrm{d}t}{\mathrm{d}x}+2t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}t^{-\frac{1}{2}}\times 2t^{\frac{1}{2}}+2t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}t}+4t\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ Correct expression in terms of y and t		A1
			(5)
(b)	$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-(6x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x}+9x^3y=x^5 \Rightarrow t^{\frac{1}{2}}\left(2\frac{\mathrm{d}y}{\mathrm{d}t}+4t\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right)-(6t+1)2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}+9t^{\frac{3}{2}}y=t^{\frac{5}{2}}$ Substitutes their expressions from part (a) and replaces x with $t^{\frac{1}{2}}$		M1
	$2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}+4t^{\frac{3}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-12t^{\frac{3}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}-2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}+9t^{\frac{3}{2}}y=t^{\frac{5}{2}}$ $\Rightarrow 4\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-12\frac{\mathrm{d}y}{\mathrm{d}t}+9y=t$ * Obtains the given answer with no errors and sufficient working shown – at least one intermediate line after substitution but check working. **Must follow full marks in (a) apart from SC below.**		A1*
			(2)

Special case in (a) and (b) for those who do not have (a) in terms of y and t only:

$$t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \ldots$$ **Scores M1.** $\ldots = 2x\frac{dy}{dt}$ **scores A0 in (a)(i)**

$$\frac{dy}{dx} = 2x\frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dt} + 2x\frac{d^2y}{dt^2}\frac{dt}{dx} = 2\frac{dy}{dt} + 4x^2\frac{d^2y}{dt^2}$$ **Scores dM1A1A0 in (a)(ii)**

$$x\frac{d^2y}{dx^2} - (6x^2+1)\frac{dy}{dx} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}}\left(2\frac{dy}{dt} + 4x^2\frac{d^2y}{dt^2}\right) - (6t+1)2x\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$$

$$\Rightarrow t^{\frac{1}{2}}\left(2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}\right) - (6t+1)2t^{\frac{1}{2}}\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}} \Rightarrow 4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = t^*$$ **Scores M1A1 in (b)**

Mark (c) and (d) together

(c)	$4m^2 - 12m + 9 = 0 \Rightarrow m = \frac{3}{2}$	Attempts to solve $4m^2 - 12m + 9 = 0$ Apply general guidance for solving a 3TQ if necessary.	M1
	$(y =)e^{\frac{3}{2}t}(At + B)$	Correct CF. No need for "$y = \ldots$" Condone $(y =)e^{\frac{3}{2}x}(Ax + B)$ here but must be in terms of t in the GS. Allow equivalents for the $\frac{3}{2}$.	A1
	$(y =)at + b \Rightarrow \frac{dy}{dt} = a \Rightarrow \frac{d^2y}{dt^2} = 0$ $\Rightarrow -12a + 9(at + b) = t$ Starts with the **correct** PI form and differentiates to obtain $\frac{dy}{dt} = a$ and $\frac{d^2y}{dt^2} = 0$ and substitutes. NB starting with a PI of $y = at$ is M0		M1
	$9a = 1 \Rightarrow a = \ldots$ $9b - 12a = 0 \Rightarrow b = \ldots$	Complete method to find a and b by comparing coefficients. **Depends on the previous method mark.**	dM1
	$y = e^{\frac{3}{2}t}(At + B) + \frac{1}{9}t + \frac{4}{27}$	Correct GS including "$y = \ldots$" and must be in terms of t (no x's). Allow equivalent exact fractions for the constants.	A1
			(5)
(d)	$y = e^{\frac{3}{2}x^2}(Ax^2 + B) + \frac{1}{9}x^2 + \frac{4}{27}$ Correct equation including "$y = \ldots$" (follow through their answer to (c)). Allow equivalent exact fractions for the constants. For the ft, the answer to (c) must be in terms of t and the answer to (d) should be the same as (c) with t replaced with x^2. If there is no final answer to (c) you can award B1ft if the equation is correct in terms of x if it follows the previous work.		B1ft
			(1)
			Total 13

Question Number	Scheme	Marks
1(a)	$\frac{2}{\sqrt{r}+\sqrt{r-2}} \times \frac{\sqrt{r}-\sqrt{r-2}}{\sqrt{r}-\sqrt{r-2}}$	M1
	$\frac{2\left(\sqrt{r}-\sqrt{r-2}\right)}{r-(r-2)} = \sqrt{r}-\sqrt{r-2}$ *	A1*
		(2)
(b)	$r=2: \ \sqrt{2}-\sqrt{2-2} \ \left(=\sqrt{2}-0\right)$ $\quad \ldots r=n-2: \ \sqrt{n-2}-\sqrt{n-4}$ $r=3: \ \sqrt{3}-\sqrt{3-2} \ \left(=\sqrt{3}-1\right)$ $\quad r=n-1: \ \sqrt{n-1}-\sqrt{n-3}$ $r=4: \ \sqrt{4}-\sqrt{4-2} \ \left(=2-\sqrt{2}\right)$ $\quad r=n: \ \sqrt{n}-\sqrt{n-2}$	M1
	$\left[\sum_{r=2}^{n}\frac{2}{\sqrt{r}+\sqrt{r-2}}=\right]\sqrt{n}+\sqrt{n-1}-1$	A1 A1
		(3)
(c)	$\left[\sum_{r=4}^{50}\frac{2}{\sqrt{r}+\sqrt{r-2}}=\right]\mathrm{f}(50)-\mathrm{f}(3)=\sqrt{50}+\sqrt{49}-1-\left(\sqrt{3}+\sqrt{2}-1\right)$	M1
	$\left(=5\sqrt{2}+7-1-\sqrt{3}-\sqrt{2}+1\right)=7+4\sqrt{2}-\sqrt{3}$	A1
		(2)
		Total 7

Notes

(a)

M1: Indicates intention to multiply either side by a correct fraction, may use $\frac{\sqrt{r-2}-\sqrt{r}}{\sqrt{r-2}-\sqrt{r}}$. The "2" may be missing. May work in reverse from right hand side to left, and this is fine.

Alternatively, may multiply the initial expression through by $\sqrt{r}+\sqrt{r-2}$ and use a sequence of equivalences (though accept with $\Rightarrow$ or nothing between lines).

A1: Fully correct proof. A result from rationalisation that is not the given answer must be seen. If using a sequences of equivalences there must be a minimal conclusion.

(b)

M1: Correct process of differences evidenced in their work, e.g. attempts any three of the 6 expression shown. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at any of $r = 0$ or 1 if they start earlier than $r = 2$.

A1: Correct algebraic terms **or** correct constant term(s) extracted. Accept unsimplified expressions such as $"-\sqrt{1}-\sqrt{0}"$

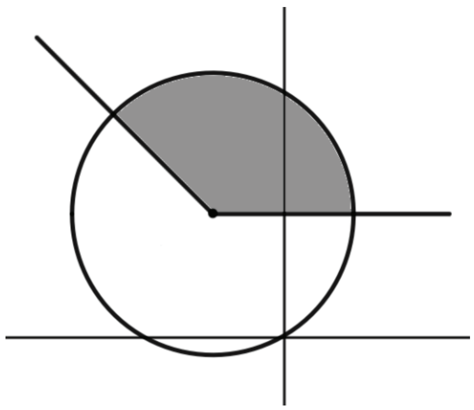
A1: Fully correct simplified expression. Must have simplified the $\sqrt{1}$ to 1

(c)

M1: Attempts f(50) – f(3) using **their answer to (b)**. Must be indication of subtraction. The $'-1'$s may be omitted, and allow if a slip is made. They may subtract the sum of their results from $r = 2$ and $r = 3$ from (b) or by using (a) again to obtain f(3) but their answer to (b) must be used for f(50). Allow from attempts starting at $r = 1$ in their summations. f(50) −f(4) is M0.

A1: Correct expression or $A = 7$, $B = 4$, $C = -1$ **following a correct answer to (b)**.

The decimal answer 10.92480344… without evidence of the M mark is 0/2

Question Number	Scheme		Marks
2(a)	$\left(\cos\frac{5\pi}{12}+\mathrm{i}\sin\frac{5\pi}{12}\right)^4=\cos\frac{20\pi}{12}+\mathrm{i}\sin\frac{20\pi}{12}$ or/and $\left(\cos\frac{\pi}{3}-\mathrm{i}\sin\frac{\pi}{3}\right)^3=\cos(-\pi)+\mathrm{i}\sin(-\pi)$		M1 / A1
	$(z_1=)\dfrac{\cos\frac{5\pi}{3}+\mathrm{i}\sin\frac{5\pi}{3}}{\cos(-\pi)+\mathrm{i}\sin(-\pi)}=\cos\left(\frac{5\pi}{3}-(-\pi)\right)+\mathrm{i}\sin\left(\frac{5\pi}{3}-(-\pi)\right)$ Alt : $(z_1=)\dfrac{\cos\frac{5\pi}{3}+\mathrm{i}\sin\frac{5\pi}{3}}{-1}=-\cos\frac{5\pi}{3}-\mathrm{i}\sin\frac{5\pi}{3}$		M1
	$=\cos\frac{8\pi}{3}+\mathrm{i}\sin\frac{8\pi}{3}\quad=\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3}$ * Alt: if denominator -1 used via e.g. $\cos\left(\frac{5\pi}{3}-\pi\right)+\mathrm{i}\sin\left(-\frac{5\pi}{3}\right)$		A1*
			(4)
(b)	$\lvert z-z_1\rvert\leqslant 1\qquad 0\leqslant\arg(z-z_1)\leqslant\frac{3\pi}{4}$		
		A circle in any position (may just see the minor arc)	M1
		A pair of **half**–lines in correct directions from their centre, one with negative gradient and one parallel to (but not) the x–axis. *If full lines are used the M marks can be implied by their shading*	M1
		Area shaded inside their circle between the two half lines from from the parallel one anticlockwise to the negative gradient line.	M1
	Fully correct shaded sector. See notes.		A1
			(4)
(c)	$\arctan\left(\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)=\ldots\qquad\frac{\pi}{3}\quad(\text{or }60^\circ)$		M1 A1
			(2)
			Total 10

Notes

(a)

M1: One correct use of de Moivre in polar (or exponential) form. Allow use of $\mathrm{e}^{\mathrm{i}\theta}$ for $\cos\theta+\mathrm{i}\sin\theta$ throughout until the final A.

A1: Both correct (unsimplified) in polar form. Accept for both marks use of

$\left(\cos\frac{\pi}{3}-\mathrm{i}\sin\frac{\pi}{3}\right)^3\to\cos\pi-\mathrm{i}\sin\pi$ but denominator directly to $\cos\pi+\mathrm{i}\sin\pi$ with no evidence of dealing with the negative between terms is A0.

M1: A correct method shown for the division of the two complex numbers. No need to simplify for this mark, a difference of argument in the trig terms is fine. Look for the subtraction of the arguments. Sight of the $\frac{8\pi}{3}$ can imply the mark if no incorrect work is seen. Note it is M0 if the arguments are added (so

$\cos\left(\frac{5\pi}{3}-\pi\right)+\mathrm{i}\sin\left(\frac{5\pi}{3}-\pi\right)$ is M0 unless the denominator has clearly been written as $\cos\pi+\mathrm{i}\sin\pi$ first.)

May write the denominator as -1 first, which is correct, score for $-\cos\frac{5\pi}{3}-\mathrm{i}\sin\frac{5\pi}{3}$.

Accept methods that convert both numbers into exact Cartesian form, apply a correct process to realise the denominator and convert back to polar form.

Do not allow going straight to $\cos\frac{2\pi}{3}+\mathrm{i}\ \sin\frac{2\pi}{3}$ for this mark as it is a given answer. Justification is required and an incorrect method is M0.

A1cso: Obtains $\cos\frac{2\pi}{3}+\mathrm{i}\ \sin\frac{2\pi}{3}$ with suitable intermediate step shown and no errors in the work. Must have scored the preceding 3 marks. This will usually be via $\cos\frac{8\pi}{3}+\mathrm{i}\ \sin\frac{8\pi}{3}$ unless equivalent suitable working has been shown to justify the correct modulus (e.g. proceeding via -1 in the denominator).

(b)

Do not be concerned about lines being dashed or dotted in this part. Accept either.

M1: A circle in any position. You may see just the minor arc of a circle, which is acceptable as long as it is clearly an arc of a circle (e.g. implied by their shading) and not just a angle demarcation.

M1: Draws or indicates a pair of half–lines from their centre (which need not be in quadrant 2) with one with negative gradient proceeding up and left and one parallel to the x–axis proceeding right but not the x-axis itself. If full lines are used this can be implied by the shading of the correct region between lines.

M1: Shades the area between their pair of rays (the second ray may have positive gradient for this mark) and inside their circle, anticlockwise from the horizontal line. The half line need not stem from the centre of the circle for this mark, and accept the x-axis as the horizontal line for this mark.

A1: Fully correct shaded sector. Must

- be in quadrants 1 and 2
- have approximately correct gradients for the half-lines (-1 and 0)
- have circle with centre in quadrant 2 and (if whole circle shown) passing roughly through the origin. If only an arc is shown apply bod as long as the position is reasonable.

Ignore any centre coordinates and axes intersections of any major sector. If extra regions are shaded they must make clear which their region R is in order to access the mark.

(c)

M1: Identifies the correct point for their sector, which must be from a circle with centre in quadrant 1 or 2 (above the real axis) and ray parallel to the real axis, and attempts the relevant angle. Look for selecting the point (c, d) at the “3 o’clock” position having identified a suitable sector and proceeding to find a relevant positive angle (e.g., allow if $\pi-\theta$ is found) (accept $\arctan\pm\frac{c}{d}$ or $\arctan\pm\frac{d}{c}$ as an attempt at the angle). Their point must be in quadrants 1 or 2. If no shading was shown in (b) allow for attempts at the relevant point of horizontal ray and circle intersection. Could use other trig.

A1 : Either correct value. Mark final answer.

Question Number	Scheme	Marks
3	$\dfrac{x+2}{x+4} \leqslant \dfrac{x}{k(x-1)}$	
(a)	$\left[\times k\left(x+4\right)^2(x-1)^2 \Rightarrow\right] \quad k(x+4)(x-1)^2(x+2) \leqslant x(x+4)^2(x-1)$ $\Rightarrow k(x+4)(x-1)^2(x+2) - x(x+4)^2(x-1) \{\leqslant 0\}$ **OR** $\dfrac{x+2}{x+4} - \dfrac{x}{k(x-1)} \{\leqslant 0\} \Rightarrow \dfrac{k(x+2)(x-1)-x(x+4)}{k(x+4)(x-1)} \{\leqslant 0\}$	M1
	$\left(x+4\right)\left(x-1\right)\left[kx^2+kx-2k-x^2-4x\right]\{\leqslant 0\}$	**d**M1
	$\left(x+4\right)\left(x-1\right)\left[\left(k-1\right)x^2+\left(k-4\right)x-2k\right] \leqslant 0$ (oe in corerct form)	A1cso
		(3)
(b)	$k=3 \Rightarrow 2x^2-x-6=0$ $[(2x+3)(x-2)=0] \quad \Rightarrow x=-\frac{3}{2}, 2$	M1
	$-4 < x \leqslant -\frac{3}{2} \qquad 1 < x \leqslant 2$	**d**M1 A1 A1
		(4)
		Total 7

Notes

(a)

M1: Attempts to multiply both sides by e.g. $k\left(x+4\right)^2(x-1)^2$ or $k^2\left(x+4\right)^2(x-1)^2$ and bring terms together OR bring terms together and attempt a common denominator. There may be slips but the intention must be clear - allow if e.g. one term is missing.

dM1: Factorises out $\left(x+4\right)\left(x-1\right)$ and/or multiplies by $\left(x+4\right)^2(x-1)^2$ (which could be implied) and expands remaining terms to get unsimplified 3 term quadratic (terms need not be collected but it must be equivalent to px^2+qx+r where at least one of p, q and r is a function of k and none are zero). **Dependent on previous M mark.**

A1cso: Correct statement or $p=k-1$, $q=k-4$, $r=-2k$ with no algebraic errors (but condone e.g. recovery of missing brackets if work is clear). It is A0 if any incorrect statement is seen e.g., use of an incorrect inequality sign. Accept with any positive multiples of the coefficients (including k or $1/k$ etc).

(b)

M1: Uses $k=3$ **in their quadratic from (a)** (implied by 2 out of three terms correct) and solves to obtain a value. Apply usual rules (may be by calculator - may need to check). If no working, obtains one consistent solution which must be real. **Alternatively** restarts from $\dfrac{x+2}{x+4} \leqslant \dfrac{x}{3(x-1)}$ and uses a valid method to obtain and solve a quadratic (see above). Marks for (a) cannot be scored in (b).

dM1: With critical values -4, 1 and two different real solutions from their quadratic ($\neq -4$ or 1), chooses the region between the two smaller values and the region between the two larger values and no other region. Could be non-strict inequalities. **Dependent on previous M mark.**

A1: Both regions correct but condone e.g incorrect strict non-strict inequalities.

A1: Both completely correct regions. Allow equivalent notations. Ignore any word between the regions but withhold this last mark if $\cap$ used with set notation.

Question Number	Scheme	Marks
4	$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=48x^2-34$	
(a)	$(\text{AE:})\quad m^2-8m+16=0$ $[\Rightarrow (m-4)^2=0]$ $\Rightarrow m=4$	M1
	$(\text{CF: } y=)\quad (A+Bx)e^{4x}$	A1
	$(\text{PI: } y=)\quad \lambda x^2+\mu x+\nu$	B1
	$y'=2\lambda x+\mu \quad y''=2\lambda$ $2\lambda-8(2\lambda x+\mu)+16(\lambda x^2+\mu x+\nu)=48x^2-34$ $16\lambda x^2=48x^2\ (-16\lambda+16\mu)x=0\quad 2\lambda-8\mu+16\nu=-34$ $\lambda=3\quad \mu=3\quad \nu=-1$	M1
	$y="(A+Bx)e^{4x}"+3x^2+3x-1$	A1ft
		(5)
(b)	$(0,4)\Rightarrow 4=A-1\quad (A=5)$	M1
	$\frac{dy}{dx}=4(A+Bx)e^{4x}+Be^{4x}+6x+3$	M1
	$21=4A+B+3\Rightarrow B=-2\quad A=5$ $[y]=(5-2x)e^{4x}+3x^2+3x-1$	M1A1
		(4)
(c)	$(x=-2\Rightarrow y=)\ 9e^{-8}+5$	M1 A1
		(2)
		Total 11

Notes

(a)
M1: Forms the auxiliary equation (condone one slip/copying error) and solves 3TQ. Usual rules. One consistent solution if no working (could be complex). Implied by a correct CF if no incorrect working shown.
A1: Correct complementary function $y=$ ... not required. May only be seen in final answer.
B1: Correct form for particular integral $y=$ ... not required.
M1: Correct method to obtain value for constants (or constant - but PI must be a quadratic - but could have 1 or 2 terms) - so differentiates twice (powers reduced) and substitutes, equates terms and solves equations. Allow if there are minor slips in the process if the holistic approach is correct.
A1ft: A correct general solution following through on their CF only - the PI must be correct. Must have " $y=$ " e.g., not " GS = "
(b)
M1: Uses (0, 4) in their answer to (a) and forms an equation in one or both of their constants.
M1: Differentiates their GS, which must contain a term "Bxe^{kx}", to obtain an expression of the correct form for their GS - product rule must be used, powers reduced, but may have errors in coefficients.
M1: Substitutes $x=0,\ \frac{dy}{dx}=21$ into their equation where their derivative is a changed function and finds values for their B (and their A if not found already).
A1: Any correct equation Condone e.g., " PS =... "
(c)
M1: Substitutes $x=-2$ into their particular solution and obtains an expression of the right form or non-zero values for p, q and r, which need not be integers for the M but should be gathered terms. Implied by a correct answer as long as it fits their answer to (b) if no method shown.
A1: Correct expression or values

Question Number	Scheme	Marks
5(a)	$w=\dfrac{z+1}{z-3}\Rightarrow wz-3w=z+1\Rightarrow wz-z=3w+1\Rightarrow z=\dfrac{3w+1}{w-1}$	M1 A1
	$(z=)\dfrac{(3u+1)+3\mathrm{i}v}{(u-1)+\mathrm{i}v}\times\dfrac{(u-1)-\mathrm{i}v}{(u-1)-\mathrm{i}v}=\ldots$	**d**M1
	$\Rightarrow x=\dfrac{3u^2-2u+3v^2-1}{(u-1)^2+v^2}\quad y=\dfrac{-4v}{(u-1)^2+v^2}$ $(y=4x\Rightarrow)\quad -4v=12u^2-8u+12v^2-4$	**dd**M1
	$\Rightarrow 3u^2+3v^2-2u+v-1=0$*	A1cso*
		(5)
(b)	$u^2-\dfrac{2}{3}u+v^2+\dfrac{1}{3}v-\dfrac{1}{3}=0\Rightarrow\left(u-\dfrac{1}{3}\right)^2-\dfrac{1}{9}+\left(v+\dfrac{1}{6}\right)^2-\dfrac{1}{36}-\dfrac{1}{3}=0$ $\Rightarrow\left(u-\dfrac{1}{3}\right)^2+\left(v+\dfrac{1}{6}\right)^2=\dfrac{17}{36}$ $\Rightarrow$ centre: $\left(\dfrac{1}{3},-\dfrac{1}{6}\right)$ $\left[\text{allow } x=\dfrac{1}{3}, y=-\dfrac{1}{6}\right]$ radius: $\sqrt{\dfrac{17}{36}}$ or $\dfrac{\sqrt{17}}{6}$	B1 B1
		(2)
		Total 7

Notes

(a)
M1: Completes an attempt to make z the subject.
A1: Correct expression.
dM1: **Dependent on previous M mark.** Replaces w with $u+\mathrm{i}v$ and indicates an appropriate attempt to rationalise the denominator. Accept if $x+\mathrm{i}y$ is used instead for this mark.
ddM1: **Dependent on both previous M marks.** Equates real and imaginary parts to obtain expressions for x and y in u and/or v only and uses $y=4x$ to form an equation in u and v only. Condone intermediate incorrect statements that are recovered for this mark, e.g. condone, missing or incorrect (real) denominator or slips with initial inclusion of i in the y if recovered. E.g. any of

$x=3u^2-2u+3v^2-1,\quad x=\dfrac{3u^2-2u+3v^2-1}{(u-1)^2-v^2},\quad y=\dfrac{-4v\mathrm{i}}{(u-1)^2+v^2}$ or similar may be recovered for the M's

A1cso*:obtains the equation (must be equal to 0) or states $k=-1$ following correct work. Do not allow recovery from incorrect statements as shown above for the final mark - all may lead to the correct answer but are A0. But if there are no incorrect statements e.g. cancelling of denominators may be implied for the A. Watch out for an incorrect $z=\dfrac{3w+1}{1-w}$ which also leads to a correct expression but loses both A marks.

Note: Alternatively methods are possible. Two are shown on the next page, but are not exhaustive.

(b)
Allow both marks from cases where $k=-1$ is guessed or achieved from fortuitous work.
B1: Correct centre **or** radius. Accept unsimplified fractions for this mark.
B1: Correct centre **and** radius - fractions must be simplified.

5(a) Alt	$z=x+\mathrm{i}y,\ y=4x \Rightarrow w=\frac{z+1}{z-3}=\frac{x+4x\mathrm{i}+1}{x+4x\mathrm{i}-3}=\frac{(x+1)+4\mathrm{i}x}{(x-3)+4\mathrm{i}x}\times\frac{(x-3)-4\mathrm{i}x}{(x-3)-4\mathrm{i}x}=\ldots$	M1
	$(u+\mathrm{i}v=)\frac{(17x^2-2x-3)-16\mathrm{i}x}{(x-3)^2+16x^2}$ oe	**A1**
	$\Rightarrow u=\frac{17x^2-2x-3}{(x-3)^2+16x^2} \quad v=\frac{-16x}{(x-3)^2+16x^2}$ $\Rightarrow 3u^2+3v^2-2u+v=\ldots$	**d**M1
	$\Rightarrow\ldots\Rightarrow 3u^2+3v^2-2u+v-1=0^*$ or states $k=-1$	**dd**M1A1cso*
		(5)
5(a) Alt II	$z=x+\mathrm{i}y,\ y=4x \Rightarrow w=\frac{z+1}{z-3}=\frac{x+4x\mathrm{i}+1}{x+4x\mathrm{i}-3}$	
	$\Rightarrow(u+\mathrm{i}v)((x-3)+4\mathrm{i}x)=(x+1)+4\mathrm{i}x\Rightarrow\ldots+\ldots\mathrm{i}=\ldots$	M1
	$u(x-3)-4vx+\mathrm{i}(v(x-3)+4ux)=(x+1)+4\mathrm{i}x$	A1
	$\Rightarrow u(x-3)-4vx=(x+1) \quad v(x-3)+4ux=4x$ $\Rightarrow x=\frac{3u+1}{u-4v-1} \quad x=\frac{3v}{v+4u-4}\Rightarrow\frac{3u+1}{u-4v-1}=\frac{3v}{v+4u-4}$	**d**M1
	$(3u+1)(v+4u-4)=3v(u-4v-1)\Rightarrow 12u^2+12v^2-8u+4v-4=0$ $\Rightarrow 3u^2+3v^2-2u+v-1=0^*$	**dd**M1A1cso*
		(5)

Notes

(a) Alt I

M1: Uses $y = 4x$ and $z = x + \mathrm{i}y$ in the expression for w and multiplies through by complex conjugate of denominator

A1: Correct expression in terms of x (oe)

dM1: Replaces w with $u + \mathrm{i}v$ and equates real and imaginary parts and substitutes into the LHS of given equation. **Dependent on previous M mark.**

ddM1: Proceeds simplify the expression putting over common denominator.They are unlikely to make much progress at this stage in this method. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression equates to 0 and gives minimal conclusion.

(a) Alt II

M1: Uses $y = 4x$ and $z = x + \mathrm{i}y$ in the expression for w and equates to $u + \mathrm{i}v$, cross multiples and expands.

A1: Correct expression with the i^2 terms simplified.

dM1: Equates real and imaginary parts and proceeds to eliminate x solving simultaneously. **Dependent on previous M mark.**

ddM1: Proceeds to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression required holds. As main scheme.

Question Number	Scheme	Marks
6(a)	$y=\sec x \Rightarrow \dfrac{dy}{dx}=\sec x\tan x$ $\dfrac{d^2y}{dx^2}=(\sec x\tan x)\tan x+\sec x(\sec^2 x)\ \left(=\sec x\tan^2 x+\sec^3 x\right)$	M1
	$\dfrac{d^2y}{dx^2}=\sec x(\sec^2 x-1)+\sec^3 x=2\sec^3 x-\sec x$ $\Rightarrow \dfrac{d^3y}{dx^3}=6\sec^2 x(\sec x\tan x)-\sec x\tan x$ **or** $\dfrac{d^3y}{dx^3}=2\sec x\tan x\sec^2 x+\tan^2 x\sec x\tan x+3\sec^2 x\sec x\tan x$	**d**M1
	$\left(\dfrac{d^3y}{dx^3}=5\sec^3 x\tan x+\sec x\tan x(\sec^2 x-1)=6\sec^3 x\tan x-\sec x\tan x\right)$ $\Rightarrow \dfrac{d^3y}{dx^3}=\sec x\tan x(6\sec^2 x-1)$	**dd**M1 A1
		(4)
(b)	$\sec\left(\dfrac{\pi}{3}\right)=2,\ \tan\left(\dfrac{\pi}{3}\right)=\sqrt{3}\Rightarrow f\left(\dfrac{\pi}{3}\right)=2,\ f'\left(\dfrac{\pi}{3}\right)=2\sqrt{3},\ f''\left(\dfrac{\pi}{3}\right)=14,\ f'''\left(\dfrac{\pi}{3}\right)=46\sqrt{3}$	M1
	$f(x)=f(a)+(x-a)f'(a)+\dfrac{(x-a)^2}{2!}f''(a)+\dfrac{(x-a)^3}{3!}f'''(a)+\ldots$ $\Rightarrow 2+2\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+7\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{23\sqrt{3}}{3}\left(x-\dfrac{\pi}{3}\right)^3$	**d**M1 A1
		(3)
(c)	$\left(\sec\dfrac{7\pi}{24}\approx\right)=2+2\sqrt{3}\left(-\dfrac{\pi}{24}\right)+7\left(-\dfrac{\pi}{24}\right)^2+\dfrac{23\sqrt{3}}{3}\left(-\dfrac{\pi}{24}\right)^3=\ldots$	M1
	$\sec\dfrac{7\pi}{24}\approx 1.636709263\ldots\approx 1.637$	A1
		(2)
		Total 9

Notes

(a)
M1: Differentiates twice and obtains expression for second derivative allowing for sign errors only
dM1: **Dependent on previous M mark.** Differentiates again and obtains expression for third derivative allowing for sign errors only (which could come from incorrect signs in trig identities)
ddM1: **Dependent on both previous M marks.** Obtains an answer of the correct form or values for p and q. Allow slips from their third derivative but any trig identities used can only have errors in sign.
A1: Fully correct expression or values for p and q
Note: If they decide not to work in sec x and tan x the main scheme still applies – see Alt next page.
Allow "meet in the middle" approaches that determine the values of p and q

(b)
M1: Uses sec x and their three derivatives to attempt to find values for $f\left(\dfrac{\pi}{3}\right)$, $f'\left(\dfrac{\pi}{3}\right)$, $f''\left(\dfrac{\pi}{3}\right)$ and $f'''\left(\dfrac{\pi}{3}\right)$ evidenced by values for each with at least 2 correct if no method seen. May be recovered by substitution of values into formula if not all listed.
dM1: **Dependent on previous M mark.** Uses a correct Taylor series with all four of their values. Series must be correct for their values but allow slips if a correct formula is quoted.

A1: Correct series with coefficients in a simplest form e.g., allow $\sqrt{12}$ for $2\sqrt{3}$, $\frac{23}{\sqrt{3}}$ for $\frac{23}{3}\sqrt{3}$ Condone absence of sec x =... or y =… Must have had a correct third derivative in (a), though need not have been simplified to the form required (and allow if there was a correct third derivative which was incorrectly simplified if the correct answer is found).

(c)

M1: Shows evidence of substitution of $\frac{7\pi}{24}$ into their series of the right form (powers of $\left(x-\frac{\pi}{3}\right)$). If only a value is given score M0 unless it is the correct 4 s.f. value for their series (1.636709263... allowing awrt 1.637 if (b) is correct). You may need to check the answer.

A1: 1.637 only (not awrt) from a correct series.

Note that sec $\frac{7\pi}{24}$ = 1.642679632...

6(a) Alt	$y=\sec x \Rightarrow y=(\cos x)^{-1} \Rightarrow \frac{dy}{dx}=-(\cos x)^{-2}(-\sin x)=\frac{\sin x}{\cos^2 x}$ $\frac{d^2y}{dx^2}=\frac{\cos^2 x\cos x-2\sin x\cos x(-\sin x)}{\cos^4 x}$	M1
	$\frac{d^2y}{dx^2}=\frac{\cos^3 x+2\cos x\left(1-\cos^2 x\right)}{\cos^4 x}=\frac{2\cos x-\cos^3 x}{\cos^4 x}=2(\cos x)^{-3}-(\cos x)^{-1}$ $\Rightarrow\frac{d^3y}{dx^3}=-6(\cos x)^{-4}(-\sin x)+(\cos x)^{-2}(-\sin x)$	**d**M1
	$\frac{d^3y}{dx^3}=\frac{6\sin x}{\cos^4 x}-\frac{\sin x}{\cos^2 x}=\frac{\sin x}{\cos^2 x}\left(\frac{6}{\cos^2 x}-1\right)=\sec x\tan x\left(6\sec^2 x-1\right)$	**dd**M1 A1
		(4)
Notes		
Mark as per main scheme allowing sign slips only - forms correct at each stage.		

Question Number	Scheme	Marks
7(a)	$z=y^{-2}\Rightarrow \frac{\mathrm{d}z}{\mathrm{d}y}=-2y^{-3}$ or $1=-2y^{-3}\frac{\mathrm{d}y}{\mathrm{d}z}$ $y^2=z^{-1}\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}z}=-z^{-2}$ $y=z^{-\frac{1}{2}}\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}z}=-\frac{1}{2}z^{-\frac{3}{2}}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}z}{\mathrm{d}x}\cdot\frac{\mathrm{d}y}{\mathrm{d}z}$ (oe) $\Rightarrow$ e.g., $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}y^3\frac{\mathrm{d}z}{\mathrm{d}x}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{\mathrm{d}z}{\mathrm{d}x}$	M1 A1
	$x\frac{\mathrm{d}y}{\mathrm{d}x}+y+4x^2y^3\ln x=0\Rightarrow$ e.g., $-\frac{1}{2}xy^3\frac{\mathrm{d}z}{\mathrm{d}x}+y+4x^2y^3\ln x=0$ $-\frac{1}{2}xz^{-\frac{3}{2}}\frac{\mathrm{d}z}{\mathrm{d}x}+z^{-\frac{1}{2}}+4x^2z^{-\frac{3}{2}}\ln x=0$	**d**M1
	$\frac{\mathrm{d}z}{\mathrm{d}x}-\frac{2}{xy^2}-8x\ln x=0$ $\frac{\mathrm{d}z}{\mathrm{d}x}-\frac{2z}{x}-8x\ln x=0$ $\Rightarrow\frac{\mathrm{d}z}{\mathrm{d}x}-\frac{2z}{x}=8x\ln x$ *	A1*
		(5)
(b)	$(\mathrm{IF}=)\ \mathrm{e}^{\int-\frac{2}{x}(\mathrm{d}x)}$	M1
	$=\mathrm{e}^{-2\ln x}\quad\left(=x^{-2}\right)$	A1
	$x^{-2}z=\int x^{-2}(8x\ln x)[\mathrm{d}x]$	M1
	E.g. Parts: $\int x^{-1}\ln x\,\mathrm{d}x$: $\left(u=\ln x,\ u'=x^{-1},v'=x^{-1},v=\ln x\right)$ $\Rightarrow I=(\ln x)^2-kI\Rightarrow I=p(\ln x)^2$ Or substitution: $t=\ln x,\ \frac{\mathrm{d}t}{\mathrm{d}x}=x^{-1}\ \Rightarrow I=k\int x^{-1}\ln x\cdot x\,\mathrm{d}t=k\int t\,\mathrm{d}t=pt^2$	M1
	$\int x^{-1}\ln x\,\mathrm{d}x=\frac{1}{2}(\ln x)^2\ [+c]$	A1
	$x^{-2}z=4(\ln x)^2+k\Rightarrow z=4x^2(\ln x)^2+kx^2$ $\Rightarrow y^2=\frac{1}{4x^2(\ln x)^2+kx^2}$ oe	A1
		(6)
		Total 11

Notes

(a)

B1: Any correct equation following differentiation of the given substitution. Could be implied.

M1: Uses a correct chain rule to obtain an equation linking $\frac{\mathrm{d}y}{\mathrm{d}x}\left(\text{or }\frac{\mathrm{d}x}{\mathrm{d}y}\right)$ **and** $\frac{\mathrm{d}z}{\mathrm{d}x}\left(\text{or }\frac{\mathrm{d}x}{\mathrm{d}z}\right)$

A1: Any correct equation

Note that the first three marks could be scored by a first step of, e.g.:

$$\frac{\mathrm{d}z}{\mathrm{d}x}=-2y^{-3}\frac{\mathrm{d}y}{\mathrm{d}x}\quad\text{or}\quad 2y\frac{\mathrm{d}y}{\mathrm{d}x}=-z^{-2}\frac{\mathrm{d}z}{\mathrm{d}x}\quad\text{or}\quad\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{\mathrm{d}z}{\mathrm{d}x}$$

dM1: **Dependent on previous M mark.** Substitutes their $\frac{\mathrm{d}y}{\mathrm{d}x}$ into differential equation (I). Need not replace fully for this mark.

A1*: Fully correct proof with no errors. There must be an intermediate line of working between the line where they substitute into DE(I) and the given answer.

Note: Use Review for any incorrect but potentially creditworthy attempts that "meet in the middle" or use (II) $\Rightarrow$ (I)

(b)

M1: Correct form for integrating factor. Allow $e^{\int \frac{k}{x}}$ and missing dx

A1: Correct IF in any form

M1: Obtains $\text{IF} \cdot z = \int \text{IF} \cdot (8x \ln x)[\text{d}x]$ Correct for their IF and z not y

M1: Must have achieved an integral of form $k\int x^{-1}(\ln x)[\text{d}x]$. Correct form following integration of $x^{-1} \ln x$ (constant not required):

$$\int x^{-1} \ln x \,\text{d}x \Rightarrow p(\ln x)^2 \; [+c]$$

The work in the scheme above is sufficient but you do not need to scrutinise the details, accept work that leads to the correct form.. Condone " $\ln x^2$ " for this mark

A1: Correct integration (constant not required) $\ln x^2$ is A0 unless recovered – must see $(\ln x)^2$ but allow $\ln^2 x$

A1: Any correct equation in $y^2 = \ldots$ form e.g., $y^2 = \left[x^2\left(4\ln^2 x + k\right)\right]^{-1}$ Constant may appear as a multiple, e.g., 8c

Question Number	Scheme	Marks
8	$r = 6(1+\cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$	
(a)	$\theta = 0,\ r = 6(1+\cos 0) \Rightarrow 12$ or $(12, 0)$	B1
		(1)
(b)	$\frac{d}{d\theta}(r\sin\theta)$: $= 6\sin\theta(1+\cos\theta) \Rightarrow$ $6\sin\theta(-\sin\theta)+6\cos\theta(1+\cos\theta)$ $= 6\sin\theta + 6\sin\theta\cos\theta \Rightarrow$ $6\cos\theta + 6(\cos^2\theta - \sin^2\theta)$ $= 6\sin\theta + 3\sin 2\theta \Rightarrow$ $6\cos\theta + 6\cos 2\theta$	M1
	$\Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$ $[(2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow]\quad \cos\theta = \frac{1}{2}\quad [\text{or} -1]$	**d**M1
	$\theta = \frac{\pi}{3} \qquad r = 6\left(1+\cos\frac{\pi}{3}\right) = 9 \quad$ or $\left(9, \frac{\pi}{3}\right)$	A1 A1
		(4)
8(c)	$[\tfrac{1}{2}]\int r^2\,d\theta = [\tfrac{1}{2}]\int 36(1+\cos\theta)^2\,[d\theta]$	M1
	$\int r^2\,d\theta = 36\int(1+2\cos\theta+\cos^2\theta)[d\theta] = 36\int(\tfrac{3}{2}+2\cos\theta+\tfrac{1}{2}\cos 2\theta)[d\theta]$	M1
	$\int(a+b\cos\theta+c\cos 2\theta)[d\theta] = a\theta \pm b\sin\theta \pm \frac{c}{2}\sin 2\theta$	M1
	$18\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_{...}^{...}$	A1
	$18\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}} = 18\left(\frac{\pi}{2}+\sqrt{3}+\frac{\sqrt{3}}{8}\right)[-0] \quad \left[= 9\pi + \frac{81}{4}\sqrt{3}\right]$	**d**M1
	E.g. $OB = 6\left(1+\cos\frac{'\pi'}{3}\right) \Rightarrow BQ = 6\left(1+\cos\frac{'\pi'}{3}\right)\sin\frac{'\pi'}{3} \quad \left[= \frac{9\sqrt{3}}{2}\right]$	M1
	$BP = 12 - 9\cos\frac{\pi}{3} = 12 - \frac{9}{2} = \frac{15}{2}$ $\Rightarrow$ area $OBPA = \frac{1}{2}\left(12+\frac{15}{2}\right)\left(\frac{9\sqrt{3}}{2}\right)$ or $\frac{1}{2}\times\frac{9}{2}\times\frac{9\sqrt{3}}{2}+\frac{9\sqrt{3}}{2}\times\frac{15}{2} \quad \left[= \frac{351}{8}\sqrt{3}\right]$	A1
	area of region $R = \frac{351}{8}\sqrt{3} - \frac{81}{4}\sqrt{3} - 9\pi = \frac{189}{8}\sqrt{3} - 9\pi$	A1
		(8)
		Total 13

Notes

(a)

B0: Correct values for θ and r or correct coordinates. Condone (0, 12).

(b)

M1: Differentiates $r\sin\theta$. Allow sign errors only. The "6" may be missing

dM1: **Dependent on previous M mark.** Uses correct identity/identities to reach a 3TQ in $\cos\theta$ and solves. Apply usual rules or by calculator must obtain at least one real consistent solution where $-1 \leqslant \cos\theta \leqslant 1$.

Accept $\theta = \frac{\pi}{3}$ following a correct derivative set equal to zero to imply the M if no incorrect working shown (by calculator or by inspection).

A1: Either r or θ correct.

A1: Both coordinates correct. Only accept $\frac{\pi}{3}$ and 9. Withhold last mark if additional answers offered and not rejected, but isw after correct answers seen if only a miscopy is made.

(c)

M1: Attempts $\int r^2 \, d\theta$ which may include the $\frac{1}{2}$ for the area formula. The multiple 36 may be missing or wrong. Condone poor squaring - allow this mark if there are only two terms.

M1: Uses $\cos^2\theta = \pm\frac{1}{2}\cos 2\theta \pm \frac{1}{2}$ and obtains an integrand of the form $a + b\cos\theta + c\cos 2\theta$ (constants may be uncollected)

M1: Integrates and obtains a form $a\theta \pm b\sin\theta \pm \frac{c}{2}\sin 2\theta$ (sign errors on trig terms only). May be two terms in θ

A1: Fully correct expression for the area of the sector after integration. Ignore limits but $\frac{1}{2}$ must have been used or appear later.

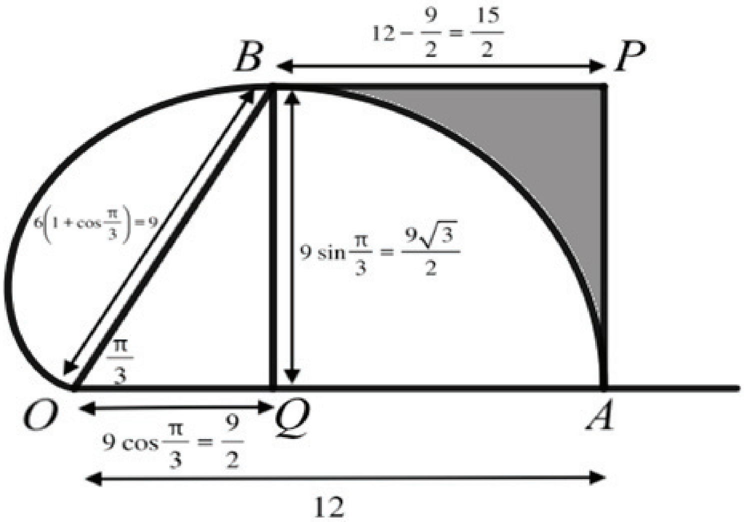
dM1: **Dependent on previous M mark.** Evidence of substitution of "correct" limits into the integral, so their $\frac{\pi}{3}\left(\text{provided } 0 < \theta < \frac{\pi}{2}\right)$ and the lower limit must be 0 and lead to zero but this can be implied by omission.

M1: Correct method for the perpendicular distance between l_1 and the initial line with their r and θ $\left(\text{provided } 0 < \theta < \frac{\pi}{2}\right)$ for point B. This may only be seen embedded in their attempt at an area, so may be implied if not explicit. E.g. look for "their r sin their $\frac{\pi}{3}$" or may find r again from the equation of curve.

A1: Correct expression for area of trapezium $OBPA$ - may be given as as a sum of separate areas, e.g. triangle OBQ + rectangle $QBPA$. Other formulations are possible.

A1: Correct answer in the correct form or $p = \frac{189}{8}$ or exact equivalent, $q = -9$

Useful diagram:



	TOTAL FOR PAPER: 75 MARKS

January 2024
WFM02 Further Pure Mathematics F2 Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>1</td><td colspan="2">$$\frac{1}{x+2} > 2x+3$$</td><td></td></tr>
<tr><td></td><td colspan="2">Examples:
$$\frac{1-(x+2)(2x+3)}{x+2} > 0 \Rightarrow 2x^2+7x+5=0$$ $$x+2 > (2x+3)(x+2)^2$$ $$\Rightarrow (x+2)\left(2x^2+7x+5\right)=0 \text{ or } 2x^3+11x^2+19x+10=0$$ $$\frac{1}{x+2}=2x+3 \Rightarrow (2x+3)(x+2)-1=2x^2+7x+5=0$$ Uses algebra to obtain a 3TQ, $(x+2)$ multiplied by a 3TQ or a 4TC. Allow slips and condone incorrect inequality signs but the first algebraic step should be otherwise appropriate so do not accept work with e.g., $(2x+3)(x+2)=0$. The “= 0” can be implied by solutions. Graphical attempts require intersections to be found algebraically. Squaring first is acceptable so allow M1 for obtaining a 5TQ $(4x^4+28x^3+73x^2+84x+35=0)$</td><td>M1</td></tr>
<tr><td></td><td>e.g., $(2x+5)(x+1)=0 \Rightarrow$
$x=-\frac{5}{2},-1$</td><td>Both -1 and $-\frac{5}{2}$ from appropriate work and no extra incorrect cvs. May only be seen in the solution set. Allow solving a 3TQ etc. by calculator.</td><td>A1</td></tr>
<tr><td></td><td>$x=-2$</td><td>Identifies -2 as a critical value. May only be seen in solution set. This is the only mark available if there is no algebraic manipulation seen. Allow from any or no working e.g., from $(2x+3)(x+2)=0$</td><td>B1</td></tr>
<tr><td></td><td colspan="2">$$\Rightarrow x < -\frac{5}{2},\ -2 < x < -1 \text{ or e.g., } (-\infty,-2.5),(-2,-1)$$ M1: For the regions $x < a,\ -2 < x < b$ with real cvs $a < -2$ and $b > -2$ but condone $b < x < -2$ as a notational slip for this mark.
Condone any non-strict inequality signs and poor notation for this mark. Not dependent but must follow an attempt at algebraic manipulation.
A1: Correct solution set in any form. Do not isw if the correct inequalities are subsequently incorrectly amended. Allow all marks even if an incorrect inequality sign was seen earlier in the working.</td><td>M1 A1</td></tr>
<tr><td></td><td colspan="2">Examples:
$-\frac{5}{2} > x$ or $-2 < x < -1$ M1 A1 $x < -\frac{5}{2}$ and $-2 < x < -1$ M1 A1
(Accept any word between the two correct regions)
$x < -\frac{5}{2},\ -1 < x < -2$ M1 A0 (notational slip)
$\left(-\infty,-\frac{5}{2}\right)\cap(-2,-1)$ M1A0 (incorrect symbol – allow “and”) $\left[-\infty,-\frac{5}{2}\right]\cup[-2,-1]$ M1A0
$x < -\frac{5}{2}\quad -2 < x \quad x < -1$ M0 A0 (insufficient)</td><td></td></tr>
<tr><td></td><td colspan="2"></td><td>(5)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 5</td></tr>
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Question Number	Scheme	Notes	Marks
2(a)	**(i)** $z=6-6\sqrt{3}\mathrm{i} \Rightarrow \lvert z\rvert=\sqrt{6^2+\left(6\sqrt{3}\right)^2}=12$	+12 only. Accept if just stated	**B1**
	(ii) e.g., $\arg z=-\arctan\frac{6\sqrt{3}}{6}$ Attempts an expression for a relevant angle. Look for $\pm\arctan\left(\pm\frac{6\sqrt{3}}{6}\right)$ or e.g., $\pm\tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$ If arctan is not seen allow e.g., $\tan\alpha=\frac{6\sqrt{3}}{6}\Rightarrow\alpha=\frac{\pi}{3}$ with α correct for their $\tan\alpha$ If using sin or cos the hypotenuse must be their 12		**M1**
	$\arg z$ or arg or argument (of z) $=-\frac{\pi}{3}$ * A correct proof with no incorrect work/statements. LHS required. **Allow** "$\theta=$" **if consistent**, e.g., $\theta=-\frac{\pi}{3}$ cannot follow "$\tan\theta=+\sqrt{3}$"		**A1***
(ii) Way 2	$z=12\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}\right)=12\left(\cos\left(-\frac{\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ or $12\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}$ or $\cos\theta=\frac{1}{2}$ or $\sin\theta=-\frac{\sqrt{3}}{2}$ [M1] $\Rightarrow \arg z=-\frac{\pi}{3}$ [A1*] M1: Factorises out 12 and writes in trig or exp form or identifies $\cos\theta=\frac{1}{2}$ and $\sin\theta=-\frac{\sqrt{3}}{2}$ A1: Acceptable statement with all work correct		
(ii) Way 3	$z=12\left(\cos\left(-\frac{\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ or $12\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}$ or $12\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}\right)=6-6\sqrt{3}\mathrm{i}$ [M1] $\Rightarrow \arg z=-\frac{\pi}{3}$ [A1*] M1: Assumes result, writes correctly for their 12 and attempts $a+ib$ form A1: Obtains $6-6\sqrt{3}\mathrm{i}$ and makes acceptable statement with all work correct		
			(3)
(b)	$z=\text{"12"}\left(\cos\left(-\frac{\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ or $\text{"12"}e^{-\frac{\pi}{3}\mathrm{i}}$ [no missing "i" unless recovered] Correct trig or exp. form with their 12. Could be implied by their z^4 in trig or exp. form e.g., $(\text{"12"}e^{-\frac{\pi}{3}\mathrm{i}})^4$ Allow equivalent values of θ e.g. $\frac{5\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{3}\right)=-\sin\left(\frac{\pi}{3}\right)$. Condone poor bracketing. Allow this mark if $+2k\pi,-2k\pi,\pm2k\pi$ appears with argument		**M1**
	$z^4=20736\left(\cos\left(-\frac{4\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{4\pi}{3}\right)\right)$ or $20736\left(\cos-\frac{4\pi}{3}+\mathrm{i}\sin-\frac{4\pi}{3}\right)$ or $20736e^{-\frac{4\pi}{3}\mathrm{i}}$ Correct z^4 in any form. 12^4 evaluated and arg. of $-\frac{4\pi}{3}$ (not just $4\times-\frac{\pi}{3}$) or $\frac{2\pi}{3}$ only although may use e.g., $\sin\left(-\frac{4\pi}{3}\right)=-\sin\left(\frac{4\pi}{3}\right)$. No "$k$"s. Condone an "unclosed" bracket. Only accept $-10368+10368\sqrt{3}\mathrm{i}$ or $20736\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}\right)$ **provided evidence of de Moivre.**		**A1**
			(2)

Question Number	Scheme	Notes	Marks
2(c)	$w=z^{\frac{1}{2}}=(\pm)\sqrt{"12"}\left(\cos\left(\frac{-\frac{\pi}{3}}{2}\right)+\text{i}\sin\left(\frac{-\frac{\pi}{3}}{2}\right)\right)$ or e.g., $(\pm)"2\sqrt{3}"e^{-\frac{\pi}{6}\text{i}}$ [no missing "i" unless recovered] Correct use of de Moivre's theorem with $-\frac{\pi}{3}$ and their 12 to attempt one square root. Allow work with argument of $\frac{5\pi}{3}$ for $-\frac{\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{6}\right)=-\sin\left(\frac{\pi}{6}\right)$. Condone poor bracketing. M0 if z^4 used for z. Allow this mark if $+2k\pi,-2k\pi,\pm 2k\pi$ appears with argument		M1
	$w=3-\sqrt{3}\text{i},-3+\sqrt{3}\text{i}$ oe A1ft: One correct exact root in $a+\text{i}b$ or $c(a+\text{i}b)$ form (a, b, c may be unsimplified but not numerical trig expressions) **ft their 12 only** i.e.$(\pm)\sqrt{"12"}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\text{i}\right)$ A1: Both exact roots (no others) correct in $a+\text{i}b$ form – a and b may be unsimplified (but not numerical trig expressions) e.g. accept $a=(\pm)\sqrt{12}\frac{\sqrt{3}}{2},(\pm)\frac{\sqrt{36}}{2}\quad b=(\mp)\frac{\sqrt{12}}{2},(\mp)\frac{2\sqrt{3}}{2}$ Accept $\pm\left(3-\sqrt{3}\text{i}\right)$ but just $\pm 3-\sqrt{3}\text{i}$ is A1 A0. Just $\pm\sqrt{3}\left(\sqrt{3}-\text{i}\right)$ is A1 A0		A1ft A1
	Note: $w^2=r^2(\cos 2\theta+i\sin 2\theta)=z\Rightarrow r,\ \theta,\ w=\ldots$ is an acceptable approach		**(3)**
Alt	$w^2=z\Rightarrow(a+\text{i}b)^2=a^2-b^2+2ab\text{i}=6-6\sqrt{3}\text{i}\Rightarrow a^2-b^2=6,\ 2ab=-6\sqrt{3}$ $b=-\frac{3\sqrt{3}}{a}\Rightarrow a^2-\frac{27}{a^2}=6\Rightarrow a^4-6a^2-27=(a^2-9)(a^2+3)=0\Rightarrow a^2=9,\ a=\pm 3,\ b=\mp\sqrt{3}$ **M1**: From a correct starting point, expands and equates real and imaginary parts to form two equations in a and b and obtains at least one value for both a and b $w=3-\sqrt{3}\text{i},-3+\sqrt{3}\text{i}$ **A1**: One correct exact root in $a+\text{i}b$ or $c(a+\text{i}b)$ form (a, b, c may be unsimplified) **A1**: Both exact roots (no others) correct in $a+\text{i}b$ form – a and b may be unsimplified		
			Total 8

Question Number	Scheme	Notes	Marks
3(a)	$\frac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}\times\frac{\sqrt{r(r+1)}-\sqrt{r(r-1)}}{\sqrt{r(r+1)}-\sqrt{r(r-1)}}$	A correct multiplier to rationalise the denominator seen or implied by correct work	**M1**
	$=\frac{r\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)}{r(r+1)-r(r-1)}=\frac{\sqrt{r(r+1)}-\sqrt{r(r-1)}}{2}$ or $A=\frac{1}{2}$ Correct expression or correct value for A. Condone poor notation if intention clear. There must be (minimal) correct supporting working.		**A1**
	Alternative: $A=\frac{r}{\left(\sqrt{r(r+1)}+\sqrt{r(r-1)}\right)\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)}=\frac{r}{r(r+1)-r(r-1)}$ or $\frac{r}{r^2+r-r^2+r}$ or $\frac{r}{2r}\Rightarrow A=\frac{1}{2}$ M1: Correctly makes A the subject A1: Correct completion with one intermediate fraction		
			(2)
(b)	$\sum_{r=1}^{n}\frac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}="\frac{1}{2}"\left(\begin{array}{c}\sqrt{1\times2}-\sqrt{1\times0}\ \left(=\sqrt{2}\ (-0)\right)\\ +\sqrt{2\times3}-\sqrt{2\times1}\ \left(=\sqrt{6}-\sqrt{2}\right)+\ldots\\ \ldots+\sqrt{(n-1)(n-1+1)}-\sqrt{(n-1)(n-1-1)}\ \left(=\sqrt{n(n-1)}-\sqrt{(n-1)(n-2)}\right)\\ +\sqrt{n(n+1)}-\sqrt{n(n-1)}\end{array}\right)$ **M1**: Applies the method of differences for $r=1$ and $r=n$ in the given expression with or without their A and obtains one correct row of these 2. **M1**: Applies the method of differences for $r=1$, $r=n$ and **either** $r=2$ or $r=n-1$ in the given expression with/without their A and obtains 2 correct rows of these 4. When considering how many rows are correct, if A has been clearly applied to any term then assess all rows as if A has been applied throughout. Condone missing bracket if their A is applied to a row e.g., "$\frac{1}{2}\times\ \sqrt{6}-\sqrt{2}$" <u>if it is recovered</u> but e.g., $\frac{\sqrt{6}}{2}-\sqrt{2}$ is an incorrect row. Ignore a row for $r=0$. Condone equivalent work with r or e.g., k used for n. **Both marks can be implied by a correct final expression with or without their A provided there are at least any two correct rows of differences** i.e., $"\frac{1}{2}"\left(\sqrt{n(n+1)}-0\right)$ or $\sqrt{n(n+1)}-0$		**M1** **M1**
	Note: row 3 is $"\frac{1}{2}"(\sqrt{12}\,(\text{or }2\sqrt{3})-\sqrt{6})$, row 4 is $"\frac{1}{2}"(\sqrt{20}\,(\text{or }2\sqrt{5})-\sqrt{12}\,(\text{or }2\sqrt{3}))$ If $\frac{1}{2}$ is fully applied the rows are: $\frac{\sqrt{2}}{2},\ \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2},\ \frac{\sqrt{12}}{2}(\text{or }\sqrt{3})-\frac{\sqrt{6}}{2},\ \frac{\sqrt{20}}{2}(\text{or }\sqrt{5})-\frac{\sqrt{12}}{2}(\text{or }\sqrt{3}),\ldots$ $\ldots,\frac{\sqrt{(n-2)(n-1)}}{2}-\frac{\sqrt{(n-2)(n-3)}}{2},\ \frac{\sqrt{n(n-1)}}{2}-\frac{\sqrt{(n-1)(n-2)}}{2},\ \frac{\sqrt{n(n+1)}}{2}-\frac{\sqrt{n(n-1)}}{2}$		
	$=\frac{1}{2}\sqrt{n(n+1)}$ oe e.g., $\frac{\sqrt{n^2+n}}{2}$	Correct expression in terms of n. No incorrect terms seen in differences work even if cancelled but condone the occasional poor bracket. There should be no "0" so e.g., $\frac{1}{2}\left(\sqrt{n(n+1)}-0\right)$ is A0 Does not require marks in (a)	**A1**
			(3)

Question Number	Scheme	Notes	Marks
3(c)	$\sum r = \frac{1}{2}n(n+1)$ e.g., sight of $k \times ... = \sqrt{\frac{1}{2}n(n+1)}$	States or uses the correct summation formula for integers	**M1**
	$\frac{k}{2}\sqrt{n(n+1)} = \sqrt{\frac{1}{2}n(n+1)} \Rightarrow \frac{k}{2} = \sqrt{\frac{1}{2}} \Rightarrow k = \sqrt{2}$	$\sqrt{2}$ only (Not $\pm$). $k = \sqrt{2}$ must not come from a clearly incorrect equation.	**A1**
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
4(a)	$y=\tan\left(\frac{3x}{2}\right)\Rightarrow y'=\frac{3}{2}\sec^2\left(\frac{3x}{2}\right)$	Any correct first derivative. Not implied by $y'\left(\frac{\pi}{6}\right)=3$	**B1**
	$\Rightarrow y''=2\times\frac{3}{2}\sec\left(\frac{3x}{2}\right)\times\sec\left(\frac{3x}{2}\right)\tan\left(\frac{3x}{2}\right)\times\frac{3}{2}$ $\left[=\frac{9}{2}\sec^2\left(\frac{3x}{2}\right)\tan\left(\frac{3x}{2}\right)\right]$	Attempts the second derivative achieving $k\sec^2\left(\frac{3x}{2}\right)\tan\left(\frac{3x}{2}\right)$ or unsimplified equivalent. Not implied by $y''\left(\frac{\pi}{6}\right)=9$	**M1**
	$\Rightarrow y'''=\frac{9}{2}\sec^2\left(\frac{3x}{2}\right)\sec^2\left(\frac{3x}{2}\right)\times\frac{3}{2}+\frac{9}{2}\tan\left(\frac{3x}{2}\right)\times2\times\frac{3}{2}\sec^2\left(\frac{3x}{2}\right)\tan\left(\frac{3x}{2}\right)$ $\left[=\frac{27}{4}\sec^4\left(\frac{3x}{2}\right)+\frac{27}{2}\sec^2\left(\frac{3x}{2}\right)\tan^2\left(\frac{3x}{2}\right)\right]$	**d**M1: Attempts third derivative using the product rule, achieving $P\sec^4\left(\frac{3x}{2}\right)+Q\sec^2\left(\frac{3x}{2}\right)\tan^2\left(\frac{3x}{2}\right)$ or unsimplified equivalent. **Requires previous M mark.** A1: Correct differentiation. Accept unsimplified. Not implied by $y'''\left(\frac{\pi}{6}\right)=54$	**dM1** **A1**
	If $\sec^2\left(\frac{3x}{2}\right)=\tan^2\left(\frac{3x}{2}\right)+1$ is used the identity must be used correctly and to score M marks expressions of consistent form should be achieved. Note that replacing $\sec^2\left(\frac{3x}{2}\right)$ in $y''\Rightarrow y'''=\frac{27}{4}\sec^2\left(\frac{3x}{2}\right)+\frac{81}{4}\sec^2\left(\frac{3x}{2}\right)\tan^2\left(\frac{3x}{2}\right)$		
	$y\left(\frac{\pi}{6}\right)=1, y'\left(\frac{\pi}{6}\right)=3, y''\left(\frac{\pi}{6}\right)=9, y'''\left(\frac{\pi}{6}\right)=54$ Attempts values (but allow numerical trig expressions) for y and their 3 derivatives at $\frac{\pi}{6}$ - accept stated values or insertion into a series of the correct form		**M1**
	$(y=)1+3\left(x-\frac{\pi}{6}\right)+\frac{9}{2!}\left(x-\frac{\pi}{6}\right)^2+\frac{54}{3!}\left(x-\frac{\pi}{6}\right)^3+\ldots$ Applies Taylor's correctly about $\frac{\pi}{6}$ with their values/numerical trig expressions. If values are not seen separately the work should imply a correct formula but allow a recognisable attempt at the series following the correct general formula stated. **Requires previous M mark.**		**dM1**
	$(y=)1+3\left(x-\frac{\pi}{6}\right)+\frac{9}{2}\left(x-\frac{\pi}{6}\right)^2+9\left(x-\frac{\pi}{6}\right)^3+\ldots$	Correct expression with coeffs. in simplest form. "$y=\ldots$" not required. **Requires all previous marks.** Score A0 if **clear evidence** of **<u>use</u>** of any wrong derivative expression.	**A1**
	If e.g. $y'''\left(\frac{\pi}{6}\right)$ is found by calculator but $y'(x)$ and $y''(x)$ were seen award 1100110 max		**(7)**
	Note: With responses that work in sin and cos throughout, to score M marks there must be no loss of form when differentiating (sign and coefficient errors only, also allowing sign errors with product/quotient formulae). Any use of identities must be correct. E.g: $y=\tan\left(\frac{3x}{2}\right)=\frac{\sin\left(\frac{3x}{2}\right)}{\cos\left(\frac{3x}{2}\right)}\Rightarrow y'=\frac{\frac{3}{2}\cos^2\left(\frac{3x}{2}\right)+\frac{3}{2}\sin^2\left(\frac{3x}{2}\right)}{\cos^2\left(\frac{3x}{2}\right)}$ $y''=\frac{\frac{9}{2}\cos^3\left(\frac{3x}{2}\right)\sin\left(\frac{3x}{2}\right)+\frac{9}{2}\cos\left(\frac{3x}{2}\right)\sin^3\left(\frac{3x}{2}\right)}{\cos^4\left(\frac{3x}{2}\right)}$ or $\frac{\frac{9}{2}\cos\left(\frac{3x}{2}\right)\sin\left(\frac{3x}{2}\right)}{\cos^4\left(\frac{3x}{2}\right)}$ or $\frac{9\sin\left(\frac{3x}{2}\right)}{2\cos^3\left(\frac{3x}{2}\right)}$ $y'''=\frac{\frac{27}{4}\cos^8\left(\frac{3x}{2}\right)+27\cos^6\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)+\frac{81}{4}\cos^4\left(\frac{3x}{2}\right)\sin^4\left(\frac{3x}{2}\right)}{\cos^8\left(\frac{3x}{2}\right)}=\frac{27}{4}+27\tan^2\left(\frac{3x}{2}\right)+\frac{81}{4}\tan^4\left(\frac{3x}{2}\right)$		

Question Number	Scheme	Notes	Marks
4(b)	$\left\{y\left(\frac{\pi}{4}\right)=\right\}1+3\left(\frac{\pi}{4}-\frac{\pi}{6}\right)+\frac{9}{2}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)^2+9\left(\frac{\pi}{4}-\frac{\pi}{6}\right)^3$ or $1+3\left(\frac{\pi}{12}\right)+\frac{9}{2}\left(\frac{\pi}{12}\right)^2+9\left(\frac{\pi}{12}\right)^3$ Substitutes $\frac{\pi}{4}$ into their expression for y of the correct form with at least the first three terms (series about $\frac{\pi}{6}$). Must have values (not unevaluated trig expressions). If only a decimal value is given then it must be the correct awrt 2.26 to score M1 (2.255314325). If there is no working they must obtain an expression with at least $a+b\pi+c\pi^2$ and correct exact ft a, b and c for their series or $1+\frac{\pi}{4}+c\pi^2$ with correct exact ft c		**M1**
	$=1+\frac{\pi}{4}+\frac{\pi^2}{32}+\frac{\pi^3}{192}$ or $1+\frac{1}{4}\pi+\frac{1}{32}\pi^2+\frac{1}{192}\pi^3$	Correct answer or values for A (32) and B (192). Can be awarded if full marks were not scored in (a).	**A1**
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5	$r^2 = 100\cos^2\theta + 20\cos\theta\tan\theta + \tan^2\theta$	Any correct expression for r^2	**B1**
	$\left\{\frac{1}{2}\right\}\int_0^{\frac{\pi}{3}} r^2\,\mathrm{d}\theta = \left\{\frac{1}{2}\right\}\int_0^{\frac{\pi}{3}}\left(100\cos^2\theta + 20\sin\theta + \tan^2\theta\right)\{\mathrm{d}\theta\}$	Attempts formula for the area with their r^2 which may not be expanded Condone missing $\frac{1}{2}$ and limits not required	**M1**
	$= \frac{1}{2}\int_0^{\frac{\pi}{3}}\left(50\left(1+\cos 2\theta\right) + 20\sin\theta + \sec^2\theta - 1\right)\{\mathrm{d}\theta\}$ **M1**: Uses $\cos^2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ **or** $\tan^2\theta = \pm\sec^2\theta \pm 1$ in their r^2 **M1**: Uses both $\cos^2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ **and** $\tan^2\theta = \pm\sec^2\theta\pm 1$ in their r^2 Both M marks can be scored without the integral and the $\frac{1}{2}$. Condone mixed variables. **A1**: Correct integral following $\cos^2\theta = \frac{1}{2}+\frac{1}{2}\cos 2\theta$ and $\tan^2\theta = \sec^2\theta - 1$. The $\cos\theta\tan\theta$ must be written as $\sin\theta$ (implied if appropriately integrated later). The $\frac{1}{2}$ is required (it may be seen later) but limits/ $\mathrm{d}\theta$ are not needed. Allow mixed variables if subsequent work recovers this.		**M1** **M1** **A1**
	$= \frac{1}{2}\left[49\theta + 25\sin 2\theta - 20\cos\theta + \tan\theta\right]_0^{\frac{\pi}{3}}$ or $\left[\frac{49}{2}\theta + \frac{25}{2}\sin 2\theta - 10\cos\theta + \frac{1}{2}\tan\theta\right]_0^{\frac{\pi}{3}}$ **M1**: Achieves **three** of the following four integrated forms: $k \to k\theta$ (at least once), $\cos 2\theta \to ...\sin 2\theta$, $\sin\theta \to ...\cos\theta$, $\sec^2\theta \to ...\tan\theta$. Ignore other terms if 3 of the above are satisfied. No $\frac{1}{2}$ or limits required. Condone mixed variables. **A1**: Correct integration including the $\frac{1}{2}$ (may be seen later). Limits not required. May be unsimplified e.g., 49θ seen as $50\theta - \theta$. Allow mixed variables if subsequent work recovers this.		**M1** **A1**
	$= \frac{1}{2}\left(\frac{49\pi}{3} + 25\sin\frac{2\pi}{3} - 20\cos\frac{\pi}{3} + \tan\frac{\pi}{3} - \left(0+0-20+0\right)\right)$ $\left\{= \frac{1}{2}\left(\frac{49\pi}{3} + \frac{25\sqrt{3}}{2} - 10 + \sqrt{3} + 20\right) \text{ or } \frac{49\pi}{6} + \frac{25\sqrt{3}}{4} - 5 + \frac{\sqrt{3}}{2} + 10\right\}$ Applies the correct limits to an expression of the form $p\theta + q\sin 2\theta + r\cos\theta + s\tan\theta$ $(p,q,r,s \neq 0)$ Allow slips but there must be a clear attempt to substitute, and they must only subtract the value of their r, e.g. if $r = -20$ work must have or imply ...– (–20) or +20. Allow mixed variables if the substitution recovers this.		**M1**
	$= \frac{1}{12}\left(98\pi + 81\sqrt{3} + 60\right)$	Correct answer or values for *a*, *b* & *c*	**A1**
	Note that there are other viable routes through the integration e.g., use of integration by parts		**(9)**
			Total 9

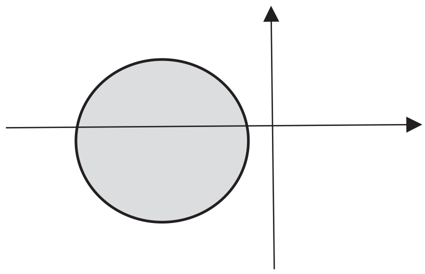
Question Number	Scheme	Notes	Marks
6	$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+13x=8e^{-3t} \qquad t \geqslant 0$		
(a)	$m^2+6m+13=0 \Rightarrow m=\frac{-6\pm\sqrt{36-52}}{2}$ $\{=-3\pm 2i\}$	Forms correct auxiliary equation and obtains a correct numerical expression for at least one root by formula or uses CTS (apply usual CTS rule below). One correct root if no working	**M1**
	CTS rule: $m^2+6m+13=0 \Rightarrow \left(m\pm\frac{6}{2}\right)^2 \pm q \pm 13=0,\ q\neq 0 \Rightarrow m=...$		
	CF examples: $(x=)\ e^{-3t}\left(A\cos 2t+B\sin 2t\right)$ or $(x=)\ Ae^{-3t}\cos(-2t)+Be^{-3t}\sin(-2t)$ or $(x=)\ Pe^{(-3+2i)t}+Qe^{(-3-2i)t}$ or $(x=)\ e^{-3t}\left(Pe^{2it}+Qe^{-2it}\right)$	Correct complementary function in any form, allow if the "$x=$" is missing or wrong and accept for this mark if the CF is given fully in terms of x instead of t.	**A1**
	PI: $\{x=\}\ \lambda e^{-3t}$	Correct form for the particular integral selected. Must include λe^{-3t} but accept with any extra terms that correctly disappear when coefficients found. Accept "PI=". If λe^{pt} is used $p=-3$ must be seen later.	**B1**
	$\frac{dx}{dt}=-3\lambda e^{-3t}$; $\frac{d^2x}{dt^2}=9\lambda e^{-3t}$ $\Rightarrow 9\lambda e^{-3t}+6\left(-3\lambda e^{-3t}\right)+13\lambda e^{-3t}=8e^{-3t}$	Differentiates a PI of **any** form twice (**provided** it has at least one constant and is a function of t) **and** substitutes into the equation. Allow only sign/coefficient errors only in the differentiation. Their PI must lead to non-zero derivatives.	**M1**
	$\Rightarrow 9\lambda-18\lambda+13\lambda=8 \Rightarrow \lambda=...\ (2)$	Proceeds to find the value of the constant **following use of a PI of the correct form**. Any unnecessary extra terms in the PI must be found to be zero	**dM1**
	$x=$"$e^{-3t}\left(A\cos 2t+B\sin 2t\right)$"$+2e^{-3t}$	Correct general solution ft on **their CF only** – any CF **provided** it has at least one constant and is in terms of t. **Must have $x=\ldots$** Do not allow if their CF is miscopied or mathematically changed	**A1ft**
	Work with a PI of the form λte^{-3t} is B0M1dM0A0 max even if $2e^{-3t}$ is obtained. Only condone incorrect variables if they are recovered but refer to the note for the first A1.		**(6)**

Question Number	Scheme	Notes	Marks
6(b)	$x=\frac{1}{2}$ at $t=0$ $\Rightarrow\frac{1}{2}=A+2\quad\left(\Rightarrow A=-\frac{3}{2}\right)$	Uses the initial condition for x in their GS to find a linear equation in one or two constants. Allow for GS = CF or CF + PI and the constant may come from the +PI	**M1**
	$x=e^{-3t}\left(A\cos 2t+B\sin 2t\right)+2e^{-3t}$ $\frac{dx}{dt}=e^{-3t}\left(-2A\sin 2t+2B\cos 2t\right)-3e^{-3t}\left(A\cos 2t+B\sin 2t\right)-6e^{-3t}$ Uses the **product rule** to differentiate their real GS obtaining an expression in terms of t of the correct form for their GS (sign and coefficient errors only – so do not allow e.g., $...e^{pt}\to...e^{qt}$). Allow for GS = CF or CF + PI and does not have to include constants. If they work with a complex function e.g., $x=Pe^{(-3+2i)t}+Qe^{(-3-2i)t}+2e^{-3t}$ progress is unlikely. This mark is not scored for work in (c)		**M1**
	$t=0,\frac{dx}{dt}=\frac{1}{2}\Rightarrow\frac{1}{2}=2B-3A-6\Rightarrow B=...(=1)$ Uses both initial conditions to find values for the 2 constants (no others) in their **GS = (CF with 2 constants) + PI(no constants)**. One constant must be found to be non-zero. **Requires both previous M marks.**		**ddM1**
	Examples: $x=e^{-3t}\left(-\frac{3}{2}\cos 2t+\sin 2t\right)+2e^{-3t}$ or $x=e^{-3t}\left(-\frac{3}{2}\cos 2t+\sin 2t+2\right)$ or $x=2e^{-3t}-\frac{3}{2}e^{-3t}\cos 2t+e^{-3t}\sin 2t$	Correct particular solution in any form in terms of t. **Must be x = ...** <u>unless</u> this was the only reason for final A0 in part (a) due to omission or e.g, "y = ..." was used	**A1**
			(4)
(c)	$\frac{dx}{dt}=e^{-3t}\left(3\sin 2t+2\cos 2t\right)-3e^{-3t}\left(-\frac{3}{2}\cos 2t+\sin 2t\right)-6e^{-3t}=0$ Sets an expression for $\frac{dx}{dt}=0$. Accept with any unfound constants provided $\frac{dx}{dt}=f(t)$		**M1**
	$\left(3\sin 2t+2\cos 2t\right)-3\left(-\frac{3}{2}\cos 2t+\sin 2t\right)-6=0$ Achieves an equation of the form $a\sin bt+c\cos bt+d=0$ <u>or equivalent with terms uncollected</u>. One of a and c non-zero and b and d non-zero. **Must follow a GS = CF + PI where two constants were found for the CF and one for the PI. Requires previous M mark**.		**dM1**
	$\cos 2t=\frac{12}{13}\Rightarrow t=0.1973955598...\Rightarrow x$ or $a=\frac{1}{2}e^{-3(0.1973...)}\left(4-3\times\frac{12}{13}+2\sin(2\times 0.1973...)\right)=..$ Finds a value of t from $\cos kt=c$ $\left(k\neq 1,\ -1<c<1\right)$ and uses their positive (or made positive) value of t to find a value of x (or a) via their PS. Accept a pair of stated values. **Requires both previous M marks.**		**ddM1**
	x or $a=0.553(1164729...)$	awrt 0.553	**A1**
			(4)
			Total 14

Question Number	Scheme	Notes	Marks
7(a) **Way 1**	$w=\dfrac{z-3}{2\text{i}-z}\Rightarrow 2\text{i}w-wz=z-3\Rightarrow z=\ldots$	Attempts to make z the subject and obtains any f(w)	**M1**
	$z=\dfrac{3+2\text{i}w}{w+1}$ or $\dfrac{-3-2\text{i}w}{-w-1}$	Any correct expression for z in terms of w	**A1**
	$=\dfrac{3+2\text{i}u-2v}{u+\text{i}v+1}\times\dfrac{u+1-\text{i}v}{u+1-\text{i}v}$ Applies $w = u + \text{i}v$ and a correct multiplier for their z seen or implied by a correct result from their z. Denominator must have had a "w". Note alternative route below.		**M1**
	$x+\text{i}y=\dfrac{3+2\text{i}u-2v}{u+\text{i}v+1}\times\dfrac{u+1-\text{i}v}{u+1-\text{i}v}=\dfrac{(3-2v)(u+1)+2uv+2u(u+1)\text{i}-(3-2v)v\text{i}}{(u+1)^2+v^2}$ $y=x+3 \text{ oe } \Rightarrow \dfrac{2u(u+1)-(3-2v)v}{(u+1)^2+v^2}=\dfrac{(3-2v)(u+1)+2uv}{(u+1)^2+v^2}+3$ Multiplies, extracts real and imaginary parts and uses them in the equation $y = x + 3$ (oe) to produce an equation in u and v only – no "i"s. Condone $y = \ldots\text{i}$ if recovered. Can follow slips with multiplier but denominator of z must have had a "w" Note: Just $2u(u+1)-(3-2v)v=(3-2v)(u+1)+2uv+3$ is M0 (lost denominators)		**M1**
	$2u(u+1)-(3-2v)v=(3-2v)(u+1)+2uv+3(u+1)^2+3v^2$ $\Rightarrow u^2+7u+v^2+v+6=0$	Expands and simplifies to obtain an equation of a circle with 4 or 5 real unlike terms. **All previous Ms required.**	**dddM1**
	Alternative for the above 3 marks (note this could be done by equating expressions for y) $x+\text{i}y=\dfrac{3+2\text{i}u-2v}{u+\text{i}v+1}\Rightarrow\left(x+\text{i}\left(x+3\right)\right)\left(u+1+\text{i}v\right)=3+2u\text{i}-2v$ **M1**: Applies $z=x+\text{i}y$, uses $y = x + 3$ and cross multiplies $x\left(u+1\right)-v\left(x+3\right)+\left(x+3\right)\left(u+1\right)\text{i}+xv\text{i}=3-2v+2u\text{i}$ $\Rightarrow ux+x-vx-3v=3-2v,\quad ux+x+3u+3+xv=2u$ $\Rightarrow x=\dfrac{3+v}{u+1-v},\quad x=\dfrac{-u-3}{u+1+v}$ **M1**: Equates real and imaginary parts and makes x the subject twice $\left(3+v\right)\left(u+1+v\right)=-\left(u+3\right)\left(u+1-v\right)\Rightarrow 3u+3+3v+uv+v+v^2=-u^2-u+uv-3u-3+3v$ $\Rightarrow u^2+v^2+7u+v+6=0$ **M1**: Equates expressions for x to obtain a circle equation with 4 or 5 real unlike terms		
	$\Rightarrow\left(u+\dfrac{7}{2}\right)^2+\left(v+\dfrac{1}{2}\right)^2=\dfrac{49}{4}+\dfrac{1}{4}-6=\dfrac{13}{2}\Rightarrow\text{centre: }\left(-\dfrac{7}{2},-\dfrac{1}{2}\right)\text{ radius: }\dfrac{\sqrt{26}}{2}\text{ or }\sqrt{\dfrac{13}{2}}$ **M1**: Extracts the centre **and/or** radius from their **circle** equation, however obtained, with 4 or 5 real unlike terms. Circle equation must not be in terms of z or w. They must get one correct coordinate (but condone wrong sign) or the correct radius for their circle. May use $u^2+v^2+2gu+2fv+c=0\Rightarrow\text{centre}:\left(-g,-f\right),\text{ radius}=\sqrt{g^2+f^2-c}$ **A1**: For a correct centre or radius **from a correct circle equation** **A1**: For correct centre and radius **from a correct circle equation** Centre as coordinates, x/u=..., y/v =... or as $-\dfrac{7}{2}-\dfrac{1}{2}\text{i}$ and allow $\left(-\tfrac{7}{2},-\tfrac{1}{2}\text{i}\right)$ Allow exact equivalents for coordinates/radius		**M1** **A1** **A1**
			(8)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="5">7(a)

Way 2</td><td>$w=\frac{z-3}{2\mathrm{i}-z}=\frac{x+\mathrm{i}y-3}{2\mathrm{i}-x-\mathrm{i}y}=\frac{x-3+\mathrm{i}(x+3)}{2\mathrm{i}-x-\mathrm{i}(x+3)}$
[Note that it is possible to replace x with $y-3$]</td><td>M1: Uses $z = x + \mathrm{i}y$ and $y = x + 3$ in the given transformation
A1: Correct expression for w in terms of x</td><td>M1
A1</td></tr>
<tr><td>$\frac{x-3+\mathrm{i}(x+3)}{-x-\mathrm{i}(x+1)}=u+\mathrm{i}v\Rightarrow x-3+\mathrm{i}(x+3)=-xu+v(x+1)-\mathrm{i}u(x+1)-\mathrm{i}vx$</td><td>Applies $w = u + \mathrm{i}v$ and multiplies</td><td>M1</td></tr>
<tr><td>$x-3=-ux+vx+v,\quad x+3=-ux-u-vx$
$x=\frac{3+v}{1+u-v},\quad x=\frac{-3-u}{1+u+v}$</td><td>Equates real and imaginary parts and makes x the subject twice</td><td>M1</td></tr>
<tr><td>$3+3u+3v+v+uv+v^2=-3-3u+3v-u-u^2+uv$
$\Rightarrow u^2+v^2+7u+v+6=0$</td><td>Equates expressions for x to obtain a circle equation with 4 or 5 real unlike terms.
All previous Ms required.</td><td>dddM1</td></tr>
<tr><td colspan="2">$\Rightarrow\left(u+\frac{7}{2}\right)^2+\left(v+\frac{1}{2}\right)^2=\frac{49}{4}+\frac{1}{4}-6=\frac{13}{2}\Rightarrow$ centre: $\left(-\frac{7}{2},-\frac{1}{2}\right)$ radius: $\frac{\sqrt{26}}{2}$ or $\sqrt{\frac{13}{2}}$
M1: Applies a correct process to extract the centre and/or radius from a circle equation, however obtained, with 4 or 5 real unlike terms. One correct coordinate (but condone wrong sign) or radius correct for their circle.
May use $u^2+v^2+2gu+2fv+c=0\Rightarrow$ centre: $(-g,-f)$, radius $=\sqrt{g^2+f^2-c}$
A1: For correct centre or radius from a correct circle equation
A1: For correct centre and radius from a correct circle equation
Centre as coordinates, $x/u=\ldots$, $y/v=\ldots$ or as $-\frac{7}{2}-\frac{1}{2}\mathrm{i}$ and allow $\left(-\frac{7}{2},-\frac{1}{2}\mathrm{i}\right)$ (8)</td><td>M1
A1
A1</td></tr>
<tr><td rowspan="6">Way 3</td><td>e.g., 3 points on line are (0,3), (1,4) and (2,5)
or $z_1=3\mathrm{i},\ z_2=1+4\mathrm{i},\ z_3=2+5\mathrm{i}$</td><td>Attempts three points/complex numbers on $y = x + 3$ with 2 correct</td><td>M1</td></tr>
<tr><td>$w=\frac{z-3}{2\mathrm{i}-z}\Rightarrow w_1=\frac{3\mathrm{i}-3}{-\mathrm{i}}\quad w_2=\frac{-2+4\mathrm{i}}{-1-2\mathrm{i}}\quad w_3=\frac{-1+5\mathrm{i}}{-2-3\mathrm{i}}$</td><td>Correct transformed complex numbers</td><td>A1</td></tr>
<tr><td colspan="2">$w_1=\frac{3\mathrm{i}-3}{-\mathrm{i}}\times\frac{\mathrm{i}}{\mathrm{i}}\quad w_2=\frac{-2+4\mathrm{i}}{-1-2\mathrm{i}}\times\frac{-1+2\mathrm{i}}{-1+2\mathrm{i}}\quad w_3=\frac{-1+5\mathrm{i}}{-2-3\mathrm{i}}\times\frac{-2+3\mathrm{i}}{-2+3\mathrm{i}}$
At least two correct multipliers to remove "i" from denominator seen or implied (one if (–1, 2) used). Requires 2 correct points/complex numbers on line</td><td>M1</td></tr>
<tr><td>$w_1=-3-3\mathrm{i}\quad w_2=-\frac{6}{5}-\frac{8}{5}i\quad w_3=-1-\mathrm{i}$</td><td>Two correct complex numbers in $a + \mathrm{i}b$ form or as points</td><td>M1</td></tr>
<tr><td>e.g., $x^2+y^2+2gx+2fy+c=0\Rightarrow\begin{array}{r}6g+6f-c=18\\ \frac{12}{5}g+\frac{16}{5}f-c=0\\ 2g+2f-c=0\end{array}$</td><td>Uses a correct general equation of a circle to form three simultaneous equations.
All previous Ms required.</td><td>dddM1</td></tr>
<tr><td colspan="2">$\Rightarrow g=\frac{7}{2},\ f=\frac{1}{2},\ c=6\Rightarrow$ centre $(-g,-f)$: $\left(-\frac{7}{2},-\frac{1}{2}\right)$ radius $=\sqrt{g^2+f^2-c}=\frac{\sqrt{26}}{2}$ or $\sqrt{\frac{13}{2}}$
M1: Solves and obtains at least one correct coordinate (but condone wrong sign) or radius for their constants
A1: Correct centre or radius from correct work
A1: Correct centre and radius from correct work (8)</td><td>M1
A1
A1</td></tr>
</table>

Question Number	Scheme	Notes	Marks
7(b) (i) & (ii)		M1: **Any circle with the whole interior indicated**. Ignore any inconsistencies with their stated centre, value for radius (which may have been negative) or circle equation. If shaded, consider the shaded area but if not allow any credible indication such as an "R" inside the circle unless they have **clearly** indicated a segment. A1: **Correct circle drawn in the correct position with whole interior shaded**. Entirely in quadrants 2 & 3 and centre if marked in Q3 (if not marked then more than half of the circle in Q3). Condone if it appears that the area above the x-axis is greater than the area below provided the centre is indicated in Q3. **Must be shaded** but does not require a label. Circumference may be dotted/dashed line. Ignore incorrect labelling of centre/axes/intersections but **requires full marks in (a).**	**M1 (B1 on ePen)** **A1 (B1 on ePen)**
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
8(a)	Allow "single fraction" to be implied by sum/difference of fractions with same denominator or a product of fractions. No further fractions in numerator/denominator.		

	$\cot 2x\{+\tan x\} = \dfrac{\cos 2x}{\sin 2x}\left\{+\dfrac{\sin x}{\cos x}\right\}$	Uses $\cot 2x = \dfrac{\cos 2x}{\sin 2x}$ or e.g., $\dfrac{\cos 2x}{2\sin x\cos x}$	**M1**
	$\dfrac{\cos 2x + 2\sin^2 x}{2\sin x\cos x} \Rightarrow$ e.g., $\dfrac{1-2\sin^2 x+2\sin^2 x}{2\sin x\cos x}$ or $\dfrac{\cos^2 x-\sin^2 x+2\sin^2 x}{2\sin x\cos x}$ $\dfrac{2\cos^2 x-1+2\sin^2 x}{\sin 2x}$ or $\dfrac{\cos 2x+1-\cos 2x}{\sin 2x}$ OR $\dfrac{\cos 2x+\tan x\sin 2x}{\sin 2x} \Rightarrow$ $\Rightarrow \dfrac{\cos 2x+\frac{\sin x}{\cos x}\times 2\sin x\cos x}{\sin 2x} \Rightarrow$ e.g., $\dfrac{1-2\sin^2 x+2\sin^2 x}{\sin 2x}$ OR $\dfrac{\cos 2x\cos x+\sin x\sin 2x}{\sin 2x\cos x} \Rightarrow$ $\dfrac{\cos x}{\sin 2x\cos x}$ or $\dfrac{\cos^3 x-\sin^2 x\cos x+2\sin^2 x\cos x}{\sin 2x\cos x}$	Uses sufficient correct identities e.g., $\cos 2x = 1-2\sin^2 x$ $\cos 2x = \cos^2 x-\sin^2 x$ $\cos 2x = 2\cos^2 x-1$ $2\sin^2 x = 1-\cos 2x$ $\cos 2x\cos x+\sin x\sin 2x = \cos(2x-x)$ to obtain a correct single fraction with numerator in terms of $\sin x$ and/or $\cos x$ or "$\cos 2x+1-\cos 2x$". A qualifying fraction must be seen before $\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x}$ Condone poor notation.	**A1 (M1 on ePen)**
	$=\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x} = \operatorname{cosec} 2x$ *	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. $\sin x^2$ for this mark.	**A1***
			(3)
Alt	$\cot 2x\{+\tan x\} = \dfrac{1-\tan^2 x}{2\tan x}\{+\tan x\}$	Uses $\cot 2x = \dfrac{1-\tan^2 x}{2\tan x}$	**M1**
	$\dfrac{1-\tan^2 x+2\tan^2 x}{2\tan x} \Rightarrow$ e.g., $\dfrac{\tan^2 x+1}{2\tan x} \Rightarrow \dfrac{\left(\frac{\sin x}{\cos x}\right)^2+1}{2\frac{\sin x}{\cos x}} \Rightarrow \dfrac{\cos x\left(\sin^2 x+\cos^2 x\right)}{2\cos^2 x\sin x}$ or $\dfrac{\tan^2 x+1}{2\tan x}\left\{\times\dfrac{\cos x}{\cos x}\right\} \Rightarrow \dfrac{\sin^2 x+\cos^2 x}{2\sin x\cos x}$ or $\dfrac{\sec^2 x}{2\tan x}$ or $\dfrac{\cos x}{2\cos^2 x\sin x}$	Uses correct identities e.g., $\tan x = \dfrac{\sin x}{\cos x}$ oe to obtain a correct single fraction in $\sin x$ and $\cos x$ but allow $\dfrac{\sec^2 x}{2\tan x}$ following use of $\sec^2 x = 1+\tan^2 x$ A qualifying fraction must be seen before $\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x}$ Condone poor notation.	**A1 (M1 on ePen)**
	$\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x} = \operatorname{cosec} 2x$ *	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. $\sin x^2$ for this mark.	**A1***
			(3)

Question Number	Scheme	Notes	Marks
8(b)	Examples:		**M1** **A1**

	$y^2 = w\sin 2x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}w}{\mathrm{d}x}\sin 2x + 2w\cos 2x$ or $y = w^{\frac{1}{2}}(\sin 2x)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}w^{\frac{1}{2}}(\sin 2x)^{-\frac{1}{2}}(2\cos 2x) + \frac{1}{2}w^{-\frac{1}{2}}\frac{\mathrm{d}w}{\mathrm{d}x}(\sin 2x)^{\frac{1}{2}}$ or $w = \frac{y^2}{\sin 2x} \Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} = \frac{2y\sin 2x\frac{\mathrm{d}y}{\mathrm{d}x} - y^2.2\cos 2x}{\sin^2 2x}$ or $w = y^2\mathrm{cosec}2x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{cosec}2x - 2y^2\mathrm{cosec}2x\cot 2x$ **M1**: Attempts the differentiation of the given substitution using the product/quotient and chain rules **and** obtains an **equation** in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}w}{\mathrm{d}x}$ of the correct form (sign/coefficient errors only and allow sign errors with quotient/product rule). This mark is not available for work in $\frac{\mathrm{d}y}{\mathrm{d}w}$ or $\frac{\mathrm{d}w}{\mathrm{d}y}$ unless appropriate work follows to achieve an equation in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}w}{\mathrm{d}x}$ of the correct form. **A1**: Correct differentiation	
	$y\frac{\mathrm{d}y}{\mathrm{d}x} + y^2\tan x = \sin x \rightarrow$ e.g., $\frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\sin 2x + 2w\cos 2x\right) + w\sin 2x\tan x = \sin x$ A recognisable attempt to eliminate y from the original equation to obtain an equation involving $\frac{\mathrm{d}w}{\mathrm{d}x}$, w and x only . Not dependent.	**M1**
	$\Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} + 2w\left(\cot 2x + \tan x\right) = \frac{2\sin x}{\sin 2x}$ $\Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} + 2w\mathrm{cosec}\ 2x = \sec x$ * Fully correct work leading to the given equation with $2w\left(\cot 2x + \tan x\right)$ or e.g., $2w\cot 2x + 2w\tan x$ clearly replaced by $2w\mathrm{cosec}\ 2x$ but allow cot $2x$ written as $\frac{1}{\tan 2x}$ or $\frac{\cos 2x}{\sin 2x}$ and/or tan x written as $\frac{\sin x}{\cos x}$ If the result in (a) is not clearly used there must be full equivalent work. Allow use of "csc $2x$"	**A1***
		(4)

Question Number	Scheme	Notes	Marks

8(c)	$\frac{dw}{dx}+2w\,\text{cosec}\,2x=\sec x \Rightarrow \text{IF}=e^{2\int \text{cosec}\,2x\,dx}=\tan x$ or $e^{-\ln(\text{cosec}\,2x+\cot 2x)} \Rightarrow \frac{1}{\text{cosec}2x+\cot 2x}$ or $\frac{1}{\cot x}$ or $\tan x$	**M1**: $e^{2\int \text{cosec}\,2x(dx)}$ condoning omission of one or both "2"s **A1**: $\tan x$ oe Allow $k\tan x$ e.g., $e^{2c}\tan x$ Not just $e^{\ln(\tan x)}$	**M1** **A1**
	$\Rightarrow w\text{"}\tan x\text{"}=\int \text{"}\tan x\text{"}\sec x\,\{dx\}$	Correctly applies their integrating factor to the equation, i.e., $\Rightarrow \text{IF}\times w=\int \text{IF}\times \sec x\,\{dx\}$ Allow equivalents for sec x. Condone "y" used for "w" for this mark.	**M1**
	$\Rightarrow w\tan x=\sec x(+c)$	Correct equation oe with or without constant.	**A1**
	Using $\text{IF}=\frac{1}{\text{cosec}2x+\cot 2x} \Rightarrow$ RHS of $\int \frac{\sec x}{\text{cosec}2x+\cot 2x}dx$ which is likely to need rewriting as $\int \tan x\sec x\,dx$ Note that IBP on $\sec x\tan x$ by writing it as $\sec^2 x\sin x$ can lead to $\sin x\tan x+\cos x(+c)$ Use Review for any attempts at integration you are unsure about.		
	e.g., $y^2=w\sin 2x$ and $w\tan x=\sec x+c \Rightarrow \frac{y^2}{\sin 2x}\tan x=\sec x+c$ $\Rightarrow y^2=\ldots \left\{\frac{\sin 2x}{\tan x}(\sec x+c)\right\}$ Substitutes for w correctly and reaches $y^2=\ldots$ Their $y^2=\ldots$ must be consistent with **their** equation in w and x that immediately followed their integration. **This mark requires both previous M marks and an attempt at integration that includes a "+ c"** A further example is: $w=\text{cosec}\,x+\frac{c}{\tan x} \Rightarrow y^2=\text{cosec}\,x\sin 2x+\frac{c\sin 2x}{\tan x}$		**ddM1**
	$\left\{\text{e.g., } y^2=\frac{2\sin x\cos^2 x}{\sin x}\left(\frac{1}{\cos x}+c\right)\Rightarrow\right\}$ $y^2=2\cos x+A\cos^2 x$ Any correct $y^2=\ldots$ equation with RHS fully in terms of $\cos x$. E.g. accept $y^2=2\cos x+2c\cos^2 x \quad y^2=\cos x(2+A\cos x) \quad y^2=2\cos^2 x\left(\frac{1}{\cos x}+c\right)$ Ignore any inconsistencies with the constant e.g., $2c$ later written as c		**A1**
			(6)
			Total 13