

Question Number	Scheme	Notes	Marks
1(a)	$\frac{10x}{x-6} = 2x+12 \Rightarrow 10x = 2x^2 - 72$ $x^2 - 5x - 36 = 0 \Rightarrow (x+4)(x-9) = 0$ $\Rightarrow x = -4, 9$	M1: Cross multiplies, forms 3TQ and solves (usual rules) A1: Both correct values	M1 A1
(2)			
(b)(i)	$-4 < x < 6, \quad x > 9$	$a < x < 6, \quad x > b$ with their $a < 0$ and $b > 6$	B1ft
(b)(ii)	$\frac{10x}{x-6} = -(2x+12) \Rightarrow$ $x^2 + 5x - 36 = 0 \Rightarrow x = -9, 4$	From a correct equation, cross multiplies, forms 3TQ and solves (usual rules)	M1
	$x < -9, \quad -4 < x < 4, \quad x > 9$	M1: Correct regions $x < \alpha, \beta < x < \gamma, \quad x > \delta$ for their values with $\alpha < \beta < 0 < \gamma < \delta$ condoning any non-strict inequality signs and $x \neq 6$ A1: Fully correct range of values in any suitable notation	M1 A1
(4)			
Total 6			

Additional Notes

(a)+(b) Answers only without an algebraic method will score M0, but the roots (product = -36, sum = ±5) are deducible from the simplified (leading coefficient 1) quadratic in this case, so accept correct answers (or allow sign error) following the correct simplified quadratic to imply the M. If an error means a quadratic does not have simple roots, a method must be shown to solve. However, allow answers following an application of the formula even if the simplification would seem to warrant a calculator.

(a)

M1: Alternative to cross multiply is putting over a common denominator of $x-6$ and forming a quadratic in the numerator, or multiplying through by $(x-6)^2$, expanding, gathering and then factoring out the $x-6$ to leave a quadratic. Method must be clear, solutions by calculator are not acceptable.

A1: Both correct values and no spurious extras.

(b) **Note on the grids (b)(ii) will appear as MAA but is being marked as MMA**

B1ft: Correct region following their values as long as they satisfy that one is less than 0 and the other greater than 6 to match the given sketch. Accept set notation or interval notation. Accept with “and” or “or” between but set notion must use union, not intersection.

M1: Must be attempting to solve the correct initial equation. Same notes as the M1 in part (a) apply.

M1: This mark is essentially for demonstrating they understand the shape of the region that is needed. They may draw on the sketch to help. The solutions to their quadratics will have to fit the indicated pattern but if they made sign errors such that they had -9 and 4 in (a) and -4 and 9 in (b) this mark is recoverable (they will have lost the A in (a) and B1ft in (b) already).

A1: Correct answer, allowing the similar recovery as noted above. Allow if scored from calculator solutions to the quadratic equation. (Same criteria.)

Note: Candidates can recover all the marks in (b)(ii) if they start again by squaring both sides, but algebra would have to be complete and non-calculator method.

$$(2x+12)^2 > \left(\frac{10x}{x-6}\right)^2 \Rightarrow 4((x-6)(x+6))^2 > 100x^2 \Rightarrow (x^2-36)^2 - 25x^2 > 0 \Rightarrow x^4 - 72x^2 + 36^2 - 25x^2 > 0$$

$$\Rightarrow x^4 - 97x^2 + 4^2 \times 9^2 > 0 \Rightarrow (x^2-16)(x^2-81) > 0 \Rightarrow \text{CVs } x = \pm 4, \pm 9 \Rightarrow -9 < x, -4 < x < 4, x > 9$$

Answers are not likely to be this concise and will probably not succeed in reaching the answers via non-calculator means.

Question Number	Scheme	Notes	Marks
2(a)	$(x+2)\frac{dy}{dx} + y = 3x^2 + 6x \Rightarrow$ $\frac{dy}{dx} + \frac{y}{x+2} = \frac{3x(x+2)}{x+2}$ $\Rightarrow \frac{dy}{dx} + \frac{y}{x+2} = 3x$	M1: Reaches $\frac{dy}{dx} + f(x)y = g(x)$ A1: Correct equation in required form. Correct answer implies both marks.	M1 A1
(2)			
(b)	$\text{IF} = \left\{ e^{\int \frac{1}{x+2} (dx)} \right\} = e^{\ln(x+2)} = x+2$	Correct attempt at the IF for their $f(x)$ – must attempt the integration but need not be fully simplified (may be implied by working)	M1
	$"(x+2)" y = \int "3" x "(x+2)" (dx)$	Applies their IF correctly with their kx	M1
	$(x+2)y = x^3 + 3x^2 (+c)$	Correct equation after integration condoning omission of the constant	A1
	$x = 4, y = 18 \Rightarrow c = 108 - 64 - 48 = -4$	Substitutes correct values to find a value for c in their integrated equation	M1
	$y = \frac{x^3 + 3x^2 - 4}{x+2} = \frac{(x+2)(x+2)(x-1)}{x+2}$ $\Rightarrow y = (x+2)(x-1) \text{ or } x^2 + x - 2$	A1: Correct particular solution with $g(x)$ in any form. Condone missing “$y = \dots$” A1: y or $g(x) = (x+2)(x-1)$ or $x^2 + x - 2$	A1 A1
(6)			
Total 8			
Note : The question can be completed via a complementary function and particular integral approach, as below. If the CF is never considered a maximum of 011000 is possible.			
(b) Alt	$(x+2)\frac{dy}{dx} + y = 0 \Rightarrow \frac{1}{y} \frac{dy}{dx} = -\frac{1}{x+2}$ $\Rightarrow \ln y = -\ln(x+2) + c$ $\Rightarrow \ln y = \ln \frac{A}{x+2} \Rightarrow y = \frac{A}{x+2}$	Considers the homogeneous equation and makes progress integrating to an equation $y = \dots$ with constant of integration.	M1
	$y = ax^2 + bx + c \Rightarrow \frac{dy}{dx} = 2ax + b$ $\Rightarrow (x+2)(2ax+b) + ax^2 + bx + c = 3x^2 + 6x$ $x^2 : 3a = 3 \quad x : 4a + 2b = 6 \quad x^0 : 2b + c = 0$ $\Rightarrow a = \dots, b = \dots, c = \dots$	Tries the PI $y = ax^2 + bx + c$, differentiates and substitutes into equation to find a, b and c	M1
	$y = x^2 + x - 2$	Correct particular integral	A1
	$y = x^2 + x - 2 + \frac{A}{x+2} \Rightarrow 18 = 16 + 4 - 2 + \frac{A}{6}$ $\Rightarrow A = \dots ; A = 0$	Uses (4, 18) in their general solution to find the constant ; Correctly shows $A = 0$	M1 A1
	$\Rightarrow y = (x+2)(x-1) \text{ or } x^2 + x - 2$	Correct answer following attempt to combine the CF and PI. Accept y or $g(x)$ as before.	A1

Additional Notes:

No need to see method for solving the cubic – if the correct answer follows directly from the unfactorized cubic that is acceptable for the final A.

Question Number	Scheme	Notes	Marks
3	$y = \arcsin 2x \quad \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} \left\{ = 2(1-4x^2)^{-\frac{1}{2}} \right\}$		
(a)	$\frac{d^2y}{dx^2} = -(1-4x^2)^{-\frac{3}{2}}(-8x) \left\{ = 8x(1-4x^2)^{-\frac{3}{2}} \right\}$	Obtains correct form for $\frac{d^2y}{dx^2}$	M1
	$\frac{d^3y}{dx^3} = (8x) \left(-\frac{3}{2} \right) (1-4x^2)^{-\frac{5}{2}}(-8x) + 8(1-4x^2)^{-\frac{3}{2}}$ or $\frac{8(1-4x^2)^{\frac{3}{2}} + 96x^2(1-4x^2)^{\frac{1}{2}}}{(1-4x^2)^3}$	Uses product/quotient rule to obtain correct form for $\frac{d^3y}{dx^3}$	dM1
	$= \frac{96x^2 + 8(1-4x^2)}{(1-4x^2)^{\frac{5}{2}}} = \frac{64x^2 + 8}{(1-4x^2)^{\frac{5}{2}}}$	Correct answer in the correct form	A1
(3)			
Allow full access to the remaining marks with an incorrect value of A			
(b)	$y(0)=0, y'(0)=2, y''(0)=0, y'''(0)=8$ $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$ $(\Rightarrow \arcsin 2x) \approx 2x + \frac{4}{3}x^3$	M1: Finds values for $y(0), y'(0), y''(0)$ and $y'''(0)$ and obtains a consistent Maclaurin expansion A1: Correct expression	M1 A1
(2)			
(c)	$e^{3x} \arcsin 2x = (1+3x+\dots) \left(2x + \frac{4}{3}x^3 + \dots \right) = \dots$	Attempts to find the expansion of e^{3x} and multiplies by their expression for $\arcsin 2x$	M1
	$= \left(1+3x+\frac{(3x)^2}{2}+\dots \right) \left(2x + \frac{4}{3}x^3 + \dots \right) = \dots$ $= 2x + 6x^2 + 9x^3 + \frac{4}{3}x^3 + \dots$	For a correct expansion of e^{3x} up to the x^2 term multiplied by their expression for $\arcsin$ with at least one correct term obtained.	A1
	$\Rightarrow e^{3x} \arcsin 2x \approx 2x + 6x^2 + \frac{31}{3}x^3$	Fully correct expression	A1
(3)			
Total 8			

Additional Notes:

(a)

M1: Attains the form $Kx(1-4x^2)^{-\frac{3}{2}}$, $K \neq 0$ or equivalent form, e.g. by quotient rule $\frac{0 - Kx(1-4x^2)^{-\frac{1}{2}}}{1-4x^2}$

dM1: Differentiates again, any correct unsimplified form is fine (chain rule must have been applied).

A1: Reaches the correct simplified answer.

Note: Full marks are available in (b) and (c) if an incorrect A was found in part (a).

(b)

M1: Attempts all the values for the derivatives and finds the Maclaurin expansion. If the formula is quoted correctly allow any values appearing in the formula as their values if they are not separately stated. If no formula is given and values not stated, the denominators in their expansion must be seen to imply a correct attempt at the formula.

A1: The $\arcsin 2x$ need not be stated, the A can be awarded for a correct expression. May have come from an incorrect A in (a).

(c)

M1: Attempts the series for e^{3x} and multiplies by their answer to part (b) – an attempt at the multiplication must be carried out but need not be complete or correct. Attempts via differentiation from scratch or

expanding the series for $(e^x)^3 = \left(1 + x + \frac{x^2}{2}\right)^3 = \dots$ will need to successfully obtain a correct unsimplified series up to the x term, ie $1 + 3x + \dots$ oe.

A1: Must have a **correct expansion of e^{3x} up to the x^2 term** (ie $1 + 3x + \frac{(3x)^2}{2}$) and multiply by their expression for $\arcsin 2x$ with at least one correct term obtained for the final series attained.

A1: Correct series.

Question Number	Scheme	Notes	Marks
4(a)	$a = 8, \quad b = \frac{1}{4}, \quad c = -18$	B1: Correct value for a B1: Correct value for b B1: Correct value for c	B1 B1 B1
(3)			
(b)	$ w _{\min} = \sqrt{8^2 + 7^2} \quad w _{\max} = \sqrt{8^2 + 15^2} + 8$	M1: A correct method for either $ w _{\min}$ or $ w _{\max}$	M1
		A1: $ w _{\min} = \sqrt{113}$ and $ w _{\max} = 25$	A1
	$\sqrt{113} \leq w \leq 25$	Correct region	A1
(3)			
(c)	$\arg w_{\min} = \arctan \frac{8}{23} = 0.335$	M1: Correct method for a relevant angle. A1: awrt 0.335 only	M1 A1
(2)			
Total 8			

Additional Notes:

(a)

B1B1B1: Values may be given on the diagram, but they must be clearly labelled/associated as the correct values. Do not accept e.g. 8 marked as a radius on the diagram without being associated as a .

Accept if the values are embedded in the relevant formula as long as they do not subsequently state incorrect values for the constants – if they state values explicitly these take precedence.

(b)

M1: For a correct method for either end of the region, they may show working on the diagram. Note for the upper limit, they may find the equation of the line through O and the centre of the circle, find the intersections of this line with the circle then attempt the distance for the furthest of these points from O .

A1: Both correct extreme values.

A1: Correct answer – must be the whole range, including $|w|$, not just state the maximum and minimum.

(c)

M1: For a correct method for either the angle directly or for another angle in a triangle on the diagram which includes the required angle or its complement. May be implied by awrt 0.33 or 0.335 or 1.2 or (degrees) awrt 19 or 71

A1: Accept anything which rounds to 0.335 and no others.

Question Number	Scheme	Notes	Marks
5(a)	$1 + \frac{2}{2r+5} = \frac{2r+5+2}{2r+5} = \frac{2r+7}{2r+5}$	Obtains $\frac{2r+7}{2r+5}$	B1
(1)			
(b)	$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right) = \sum_{r=1}^n \log_3 \left(\frac{2r+7}{2r+5} \right) = \sum_{r=1}^n (\log_3 (2r+7) - \log_3 (2r+5))$ $\begin{array}{l} r=1 \quad \cancel{\log_3 9} (=2) \quad - \quad \log_3 7 \\ r=2 \quad \log_3 \cancel{11} \quad - \quad \cancel{\log_3 9} (=2) \\ \dots \\ r=n \quad \log_3 (2n+7) \quad - \quad \cancel{\log_3 (2n+5)} \end{array}$		M1
	$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right) = \log_3 (2n+7) - \log_3 7 = \log_3 \left(\frac{2n+7}{7} \right)$	Correct expression in correct form	M1 A1
(3)			
Alt (b)	$\sum_{r=1}^n \log_3 \left(\frac{2r+7}{2r+5} \right) = \log_3 \frac{9}{7} + \log_3 \frac{11}{9} + \dots + \log_3 \frac{2n+7}{2n+5}$ $= \log_3 \left(\frac{\cancel{9}}{7} \times \frac{\cancel{11}}{\cancel{9}} \times \dots \times \frac{2n+7}{\cancel{2n+5}} \right)$	Combines sum of log's to show cancelling fractions or implied cancelling.	M1
	$= \log_3 \left(\frac{2n+7}{7} \right)$	Cancels their fractions to achieve correct form, A1 for correct answer.	M1 A1
	(3)		
(c)	$\sum_{r=n+2}^{10n} \log_3 \left(1 + \frac{2}{2r+5} \right) = \log_3 \left(\frac{2(10n)+7}{7} \right) - \log_3 \left(\frac{2(n+1)+7}{7} \right)$	Forms $g(10n) - g(n+a)$ where $a = 1, 2$	M1
	$\Rightarrow \frac{20n+7}{2n+9} = 3^2$	Correct removal of logs from their equation, from an attempt at difference of sums	M1
	$\begin{array}{l} 20n+7 = 18n+81 \\ 2n = 74 \\ n = 37 \end{array}$	dM1: Solves an equation of the correct form. Requires previous method mark A1: $n = 37$ only	dM1 A1
(4)			
Total 8			

Additional Notes

(a)

B1: Correct answer.

(b)

M1: For a valid attempt at the method of differences evidenced by at least two rows to show one cancelling pair of terms, or by correct extraction for the non-cancelling terms with clear indication of the cancelling (e.g. by extraction of the terms if no crossing out seen).

M1: Extracts their non-cancelling terms for their summation and combines the terms to a single log term.

A1: Correct expression in form required.

Note M0M1A1 is possible if the correct answers is attained by implication with the method of differences not clearly being shown.

Alt Method:

M1: Combines the log terms in the summations to a single log expression with cancelling fractions shown.

M1: Cancels the common terms in their expression to simplify to the required form.

A1: Correct expression in form required.

Note the combination of logs is not always being clearly shown in this method. Answers such as

$$r=1 \quad \log_3 \left(\frac{\cancel{9}}{7} \right)$$

$$r=2 \quad \log_3 \left(\frac{\cancel{11}}{\cancel{9}} \right)$$

⋮

$$r=n \quad \log_3 \left(\frac{2n+7}{\cancel{2n+5}} \right) \Rightarrow \log_3 \left(\frac{2n+7}{7} \right)$$

Can score M0M1A1

(c)

M1: Correct attempt at finding the sum from $n+2$ to $10n$. Accept $g(10n) - g(n+a)$ where $a = 1$ or 2 for a valid attempt at the method, with their answer to (b), where $g(n)$ is the answer to (b).

Allow for answers which “use” part (b) to deduce the new first and last terms/new non-cancelling terms and proceed to form the correct expression.

M1: Correct method to eliminate logs and achieve an equation in n having made some attempt at a difference in summations to evaluate the sum. (Must be 3^2 and not 2^3) Note: solutions via calculator without showing removal of logs will score M0dM0A0.

dM1: Depends on previous M. Solves the resulting equation in n

A1: Correct

Question Number	Scheme	Notes	Marks
6(a)	$x = r \cos \theta = \cos \theta (\sqrt{3} + \tan \theta)$	Uses $x = r \cos \theta$	M1
	$x = \sqrt{3} \cos \theta + \sin \theta \Rightarrow \frac{dx}{d\theta} = \cos \theta - \sqrt{3} \sin \theta = 0$ or $\frac{dx}{d\theta} = \cos \theta \sec^2 \theta - \sin \theta \tan \theta - \sqrt{3} \sin \theta = 0$	Obtains correct form for $\frac{dx}{d\theta}$ and sets equal to 0. May use product rule on $\cos \theta \tan \theta$	dM1
	$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	A1
	$r = \sqrt{3} + \tan \frac{\pi}{6} = \sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$	$r = \frac{4\sqrt{3}}{3}$ or $\frac{4}{\sqrt{3}}$	A1
(4)			
(b)	$\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (\sqrt{3} + \tan \theta)^2 d\theta = \dots$	Uses $\frac{1}{2} \int r^2 d\theta$ and attempts to expands the bracket (accept with just two terms). The $\frac{1}{2}$ may be seen later.	M1
	$\int \dots \tan^2 \theta d\theta \rightarrow \int \dots \pm 1 \pm \sec^2 \theta d\theta$	Applies $\tan^2 \theta = \pm \sec^2 \theta \pm 1$ to the integral (Note: could use parts to integrate – see notes below.)	M1
	$\int \dots \sec^2 \theta d\theta \rightarrow \dots \tan \theta$	Correct attempt to integrate $\sec^2 \theta$ to $k \tan \theta$ (see notes for attempts via parts). Depends on previous M	dM1
	$\int \dots \tan \theta d\theta \rightarrow \dots \ln \cos \theta$ or $\dots \ln \sec \theta$	Correct attempt to integrate $\tan \theta$ to $k \ln \cos \theta$ (oe) within their expression.	M1
	$= \left\{ \frac{1}{2} \right\} \int (3 + 2\sqrt{3} \tan \theta + \tan^2 \theta) d\theta =$ $= \left\{ \frac{1}{2} \right\} \left(2\theta - 2\sqrt{3} \ln \cos \theta + \tan \theta \right)$ or $\theta + \sqrt{3} \ln \sec \theta + \frac{1}{2} \tan \theta$ (oe)	Fully correct integration of $\int r^2 d\theta$ with or without the $\frac{1}{2}$ *see notes for an Alt	A1
	sector area $= \frac{1}{2} \times \left(\frac{4\sqrt{3}}{3} \right)^2 \times \frac{\pi}{6}$	Correct ft expression for sector	B1ft
	$\frac{4\pi}{9} - \left[\left(\frac{\pi}{6} - \sqrt{3} \ln \cos \frac{\pi}{6} + \frac{1}{2} \tan \frac{\pi}{6} \right) - (0 - 0 + 0) \right]$	Their sector – their integration from an attempt at $\frac{1}{2} \int r^2 d\theta$ with their limit applied appropriately	M1
	$= \frac{4\pi}{9} - \frac{\pi}{6} + \sqrt{3} \ln \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}$ $= \frac{5}{18} \pi + \frac{\sqrt{3}}{2} \left(\ln \frac{3}{4} - \frac{1}{3} \right)$	Correct answer in the correct form	A1
(8)			
Total 12			

Additional Notes:

(b)

NB Some candidates have learned $\int \tan^2 \theta d\theta = \tan \theta - \theta$ and so do not show the “ $\sec^2 \theta - 1$ ” step. Score

M1 for $\tan^2 \theta \rightarrow \pm \tan \theta \pm \theta$ and dM1 for $\tan^2 \theta \rightarrow \tan \theta - \theta$ in such cases.

Use of parts for the integral:

$$\int \tan^2 \theta d\theta = \int \sin \theta \frac{\sin \theta}{\cos^2 \theta} d\theta = \sin \theta \frac{1}{\pm \cos \theta} - \int \pm \cos \theta \frac{1}{\pm \cos \theta} d\theta \quad \text{M1}$$

$$= \pm \tan \theta \pm \theta \quad \text{dM1}$$

Or

$$\int \tan^2 \theta d\theta = \int \sin^2 \theta \sec^2 \theta d\theta = \sin^2 \theta \tan \theta - \int 2 \sin \theta \cos \theta \tan \theta d\theta \quad \text{M1}$$

$$= \sin^2 \theta \tan \theta - \int 1 - \cos 2\theta d\theta = \sin^2 \theta \tan \theta - \theta + \frac{1}{2} \sin 2\theta \quad \text{dM1}$$

Any variations mark similarly as M1 for first stage of parts applied (allowing for sign errors in trig terms), dM1 for fully integrating as long as the first stage of parts achieves an integral that can be achieved more easily. Watch for the alternate form in the second case.

The B1ft may arise from attempts to use polar areas but must reach a correct expression for the area to

access the mark: $\left\{ \frac{1}{2} \int_0^{\frac{\pi}{6}} \left(\frac{4\sqrt{3}}{2} \right)^2 d\theta = \frac{1}{2} \left(\left(\frac{4\sqrt{3}}{2} \right)^2 \times \frac{\pi}{6} \right) - (0) \right\}$

For the M mark for limits, the “ -0 ” for the lower limit may be implied (no need to see it). If no method is shown and only values given from calculator usage, then allow b.o.d. that substitution was correct as long as the correct limits were associated with the integral.

Question Number	Scheme	Notes	Marks
7(a)	$x = e^u \Rightarrow \frac{dx}{du} = e^u = x \text{ or } u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x} \frac{dy}{du} \text{ or } \frac{dy}{dx} = \frac{dy}{du} \times e^{-u} \text{ oe}$	Correct form for equation in $\frac{dy}{dx}$ and $\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = \frac{1}{x} \frac{d^2y}{du^2} \frac{du}{dx} - \frac{1}{x^2} \frac{dy}{du} \text{ or}$ $\frac{d^2y}{dx^2} = e^{-u} \frac{d^2y}{du^2} \frac{du}{dx} - e^{-2u} \frac{dy}{du} \text{ oe}$	Correct form for an equation linking $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{du^2}$	M1
	$2x^2 \left(\frac{1}{x^2} \frac{d^2y}{du^2} - \frac{1}{x^2} \frac{dy}{du} \right) + 3x \left(\frac{1}{x} \frac{dy}{du} \right) - y = 27x^2$	Substitutes to eliminate $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ and $\frac{du}{dx}$	M1
	$2 \frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} *$	Fully correct proof	A1*
(4)			
(b)	$2m^2 + m - 1 = (2m - 1)(m + 1) = 0 \Rightarrow m = \frac{1}{2}, -1$	Forms auxiliary equation and solves (usual rules)	M1
	$(y =) Ae^{\frac{1}{2}u} + Be^{-u}$	Correct CF for their two real values of m	M1
	$y = \lambda e^{2u}, y' = 2\lambda e^{2u}, y'' = 4\lambda e^{2u}$ $\Rightarrow 8\lambda + 2\lambda - \lambda = 9\lambda = 27 \Rightarrow \lambda = 3$	With correct form of PI, differentiates twice and substitutes to find a value for λ	M1
	$\left(y = Ae^{\frac{1}{2}u} + Be^{-u} + 3e^{2u} \right)$ $y = Ax^{\frac{1}{2}} + Bx^{-1} + 3x^2$	Correct GS in terms of x with no exponentials. Must include the $y =$	A1
(4)			
(c)	$\frac{dy}{dx} = \frac{1}{2} Ax^{-\frac{1}{2}} - Bx^{-2} + 6x$	Correct form for $\frac{dy}{dx}$ from GS of form $y = Ax^p + Bx^q + rx^2$	M1
	$x = \frac{1}{4}, y = \frac{11}{16} \Rightarrow \frac{11}{16} = \frac{1}{2}A + 4B + \frac{3}{16}$ $\text{and } x = \frac{1}{4}, \frac{dy}{dx} = 1 \Rightarrow 1 = A - 16B + \frac{3}{2}$	Uses $x = \frac{1}{4}$ with $y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$ to obtain two equations in the constants (from CF)	M1
	$\frac{1}{2}A + 4B = \frac{1}{2}, A - 16B = -\frac{1}{2}$ $\Rightarrow A = \frac{1}{2}, B = \frac{1}{16}$	Having found two equations in two CF constants from using the conditions, solves for A and B . Requires previous M mark	dM1
	$y = \frac{1}{2} \left(\frac{1}{8} \right)^{\frac{1}{2}} + \frac{1}{16} \left(\frac{1}{8} \right)^{-1} + 3 \left(\frac{1}{8} \right)^2 = \dots$	Substitutes $x = \frac{1}{8}$ into their PS and obtains a value for y . Requires previous 2 M marks	ddM1
	$= \frac{\sqrt{2}}{8} + \frac{1}{2} + \frac{3}{64} = \frac{\sqrt{2}}{8} + \frac{35}{64} = \frac{1}{64} (8\sqrt{2} + 35)$	$\frac{1}{64} (8\sqrt{2} + 35)$ only	A1
(5)			

Additional Notes:

(a)

M1: Applies the chain rule with an attempt at $\frac{du}{dx}$ or $\frac{dx}{du}$ for form an equation linking $\frac{dy}{dx}$ and $\frac{dy}{du}$

M1: Attempts the second derivative with product and chain rules to form an equation linking

$\frac{d^2y}{dx^2}$ and $\frac{d^2y}{du^2}$ of correct form.

M1: Substitutes to eliminate $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ and $\frac{du}{dx}$ from the equation to get an equation in $\frac{d^2y}{du^2}$ and $\frac{dy}{du}$ as the only derivatives.

A1: Completely correct proof, with intermediate work and no errors.

(b)

M1: Forms and solves the auxiliary equation. For solving the auxiliary equation no method need be shown. The M can be implied by the correct values.

M1: Correct complementary function for their solutions as long as they are distinct real roots. The $y =$ is not needed and may be mislabelled as $x =$ or have x in place of u for this mark.

M1: Full process to find the particular integral, with correct form used, differentiated twice and substituted to find the value of the constant.

A1: Correct general solution in terms of x (u must be eliminated) and including the $y =$. Allow if the GS in terms of x is stated at the beginning of part (c) instead of in (b).

(c)

M1: Differentiates the general solution to the correct form.

M1: Uses the given conditions to form simultaneous equations in the constants of their equations.

dM1: Depends on previous M. Solves their two simultaneous equations (must have formed two equations from the conditions).

ddM1: Depends on previous two M's. Uses their particular solution to find the value for y when $x = \frac{1}{8}$

Condone a slip when substituting as long as it is clear they are attempting the correct x value.

A1: Correct answer only.

NB Part (c) can be done in terms of u if they do not convert back to an equation in x

M1: Differentiates the GS in terms of u : $\frac{dy}{du} = \frac{A}{2}e^{\frac{1}{2}u} - Be^{-u} + 6e^{2u}$

M1: Forms simultaneous equations using $u = \ln \frac{1}{4}$ (oe) and the value for y and $\frac{dy}{du} = x \frac{dy}{dx} = \frac{1}{4}$. So

$$\frac{11}{16} = Ae^{\frac{1}{2}\ln\frac{1}{4}} + Be^{-\ln\frac{1}{4}} + 3e^{2\ln\frac{1}{4}} = \frac{A}{2} + 4B + \frac{3}{16} \quad \text{and} \quad \frac{1}{4} = \frac{A}{2}e^{\frac{1}{2}\ln\frac{1}{4}} - Be^{-\ln\frac{1}{4}} + 6e^{2\ln\frac{1}{4}} = \frac{A}{4} - 4B + \frac{3}{8}$$

dM1: Solves the equations. $\Rightarrow A = \dots, B = \dots$

ddM1: Substitutes $u = \ln \frac{1}{8}$ into their equation in terms u to find y .

A1: cao.

Note that they must be using attempts at u , not just x values, in such an approach.

Note: where candidates confuse u and x 's accept they must be substituting in the correct things, so using $\frac{dy}{du} = 1$ is 2nd M0, but if they state $\frac{dy}{dx}$ = function of x following an attempt $\frac{dy}{du}$ first then replacing u , you

can allow the second M for using $\frac{dy}{dx} = 1$ in this (the first M will be lost).

Question Number	Scheme	Notes	Marks
8(a)	$z^n = \cos n\theta + i \sin n\theta$ or $z^{-n} = \cos(-n\theta) + i \sin(-n\theta)$ or $z^{-n} = \cos n\theta - i \sin n\theta$	One correct use of de Moivre on either z^n or z^{-n} seen or implied.	M1
	$z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta$ $= 2 \cos n\theta^*$	Fully correct proof with LHS and RHS both seen and expressions explicitly added.	A1*
	Alternative: $z^n + \frac{1}{z^n} = e^{in\theta} + e^{-in\theta} \quad (\text{M1}) \quad = 2 \left(\frac{1}{2} (e^{in\theta} + e^{-in\theta}) \right) = 2 \cos n\theta \quad (\text{A1*})$		
(2)			
(b)	$\left(z + \frac{1}{z}\right)^4 = z^4 + 4z^3\left(\frac{1}{z}\right) + 6z^2\left(\frac{1}{z}\right)^2 + 4z\left(\frac{1}{z}\right)^3 + \left(\frac{1}{z}\right)^4$	Expands $\left(z + \frac{1}{z}\right)^4$ with correct binomial coefficients.	M1
	$\left(z + \frac{1}{z}\right)^4 = z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $\Rightarrow (2 \cos \theta)^4 = 2 \cos 4\theta + 4(2 \cos 2\theta) + 6$	Correct use of $z^n + \frac{1}{z^n} = 2 \cos n\theta$ shown for both terms	M1
	$16 \cos^4 \theta = 2 \cos 4\theta + 8 \cos 2\theta + 6$ $\Rightarrow \cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4 \cos 2\theta + 3)$	M1: Achieves expression for $\cos^4 \theta$ of the correct form. A1: Correct expression.	M1 A1
(4)			
(c)	$\pi \int y^2 dx = \pi \int \cos^4 x (1 + \sin x) dx$	Uses volume of revolution formula	M1
	$= \{\pi\} \int (\cos^4 x + \sin x \cos^4 x) dx$ $= \{\pi\} \int \left(\frac{1}{8}(\cos 4x + 4 \cos 2x + 3) + \sin x \cos^4 x \right) dx$ $= \{\pi\} \left[\frac{1}{8} \left(\frac{1}{4} \sin 4x + 2 \sin 2x + 3x \right) - \frac{1}{5} \cos^5 x \right]$	M1: Uses their answer to (b) and integrates it to the correct form M1: ... $\sin x \cos^4 x \rightarrow \dots \cos^5 x$ A1: Fully correct integration • See notes	M1 M1 A1
	$= \{\pi\} \left[\left(\frac{1}{8} \left(0 + 2 + \frac{3\pi}{4} \right) - \frac{1}{5} \left(\frac{\sqrt{2}}{2} \right)^5 \right) - \left(\frac{1}{8} \left(0 + 0 - \frac{3\pi}{2} \right) - 0 \right) \right]$	Applies limits $\frac{\pi}{4}$ and $-\frac{\pi}{2}$ appropriately to an attempt at the integral. Depends on one of the two preceding Ms being scored.	dM1
	$= \pi \left(\frac{1}{4} + \frac{3\pi}{32} - \frac{\sqrt{2}}{40} + \frac{3\pi}{16} \right) = \pi \left(\frac{1}{4} + \frac{9\pi}{32} - \frac{\sqrt{2}}{40} \right)$ $= \frac{\pi}{160} (40 + 45\pi - 4\sqrt{2})$	Correct answer in correct form	A1
(6)			
Total 12			
PAPER TOTAL: 75			

Additional Notes:

(a)

M1: One correct use of de Moivre on either z^n or z^{-n} seen or implied. May be done separately or within working.

A1: Correctly completes the proof with no errors seen. The LHS and RHS of the identity must both be seen and expressions explicitly added and the $-i \sin n\theta$ seen. Condone missing brackets in e.g. $\cos - n\theta$ terms if intent is clear.

Minimum for sufficient working is $z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta = 2 \cos n\theta$

Alternatively accept if exponential forms are used.

M1: Applies $\cos \theta + i \sin \theta = e^{i\theta}$ (may be implied) and substitutes into the given equation, with correct index work on both terms.

A1: Correctly rearranges to make the $\cos(n\theta)$ identity clear and simplifies to the given result. The minimum requirement is all four statements seen in the MS.

(b)

M1: Attempts to expand $(z + z^{-1})^4$. Correct binomial coefficients must be used, terms need not be simplified. Condone at most one slip in powers.

M1: Gathers the relevant terms (must see the grouping) and clearly applies $z^n + \frac{1}{z^n} = 2 \cos n\theta$ to both terms to obtain each in $\cos n\theta$ terms. The "2" must be seen or implied by correct working.

M1: Sets their expression equal to $(2 \cos \theta)^4$ and makes $\cos^4 \theta$ the subject with correct form.

A1: Correct answer with no clearly incorrect steps throughout (but condone minor slips which are recovered).

Note: M1M0M1A1 is possible in this part if the groups of terms is never shown but they jump directly to the expression in $\cos n\theta$ terms.

(c) The π is only required for the first and last marks.

M1: For the correct use of the volumes of revolution formula applied to the problem, but condone a slip in squaring if the intention is clear (e.g. accept $\cos^2 x(1 + \sin x)$ as the attempt to use y^2). The π must be included at some stage but may be added later. Limits not required for this mark.

M1: Applies the answer to part (b) and integrates to obtain an integral of $\cos^4 \theta$ the correct form. Slips in coefficients may have been made. The π and limits are not required.

M1: Correct form for $\int \sin x \cos^4 x \, dx = K \cos^5 x$ The π and limits are not required.

Note an alternative here is to use part (b) again and the product to sum rule:

$$\begin{aligned} \int \cos^4 x \sin x \, dx &= \frac{1}{8} \int \cos 4x \sin x + 4 \cos 2x \sin x + 3 \sin x \, dx \\ &= \frac{1}{8} \int \frac{1}{2} (\sin 5x - \sin 3x) + 2 (\sin 3x - \sin x) + 3 \sin x \, dx \\ &= \frac{1}{8} \left[\frac{1}{2} \left(-\frac{1}{5} \cos 5x + \frac{1}{3} \cos 3x \right) - \frac{2}{3} \cos 3x + \frac{1}{2} \cos x - 3 \cos x \right] \end{aligned}$$

A1: Fully correct integration of both terms. The π and limits are not required.

dM1: Depends on at least one of the two preceding Ms. Applies correct limits to the integral as long as a valid attempt at the integral has been made.

A1: Correct answer in correct form.

NB Allow SC M1M0M1A0dM1A1 for those who expand from scratch instead of using the result from (b).

Alt (b)	$[2 \cos 4\theta] = (\cos \theta + i \sin \theta)^4 + (\cos \theta - i \sin \theta)^4$ $= c^4 + 4c^3is + 6c^2(is)^2 + 4c(is)^3 + (is)^4$ $+ c^4 - 4c^3is + 6c^2(is)^2 - 4c(is)^3 + (is)^4$	Uses the identity, applies De Moivre and expands at least one set of brackets with correct binomial coefficients.	M1
	$\Rightarrow 2 \cos 4\theta = 2 \cos^4 \theta - 12 \cos^2 \theta \sin^2 \theta + 2 \sin^4 \theta$	Correct use of $z^4 + z^{-4} = 2 \cos 4\theta$ to get only real terms in sine and cosine.	M1
	$\Rightarrow \cos^4 \theta = \cos 4\theta + 6 \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right) - \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right)^2$ $= \cos 4\theta + \frac{3}{2} (1 - \cos^2 2\theta) - \frac{1}{4} (1 - 2 \cos 2\theta + \cos^2 2\theta)$ $= \cos 4\theta - \frac{3}{2} \left(\frac{1}{2} + \frac{1}{2} \cos 4\theta \right) + \frac{1}{2} \cos 2\theta - \frac{1}{4} \left(\frac{1}{2} + \frac{1}{2} \cos 4\theta \right) + \frac{5}{4}$ $= \frac{1}{8} [8 \cos 4\theta - 6(1 + \cos 4\theta) + 4 \cos 2\theta - (1 + \cos 4\theta) + 10]$ $\Rightarrow \cos^4 \theta = \frac{1}{8} (\cos 4\theta + 4 \cos 2\theta + 3)$ M1: Achieves expression for $\cos^4 \theta$ of the correct form A1: Correct expression		M1 A1

Note the question says “hence” so answers without reference to (a) will score at most one mark (e.g. purely by De Moivre). But some attempts may use the result of (a) as shown above. In which case mark as indicated.

M1: For a correct binomial expansion applied to the problem – so e.g. sets up

$(\cos \theta + i \sin \theta)^4 + (\cos \theta - i \sin \theta)^4$ and expands at least one bracket with correct coefficients. This mark may be scored for expansion of $(\cos \theta + i \sin \theta)^4$ from an approach by De Moivre using $\operatorname{Re}(\cos \theta + i \sin \theta)^4$. This may come from other approaches that use a binomial expansion.

M1: Evidence of the use of both sides of part (a), cancelling imaginary terms to get a real equation in cosine and sines only. Note that they must be using part (a) for this mark to be scored.

M1: Having used part (a), uses correct trigonometric identities to eliminate the $\sin \theta$ terms and reduce the powers to single power $\cos 2\theta$ terms – ie achieves correct form for $\cos^4 \theta$. There may be alternate identities used (e.g. Pythagorean before using double angle identities)

A1: Correct answer.