Question Number	Scheme	Notes	Marks
1(a)	$r = 4\sqrt{2}$	Correct r Accept $\pm 4\sqrt{2}$ or $\pm\sqrt{32}$ or other processed equivalents.	B1
	$\theta = -\pi + \arctan\left(\dfrac{2\sqrt{6}}{2\sqrt{2}}\right) = -\pi + \dfrac{\pi}{3} = -\dfrac{2\pi}{3}$	Complete method to find an argument in the correct quadrant for their r	M1
	$\Rightarrow -2\sqrt{2} - \left(2\sqrt{6}\right)\mathrm{i} = 4\sqrt{2}\mathrm{e}^{-\frac{2\pi}{3}\mathrm{i}}$	Correct answer in the form specified. Accept with $\sqrt{32}$	A1
			(3)
(b)	$z^5 = -2\sqrt{2} - \left(2\sqrt{6}\right)\mathrm{i}$		
	$z = \left(4\sqrt{2}\right)^{\frac{1}{5}} \mathrm{e}^{\frac{-\frac{2\pi}{3}+2k\pi}{5}\mathrm{i}}$	Attempts De Moivre correctly with either all roots indicated or simplified r with one argument. See additional notes.	M1
	$\sqrt{2}\mathrm{e}^{-\frac{2\pi}{15}\mathrm{i}}$, $\sqrt{2}\mathrm{e}^{\frac{4\pi}{15}\mathrm{i}}$, $\sqrt{2}\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$, $\sqrt{2}\mathrm{e}^{-\frac{8\pi}{15}\mathrm{i}}$, $\sqrt{2}\mathrm{e}^{-\frac{14\pi}{15}\mathrm{i}}$	Three correct roots provided coefficients of π for θ are single fractions, allow for $0 < \theta < 2\pi$ Allow $\left(4\sqrt{2}\right)^{\frac{1}{5}}$ oe for $\sqrt{2}$ and correct roots in trig form	A1
		All five correct roots and no extra. Must have $-\pi < \theta \leqslant \pi$. Allow e.g., $\sqrt{2}\mathrm{e}^{\frac{10\pi}{15}\mathrm{i}}$ for $\sqrt{2}\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$	A1
			(3)
			Total 6

Additional Notes:

(a)

B1: Accept positive or negative root. Allow if seen in part (b).

M1: Requires a method to achieve an argument in the correct quadrant for their r. Accept attempts at $\arctan\left(\dfrac{2\sqrt{6}}{2\sqrt{2}}\right) \pm \pi$ or $\arctan\left(\dfrac{2\sqrt{2}}{2\sqrt{6}}\right) \pm \pi$ for attempts where $r > 0$, or equivalent methods via arccos or arcsin (must use r as the denominator). May be implied by sight of any of $-\dfrac{2\pi}{3}$, $\dfrac{4\pi}{3}$, $\dfrac{7\pi}{6}$ or $-\dfrac{5\pi}{6}$ if no method is shown.

In cases where a negative r is given, accept attempts at just $\arctan\left(\dfrac{2\sqrt{6}}{2\sqrt{2}}\right)$ or $\arctan\left(\dfrac{2\sqrt{2}}{2\sqrt{6}}\right)$ provided they do not subsequently adjust to a different quadrant.

A1: Correct answer in the form specified, with $-\pi < \theta < \pi$. Must be given in correct form, but accept answer with negative r, i.e. $-4\sqrt{2}\mathrm{e}^{\frac{\pi}{3}\mathrm{i}}$ Accept with exact processed equivalents for r.

Note: Correct answer with no working shown will score B1M1(implied)A1.

(b)
M1: Applies de Moivre correctly with their answer to (a) which could be in trig form, to reach either simplified modulus with one correct argument for their (a), or multiple arguments with unsimplified modulus.

Condone missing "i" for the M.

Note: If $\pm 2k\pi$ is not used or is used incorrectly this mean you can score M1A0A0 for just $\sqrt{2}e^{-\frac{2\pi}{15}i}$

A1: Accept for three correct distinct roots allowing arguments in the range $-\pi < \theta < \pi$ or $0 < \theta < 2\pi$ for this mark. The modulus need not be simplified for this mark.
A1: All 5 correct roots in the form specified.

Question Number	Scheme	Notes	Marks
2	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-4\frac{\mathrm{d}y}{\mathrm{d}x}-5y^2=\mathrm{e}^{x^2-9}$		
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-4\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-10y\frac{\mathrm{d}y}{\mathrm{d}x}=2x\mathrm{e}^{x^2-9}$	M1: Differentiates to obtain three correct terms A1: Fully correct differentiated equation	M1 A1
	$y(3)=2,\ y'(3)=-1$ $y''(3)-4(-1)-5\times2^2=1\Rightarrow y''(3)=17$ $y'''(3)-4(17)-10(2)(-1)=6\Rightarrow y'''(3)=54$	Uses the given equation and their equation to obtain values for $y''(3)$ and $y'''(3)$	M1
	$(y=)\,y(3)+y'(3)(x-3)+\frac{y''(3)}{2}(x-3)^2+\frac{y'''(3)}{3!}(x-3)^3\Rightarrow$ $2-1(x-3)+\frac{"17"}{2}(x-3)^2+\frac{"54"}{3!}(x-3)^3$	A correct unsimplified Taylor series for their $y''(3)$ and $y'''(3)$	M1
	$(y=)\,2-(x-3)+\frac{17}{2}(x-3)^2+9(x-3)^3$	Correct simplified series. "$y=$ ..." is not required	A1
			(5)
			Total 5

Additional Notes

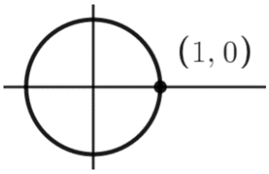
M1: Attempts to differentiate using the chain rule at least once – at least three terms correct, allowing for slips in rearranging before differentiation.

A1: Any correct third derivative expression. They may have rearranged first.

M1: Finds the values of the second and third derivatives using the given equation and their third derivative expression (which must have at least three terms). Condone one slip in substitutions as long as evidence of the correct values used is seen. If substitution is not seen, correct values can imply the method.

M1: Applies the Taylor Series formula correctly with the given values of y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ and with their values for the second and third derivatives – the first two terms must be correct. If the values of their second and third derivatives are not explicitly stated but embedded in the series, it must be clear that a correct attempt at the series has been used – the denominators 2 and 3! (or 6) must be seen – but their values need not be correct. Condone a miscopy of one of **their** values if it is clearly a correct attempt to apply their value to the series.

A1: Correct fully simplified series expansion. The $y=$ is not required and condone if f(x) is used.

Question Number	Scheme	Notes	Marks
3(a)		Circle centred at origin with radius of 1 correctly indicated in any way. May use $\pm\mathrm{i}$ or ± 1 in appropriate position on imaginary axis.	B1
			(1)
(b)	$w=\dfrac{9\mathrm{i}z-\mathrm{i}}{z+1}$ $\Rightarrow wz+w=9\mathrm{i}z-\mathrm{i}\Rightarrow z(w-9\mathrm{i})=-w-\mathrm{i}$ $\Rightarrow z=\dfrac{-w-\mathrm{i}}{w-9\mathrm{i}}=\dfrac{w+\mathrm{i}}{9\mathrm{i}-w}$ oe	M1: Makes z the subject A1: Correct expression oe	M1 A1
	$\lvert z\rvert=\left\lvert\dfrac{-w-\mathrm{i}}{w-9\mathrm{i}}\right\rvert\Rightarrow\lvert w-9\mathrm{i}\rvert=\lvert w-(-\mathrm{i})\rvert$	Uses $\lvert z\rvert=1$ and obtains $\lvert\pm w\pm z_1\rvert=\lvert\pm w\pm z_2\rvert$ **Requires previous M mark.**	**d**M1
	$\Rightarrow v=\text{"4"}$ or $u^2+(v-9)^2=u^2+(v+1)^2$ $-18v+81=2v+1$ $20v=80$ $v=4$	Obtains $v=k$ with k correct for their $\lvert w-p\mathrm{i}\rvert=\lvert w-q\mathrm{i}\rvert$ oe or substitutes $u+\mathrm{i}v$ for w, applies Pythagoras correctly and obtains a value for v Allow "$y=$ ..." for this mark. Condone $v=4\mathrm{i}$ for this mark. **Requires both previous M marks.**	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)
			Total 6
Alt 1: using points on $\lvert z\rvert=1$	$z=1\Rightarrow w=\dfrac{9\mathrm{i}-\mathrm{i}}{1+1},\quad z=\mathrm{i}\Rightarrow w=\dfrac{9\mathrm{i}^2-\mathrm{i}}{\mathrm{i}+1}$	M1: Uses two correct values of z in the formula to obtain two expressions for w A1: Both expressions for w correct	M1 A1
	$w=4\mathrm{i},\quad w=\dfrac{-\mathrm{i}-9}{1+\mathrm{i}}\times\dfrac{1-\mathrm{i}}{1-\mathrm{i}}=\dfrac{-10+8\mathrm{i}}{2}=-5+4\mathrm{i}$	Simplifies expressions to $a+b\mathrm{i}$ form including correct method to rationalise denominators. **Requires previous M mark.**	**d**M1
	$\Rightarrow v=\text{"4"}$	Obtains $v=b$ with b correct for their $a+b\mathrm{i}$ forms. Allow "$y=$ ..." for this mark. Condone $v=4\mathrm{i}$ for this mark. **Requires both previous M marks.**	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)
Alt 2: using u+iv	$w=\dfrac{9\mathrm{i}z-\mathrm{i}}{z+1}$ $\Rightarrow wz+w=9\mathrm{i}z-\mathrm{i}\Rightarrow z(w-9\mathrm{i})=-w-\mathrm{i}$	M1: Makes z the subject A1: Correct expression oe	M1 A1

	$\Rightarrow z = \frac{-w-\mathrm{i}}{w-9\mathrm{i}} = \frac{w+\mathrm{i}}{9\mathrm{i}-w}$ oe		
	$\Rightarrow z = \frac{-u-\mathrm{i}v-\mathrm{i}}{u+\mathrm{i}v-9\mathrm{i}} \times \frac{u-(v-9)\mathrm{i}}{u-(v-9)\mathrm{i}}$ $= \frac{-u^2-(v+1)(v-9)-10u\mathrm{i}}{u^2+(v-9)^2}$ $\Rightarrow x = -\frac{u^2+(v+1)(v-9)}{u^2+(v-9)^2} \quad y = -\frac{10u}{u^2+(v-9)^2}$	Replaces w with $u + v\mathrm{i}$ and rationalises the denominator and extracts real and imaginary parts	**d**M1
	$x^2+y^2=1 \Rightarrow \frac{\left(u^2+(v+1)(v-9)\right)^2+(10u)^2}{\left(u^2+(v-9)^2\right)^2}=1$ $\Rightarrow \ldots \Rightarrow (v-4)\left(u^2+v^2-18v+81\right)=0$	Applies $x^2+y^2=1$ and simplifies to identify a factor $v-k=0$ Same conditions as main scheme.	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)
Alt 3: using x+iy	$w = \frac{9\mathrm{i}(x+\mathrm{i}y)-\mathrm{i}}{x+\mathrm{i}y+1} \times \frac{x+1-\mathrm{i}y}{x+1-\mathrm{i}y} = \ldots$ $= \frac{-9y(x+1)+(9x-1)y+\left((9x-1)(x+1)+9y^2\right)\mathrm{i}}{(x+1)^2+y^2}$	M1: Replaces z by x + iy and rationalises the denominator. A1: Correct expression oe	M1 A1
	$= \frac{-10y+\left(9x^2+8x-1+9y^2\right)\mathrm{i}}{1+2x+x^2+y^2}$ $= \frac{-10y+(8x-1+9)\mathrm{i}}{1+2x+1} \left[= \frac{-5y+4(x+1)\mathrm{i}}{x+1}\right]$	Expands and applies $x^2+y^2=1$ to simplify to linear coefficients	**d**M1
	$= \frac{-5y}{x+1}+4\mathrm{i} \Rightarrow v=4$	Simplifies to a constant imaginary term and deduces the equation. Same conditions as main scheme.	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)

Additional Notes

(a)

B1: Draws the correct circle, allow tolerance with shape, look for the intent, and indicates the radius is 1. Accept ± 1 on the imaginary axis or incorrect notation of coordinates as long as the radius of 1 is implied. May be drawn as the radius. Accept r =1 stated next to the diagram, but not $x^2+y^2=1$ without further indication of the radius being 1.

(b)

Note for the main scheme ddM1 first deduction is from using the perpendicular bisector of 9i and −i.

Question Number	Scheme	Notes	Marks
4	$4\dfrac{d^2y}{dx^2}-11\dfrac{dy}{dx}-3y=78e^{3x} \quad y=\dfrac{9}{2},\ \dfrac{dy}{dx}=0$ at $x=0$		
(a)	$y=\lambda xe^{3x} \Rightarrow$ $y'=\lambda e^{3x}+3\lambda xe^{3x},\quad y''=3\lambda e^{3x}+3\lambda e^{3x}+9\lambda xe^{3x}$	Correct first and second derivatives (need not be simplified).	B1
	$4\left(6\lambda e^{3x}+9\lambda xe^{3x}\right)-11\left(\lambda e^{3x}+3\lambda xe^{3x}\right)-3\left(\lambda xe^{3x}\right)=78e^{3x}$ $\Rightarrow 13\lambda e^{3x}=78e^{3x}\Rightarrow \lambda=6$	M1: Substitutes their attempts at the derivatives of form $\alpha e^{3x}+\beta xe^{3x}$ into the differential equation and solves for λ A1: Correct value from fully correct	M1 A1cso
			(3)
(b)	$4m^2-11m-3=0\Rightarrow(4m+1)(m-3)=0$ $\Rightarrow m=-\dfrac{1}{4},\ 3$	Forms auxiliary equation and solves (usual rules)	M1
	CF: $y=Ae^{-\frac{1}{4}x}+Be^{3x}$	For $Ae^{m_1x}+Be^{m_2x}$ with their real and different values of m	M1
	$y=Ae^{-\frac{1}{4}x}+Be^{3x}+"6"xe^{3x}$	Correct GS ft their non-zero λ including "$y=$ …"	A1ft
			(3)
(c)	$\dfrac{dy}{dx}=-\dfrac{1}{4}Ae^{-\frac{1}{4}x}+3Be^{3x}+6e^{3x}+18xe^{3x}$ or ft	M1: Differentiates to obtain an expression of the correct form A1: Correct ft expression	M1 A1ft
	$x=0,\ y=\dfrac{9}{2}\Rightarrow\dfrac{9}{2}=A+B$ $x=0,\ \dfrac{dy}{dx}=0\Rightarrow 0=-\dfrac{1}{4}A+3B+6$ $\Rightarrow A=\ldots,\quad B=\ldots$	Substitutes the given values into their equations and solves simultaneously for A and B	M1
	$y=6e^{-\frac{1}{4}x}-\dfrac{3}{2}e^{3x}+6xe^{3x}$	Fully correct particular solution (including "$y=$ …" unless this was penalised in part (b))	A1
			(4)
			Total 10

Additional Notes
You may mark this question as a whole. Do not be concerned about part labelling as many will answer out of order, e.g. finding CF before PI.
(a) **Note: This appears on ePEN as MMA but we are marking it as BMA**
B1: Correct first and second derivatives.
M1: Full process to find the constant. The derivatives need not have been correct for this mark but must have form $\alpha e^{3x}+\beta xe^{3x}$ $(\alpha,\beta\neq 0)$, look for the correct process.

A1cso: Correct value from fully correct work. The xe^{3x} must have cancelled in their equation giving only $\lambda=6$.
(b)
M1: Correct attempt to solve the auxiliary equation. If no method is shown the correct values can imply the attempt.
M1: Correct CF for their values, which must be real and distinct.
A1: Correct solution following through their value for λ. Must include the $y=$..

(c)

M1: Differentiates to correct form $Pe^{-\frac{x}{4}}+Qe^{3x}+Rxe^{3x}+"6"e^{3x}\quad (P,Q,R\neq 0)$, condoning at most one slip in powers. Must involve the product rule.
A1ft: Correct expression following through on their value for "6".
M1: For a full process to set up and solve simultaneous equations in A and B (or their variables) using the given values. Do not be concerned about algebraic slips, as long as two equations are set up then allow for any values following them.
A1: Fully correct, but only penalise a missing $y=\ldots$ in the first instance it would otherwise gain a mark. Allow from solutions which gained $\lambda=6$ in (a) even if from incorrect work.

Question Number	Scheme	Notes	Marks
5(a)	$\frac{2}{r(r+1)(r+2)} = \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$ $2 = A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ $r = 0: \quad 2 = 2A \Rightarrow A = 1$ $r = -1: \quad 2 = -B \Rightarrow B = -2$ $r = -2: \quad 2 = 2C \Rightarrow C = 1$	Any valid method to obtain at least one constant with partial fractions of the correct form. Could be implied	M1
	$\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}$	Correct partial fractions	A1
			(2)
(b)	$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \begin{cases} \frac{1}{1} - \frac{2}{2} + \cancel{\frac{1}{3}} + \\ \frac{1}{2} - \cancel{\frac{2}{3}} + \cancel{\frac{1}{4}} + \\ \cancel{\frac{1}{3}} - \cancel{\frac{2}{4}} + \cancel{\frac{1}{5}} + \\ \ldots + \ldots \\ \cancel{\frac{1}{n-1}} - \cancel{\frac{2}{n}} + \frac{1}{n+1} + \\ \cancel{\frac{1}{n}} - \frac{2}{n+1} + \frac{1}{n+2} \end{cases}$	Attempts at least three rows with their partial fractions of a correct form (see notes). Implied by correct expression	M1
	$= 1 - 1 + \frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$	A1: Correct constant terms or algebraic terms extracted A1: Fully correct expression	A1 A1
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2} = \frac{(n+1)(n+2) - 2(n+2) + 2(n+1)}{2(n+1)(n+2)}$	Combines terms of the form p, $\frac{q}{n+b}$, and $\frac{r}{n+c}$ and no others into a single fraction	M1
	$= \frac{n^2 + 3n + 2 - 2n - 4 + 2n + 2}{2(n+1)(n+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$	Correct expression	A1
			(5)
(c)	$\frac{n(n+3)}{2(n+1)(n+2)} > \frac{7}{15}$ $15n^2 + 45n > 14n^2 + 42n + 28$	Forms inequality or equation, cross-multiplies or puts over common denominator and expands brackets	M1
	$n^2 + 3n - 28 > 0$ $(n+7)(n-4) > 0$ $\Rightarrow n = \ldots$ or $n > \ldots \ (n > 4)$	Forms and solves 3TQ inequality or equation (usual rules)	dM1
	$n = 5$	5 only	A1
			(3)
			Total 10

Additional Note

(a)

M1: Must be correct partial fraction form with three fractions. Note that $\frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ is not a complete partial fraction form so is M0, but this form can be used to score full marks in part (b).

A1: Correct form.

(b)
M1: For evidence of correctly applying the method of differences to the question. This may be implied by extraction of the correct terms if no/incomplete method is seen. If method is shown they should give at least three lines of fractions to show cancelling.
A1: For either the correct constant terms at the start or the correct algebraic terms at the end extracted from the differences.
A1: All correct terms extracted (and not others)
Note that $\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}$ will give $\frac{1}{2}-\frac{1}{6}+\frac{1}{6}-\frac{1}{12}+\ldots-\frac{1}{(n+1)(n+2)}=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$ which can score M1A1A1, and then the next M for combining these.
M1: Combines the terms to a single fraction provided terms of suitable forms were extracted – as shown in scheme. The denominator must be correct but allow a slip in the numerator.
A1: Correct answer from correct work.
(c)
M1: Sets up and forms the required inequality, or accept as an equality formed, and proceeds to form an unsimplified quadratic equation or inequality by cross multiplying or by **correctly** putting terms over a common denominator and expanding the $n(n+3)$ and $(n+1)(n+2)$ brackets.

dM1: Solves the resulting 3 term quadratic (accept if just one root found). Note: answers which try to solve for the whole solution set for n will carry this out as part of the working, and can score full marks. Solutions to the quadratic by calculator are acceptable – you may need to check and accept if only one root of their quadratic is given.
A1: Correct answer only.
Note: Trial and Improvement or calculator methods which do not form a quadratic inequality or equation will score no marks.

Question Number	Scheme	Notes	Marks
6	$2\dfrac{dy}{dx}+(\cot x)y+(\tan x\sec x)y^3=0$		
(a)	$z=\dfrac{1}{y^2}\Rightarrow\dfrac{dz}{dx}=-\dfrac{2}{y^3}\dfrac{dy}{dx}$ or $y^2=\dfrac{1}{z}\Rightarrow 2y\dfrac{dy}{dx}=-\dfrac{1}{z^2}\dfrac{dz}{dx}\left\{=-y^4\dfrac{dz}{dx}\right\}$ or $y=z^{-\frac{1}{2}}\Rightarrow\dfrac{dy}{dx}=-\dfrac{1}{2}z^{-\frac{3}{2}}\dfrac{dz}{dx}$	Any correct equation in $\dfrac{dy}{dx}$ and $\dfrac{dz}{dx}$ (may be implied)	B1
	$-y^3\dfrac{dz}{dx}+(\cot x)y+(\tan x\sec x)y^3=0$	Substitutes to obtain an equation with $\dfrac{dz}{dx}$ as the only derivative	M1
	$-\dfrac{dz}{dx}+\dfrac{\cot x}{y^2}+\tan x\sec x=0$ $\dfrac{dz}{dx}-(\cot x)z=\tan x\sec x$ *	Fully correct proof with intermediate step after substitution	A1*
			(3)
(b)	$\text{I.F.}=e^{\int-\cot x\,dx}$	$\text{I.F.}=e^{\int\pm\cot x\,dx}$	M1
	$=e^{-\ln\sin x}=\dfrac{1}{\sin x}$	Correct processed I.F. in any form	A1
	$\dfrac{z}{\sin x}=\displaystyle\int\dfrac{1}{\sin x}\times\dfrac{\sin x}{\cos x}\times\dfrac{1}{\cos x}dx$	Applies $\text{I.F.}\times z=\int\text{I.F.}\times\tan x\sec x\,dx$ Condone y for z	M1
	$\dfrac{z}{\sin x}=\displaystyle\int\sec^2 x\,dx\Rightarrow\dfrac{z}{\sin x}=\tan x\ (+c)$	Correct equation in z and x Condone missing "$+c$"	A1
	$z=\sin x(\tan x+c)\Rightarrow y^2=\dfrac{1}{\sin x(\tan x+c)}$ oe	Fully correct general solution. Accept equivalent forms.	A1
			(5)
(c)	$\dfrac{4\sqrt{3}}{3}=\dfrac{1}{\frac{1}{2}\left(\frac{\sqrt{3}}{3}+c\right)}\Rightarrow\dfrac{\sqrt{3}}{3}+c=\dfrac{\sqrt{3}}{2}\Rightarrow c=\ldots\left(\dfrac{\sqrt{3}}{6}\right)$	Correctly substitutes the given values to find a value for c	M1
	$x=\dfrac{\pi}{3}\Rightarrow y^2=\dfrac{1}{\frac{\sqrt{3}}{2}\left(\sqrt{3}+\frac{\sqrt{3}}{6}\right)}=\dfrac{4}{7}$ $\Rightarrow y=\pm\dfrac{2\sqrt{7}}{7}$ oe	**d**M1: Substitutes $x=\dfrac{\pi}{3}$ and their c into their equation and reaches $y^2=\ldots$ or $y=$ with trig terms evaluated. A1: Both correct exact values. The negative value must not be rejected.	**d**M1 A1
			(3)
			Total 11

Additional Notes:

Condone poor notation of $\sin^{-1}x$ for cosec x as long as it is treated correctly.

(a) Note for A mark you may condone a missing "=0" in intermediate lines as long as it is implied by working.

(b) Note an equivalent form for the answer is $y^2=\dfrac{\cos x}{\sin^2 x+K\sin 2x}$ which will lead to $c=\dfrac{\sqrt{3}}{12}$ in (c)

Question Number	Scheme	Notes	Marks
7	$r=\sqrt{3}\sin\theta\ \ (0 \leqslant \theta \leqslant \pi)$ $\qquad r=3\cos\theta\ \ \left(-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}\right)$		
(a)	$\sqrt{3}\sin\theta=3\cos\theta \Rightarrow \tan\theta=\sqrt{3}$ $\Rightarrow \theta=\dfrac{\pi}{3},\quad r=\dfrac{3}{2}$ or $\left(\dfrac{3}{2},\dfrac{\pi}{3}\right)$	B1: Either correct coordinate B1: Both correct coordinates and no others	B1 B1
			(2)
(b)	$\int\left(\sqrt{3}\sin\theta\right)^2 \mathrm{d}\theta = K\int \pm\dfrac{1}{2}\pm\dfrac{1}{2}\cos 2\theta\,\mathrm{d}\theta=\ldots$ or $\int\left(3\cos\theta\right)^2 \mathrm{d}\theta = M\int \pm\dfrac{1}{2}\pm\dfrac{1}{2}\cos 2\theta\,\mathrm{d}\theta=\ldots$	Uses one of $\sin^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ or $\cos^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ in an attempt to integrate r^2 for one of the equations.	M1
	$\int\left(\sqrt{3}\sin\theta\right)^2 \mathrm{d}\theta = K\int \dfrac{1}{2}-\dfrac{1}{2}\cos 2\theta\,\mathrm{d}\theta=\ldots$ and $\int\left(3\cos\theta\right)^2 \mathrm{d}\theta = M\int \dfrac{1}{2}+\dfrac{1}{2}\cos 2\theta\,\mathrm{d}\theta=\ldots$	Uses both $\sin^2\theta=\frac{1}{2}-\frac{1}{2}\cos 2\theta$ and $\cos^2\theta=\frac{1}{2}+\frac{1}{2}\cos 2\theta$ in an attempt to integrate r^2 for both of the equations to correct form.	M1
	$\int\left(\sqrt{3}\sin\theta\right)^2 \mathrm{d}\theta = \dfrac{3}{2}\left(\theta-\dfrac{1}{2}\sin 2\theta\right)$ and $\int\left(3\cos\theta\right)^2 \mathrm{d}\theta = \dfrac{9}{2}\left(\theta+\dfrac{1}{2}\sin 2\theta\right)$	Fully correct expression after integration ignoring limits (accept equivalent expressions)	A1
	$\dfrac{1}{2}\int_0^{\frac{\pi}{3}}\left(\sqrt{3}\sin\theta\right)^2 \mathrm{d}\theta = \dfrac{3}{4}\left["\theta-\dfrac{1}{2}\sin 2\theta"\right]_0^{\frac{\pi}{3}}=\ldots$ or $\dfrac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\left(3\cos\theta\right)^2 \mathrm{d}\theta = \dfrac{9}{4}\left["\theta+\dfrac{1}{2}\sin 2\theta"\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}=\ldots$	Full process for at least one of the two areas required, including correct area formula, with their attempt to apply the double angle formula, and evidence of correct limits applied.	M1
	Area of $R=\dfrac{1}{2}\int_0^{\frac{\pi}{3}} 3\sin^2\theta\,\mathrm{d}\theta+\dfrac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\cos^2\theta\,\mathrm{d}\theta$ $=\dfrac{3}{4}\left[\dfrac{\pi}{3}-\dfrac{1}{2}\sin\dfrac{2\pi}{3}-(0-0)\right]+\dfrac{9}{4}\left[\dfrac{\pi}{2}+0-\left(\dfrac{\pi}{3}+\dfrac{1}{2}\sin\dfrac{2\pi}{3}\right)\right]$	Full and correct process for the area of R. Formulae and limits must all be correct, and evidence of limits applied, with areas added.	DM1
	$\dfrac{\pi}{4}-\dfrac{3\sqrt{3}}{16}+\dfrac{3\pi}{8}-\dfrac{9\sqrt{3}}{16}=\dfrac{5\pi}{8}-\dfrac{3\sqrt{3}}{4}$	Correct answer. Allow $a=\dfrac{5}{8},\quad b=-\dfrac{3}{4}$	A1
			(6)
			Total 8

Additional Notes

(a)

B1: One correct coordinate, do not be concerned about the method for this mark. Allow for $\theta=\dfrac{\pi}{3}$ seen and isw if they go on to use $\theta=\dfrac{\pi}{3}$ for this mark.

B1: Both coordinates correct, and no extras. Again, do not be concerned about the method. Condone if given in the wrong order in a coordinate pair.

(b)

M1: Starts correct process to find one of the two areas. Must be using r^2 for one of the two curves, attempt a double angle formula $\sin^2\theta = \pm\frac{1}{2} \pm \frac{1}{2}\cos 2\theta$ or $\cos^2\theta = \pm\frac{1}{2} \pm \frac{1}{2}\cos 2\theta$ and any attempt to integrate the result (no matter how poor, ie a changed function). Condone errors with the coefficient not being squared.

M1: A full process to find the integral of r^2 for **both** curves with **correct** double angle formulae used, so $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$ and reaches correct for each, but again allow if slips occur in the coefficient.

A1: Fully correct integration for r^2 for **both** curves. The coefficients must have been correctly dealt with. Note the $\frac{1}{2}$ may be included throughout.

M1: For a full correct process to find at least one of the areas. Must be using $\frac{1}{2}\int_{\alpha}^{\beta} r^2\,\mathrm{d}\theta$ including the $\frac{1}{2}$ and with **correct** limits for the area applied (condone not showing subtracted 0's), but condone slips in the coefficients when squaring and condone if incorrect double angle formula attempts were made.

Note: Some candidates may attempt to find only one area and double the result. If the area formula is clearly seen or applied before doubling this is acceptable as an attempt for the full process for one area.

DM1: Depends on all previous method marks. Fully correct process for **both** areas and adds the results. This may be done separately or in one expression throughout the working.

A1: Correct answer.

Question Number	Scheme	Notes	Marks
8(a)	For intersection when $x > 0$: $\frac{15x}{x+4} = x-2$ $15x = x^2 + 2x - 8$ $x^2 - 13x - 8 = 0$ $\Rightarrow x = \frac{13+\sqrt{201}}{2}$	M1: Solves 3TQ obtained from a correct equation (usual rules) for $x > 0$ A1: Correct exact positive value (condone negative root also included)	M1 A1
	For intersections when $x < 0$: $\frac{15x}{-x+4} = x-2$ $15x = -x^2 + 6x - 8$ $x^2 + 9x + 8 = 0$ $(x+8)(x+1) = 0$ $\Rightarrow x = -8, \quad -1$	M1: Solves 3TQ obtained from a correct equation (usual rules) for $x < 0$ A1: Correct values	M1 A1
	$-8 < x < -1, \quad x > \frac{13+\sqrt{201}}{2}$	M1: For $a < x < b,\ x > c$ oe with cvs $a < b < 0 < c$ from the correct equations. A1: Fully correct set of values in any form	M1 A1
			(6)
(b)	$\frac{15x}{x+4} = -x+2$ $15x = -x^2 - 2x + 8$ $x^2 + 17x - 8 = 0$ $\Rightarrow x = \frac{-17+\sqrt{321}}{2}$	M1: Solves 3TQ obtained from a correct equation (usual rules) A1: Correct exact positive value (condone negative root also included)	M1 A1
	$x < "-8", \quad x > "\frac{13+\sqrt{201}}{2}"$	Correct regions ft their $a < 0$ and $c > 0$	B1ft
	$x < "-8", \quad "-1" < x < \frac{-17+\sqrt{321}}{2}, \quad x > "\frac{13+\sqrt{201}}{2}"$	Correct set of values in any form. Allow ft on their cvs a, b and c provided $a < b < 0$ and $c > \frac{-17+\sqrt{321}}{2} (= 0.458...)$	A1ft
			(4)
			Total 10

Additional notes.
"Usual Rules" : as well as the general information, we will accept here solutions by calculator having reached a quadratic equation but they must be correct solutions for the stated quadratic if no method is shown (you may need to check).
(a)
Accept with any inequalities used instead of equalities for the solving of the intersection points. For methods via putting over common denominators score the Ms for the process of putting over a common denominator and solving for the numerator = 0.
May see attempts via squaring

$\frac{15x}{|x|+4}=x-2 \Rightarrow |x|=\frac{15x}{x-2}-4=\frac{11x+8}{x-2} \Rightarrow x^2=\left(\frac{11x+8}{x-2}\right)^2$ then cross multiplies (or puts over a common denominator $(x-2)^2$) to get

$\Rightarrow x^2(x^2-4x+4)-\left(121x^2+176x+64\right)=x^4-4x^3-117x^2-176x-64=0$. This will probably then be solved by calculator, which as algebra has been used will be accepted. Score M1 for the process reaching quartic, A1 correct quartic, M1 for solving the quartic (look for attempt to factorise to linear terms with correct product of roots, or accept answer from calculator – may need to check), A1 for at least the three relevant roots (and condone if x =2 is also include for this mark).

For the final M they must be producing a solution set of the correct form from their CVs from attempts at the correct equations (but may have had incorrect method for solving), so if extra values from denominators are used it will be M0 (regardless of whether the form of the answer is correct).

FYI $\frac{-13-\sqrt{201}}{2}\approx -13.6$ is being used often as one of the limits, but note it is less than −8, so $-8<x<\frac{-13-\sqrt{201}}{2}$ used is M0 but you may condone the "$-8<x$" for the B1ft in (b).

Final A1: Accept any correct equivalent notation. Accept with "and" or "or" written between inequalities, but if using set notation must use the union. E.g. in interval notation $(-8,-1)\cup\left(\frac{13+\sqrt{201}}{2},\infty\right)$. Use of square brackets is A0.

(b)

Attempts at squaring may be seen again here. They will need a full process leading solving a quartic. Again for the A, accept if other roots are also included at this stage.

B1ft: For the follow through same "outside" ends of the region matching their answer to (b) in their **final** answer.

A1ft: Accept answers of the correct form providing their answer to (a) was of the correct form as specified in the scheme and their c is $c>\frac{-17+\sqrt{321}}{2}(=0.458...)$.

Question Number	Scheme	Notes	Marks
9(a)	$\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$ $= \cos^5\theta + {}^5C_1\cos^4\theta(\mathrm{i}\sin\theta) + {}^5C_2\cos^3\theta(\mathrm{i}\sin\theta)^2 + {}^5C_3\cos^2\theta(\mathrm{i}\sin\theta)^3 + {}^5C_4\cos\theta(\mathrm{i}\sin\theta)^4 + (\mathrm{i}\sin\theta)^5$		M1
	$(\sin 5\theta =) 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$	M1: Equates imaginary parts A1: Correct expression	M1 A1
	$= 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$ $= 5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta$ $= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$	M1: Uses $\cos^2\theta = 1-\sin^2\theta$ and reaches an expression of the correct form A1: Correct expression or $a=16,\ b=-20,\ c=5$ $\sin 5\theta$ must have been seen at some point	M1 A1
			(5)
(b)	$\left(\text{For } \theta = \frac{k\pi}{5},\ k \in \square,\ \sin 5\theta = \sin k\pi = 0 \Rightarrow\right)$ $16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = 0$	Sets their answer to (a) = 0 Could be implied	M1
	$\sin\theta(16\sin^4\theta - 20\sin^2\theta + 5) = 0$ $\Rightarrow \sin^2\theta = \frac{-(-20)\pm\sqrt{(-20)^2 - 4(16)(5)}}{2(16)} = \frac{20\pm\sqrt{80}}{32}$	Solves their quadratic in $\sin^2\theta$ to find both roots (usual rules)	M1
	$\Rightarrow \sin^2\frac{k\pi}{5} = \frac{5\pm\sqrt{5}}{8},\ 0$	A1: $\frac{5\pm\sqrt{5}}{8}$ oe B1: 0	A1, B1
			(4)
			Total 9

Additional Notes

(a)

M1: Attempts to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$ to obtain an expression of the correct form for the three imaginary terms. May use $c + \mathrm{i}s$ notation. Need only see the imaginary terms, condone missing or incorrect real terms. Binomial coefficients may be as expressions or values, if values only used they must be correct. The powers of i may be processed in the initial expansion. Allow invisible brackets if recovered.

M1: Extracts the imaginary terms from the expansion. The equating to $\sin 5\theta$ may be tacit at this stage.

A1: Correct expression for $\sin 5\theta$ (though the $\sin 5\theta$ need not be seen yet) in terms of $\sin\theta$ and $\cos\theta$.

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to eliminate $\cos\theta$ from the expression to get to an expanded equation in $\sin\theta$ only. Terms need not be gathered at this stage, but the simplification may be done in one step.

A1: Correct answer including the $\sin 5\theta$ having been seen identified with the expression at some stage if not included at the end. Accept values for *a*, *b* and *c* stated following reaching a suitable equation in $\sin\theta$

(b)

M1: Recognition of the correct process seen by either their answer to (a) equal to zero, which by be implied, or by an attempt to solve it (the "=0" being implied, accept one correct root for their equation as evidence).

M1: Correct method to solve the quadratic in $\sin^2\theta$ after factoring out the $\sin\theta$ - both roots must be given. Condone for this mark if they mistakenly mislabel the solution to the quadratic in $\sin^2\theta$ as $\sin\theta$, but do not

allow the A if they go on square their values. Note answer stated directly from the quintic score M0. They must at least identify the quadratic in $\sin^2\theta$ before reaching the answers (from calculator).
A1: Correct answers only but accept equivalent forms.
B1: For 0 identified as a value for $\sin^2\frac{k\pi}{5}$ - this can come from anywhere (method not required).