Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1.

$$\mathbf{A} = \begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix}$$

where a is a non-zero constant and $a \neq 3$

(a) Determine $\mathbf{A}^{-1}$ giving your answer in terms of a.

(2)

Given that $\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a.

(3)

(a) $\det(A) = -6 + 2a$

$$A^{-1} = \frac{1}{-6+2a}\begin{pmatrix} -2 & -a \\ 2 & 3 \end{pmatrix}$$

(b) $$\begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix} + \frac{1}{-6+2a}\begin{pmatrix} -2 & -a \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$3 + \frac{-2}{-6+2a} = 1$$

$$3 + \frac{2}{6-2a} = 1$$

$$6 - 2a = -1$$

$$7 = 2a$$

$$a = 3.5$$

Leave blank

2.
$$f(x) = 7\sqrt{x} - \frac{1}{2}x^3 - \frac{5}{3x} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [2.8, 2.9]

(2)

(b) (i) Find $f'(x)$.

(ii) Hence, using $x_0 = 2.8$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(4)

(c) Use linear interpolation once on the interval [2.8, 2.9] to find another approximation to α. Give your answer to 3 decimal places.

(3)

(a)

2.8 → $\frac{14}{5}$	$7\sqrt{\text{Ans}} - .5\text{Ans}^3 - \frac{5}{3\text{Ans}}$ A 0.1420022762 > 0
2.9 → $\frac{29}{10}$	$7\sqrt{\text{Ans}} - .5\text{Ans}^3 - \frac{5}{3\text{Ans}}$ B -0.8486421875 < 0

f(x) is continuous and there is a sign change.
Hence there is a root between 2.8 and 2.9.

(b) $f(x) = 7x^{\frac{1}{2}} - \frac{1}{2}x^3 - \frac{5}{3}x^{-1}$

(i) $f'(x) = \frac{7}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^2 + \frac{5}{3}x^{-2}$

(ii) $x_1 = 2.8 - \frac{f(2.8)}{f'(2.8)} = 2.8 - \frac{0.142002276\ldots}{-9.4557649\ldots} = 2.815$

(c) 2.8 — α — 2.9 ; A — 0 — B

$$\frac{2.9-2.8}{B-A} = \frac{2.8-\alpha}{A-0}$$

$\frac{2.9-2.8}{\text{B}-\text{A}}$ -0.1009443889	2.8−Ans×A 2.814334333

$\alpha = 2.814$

Leave blank

3. The quadratic equation

$$2x^2 - 5x + 7 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$ **(1)**

$\frac{5}{2}$ $\frac{7}{2}$

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{25}{4} - \frac{14}{2} = -\frac{3}{4}$

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3(\alpha+\beta)\alpha\beta = \frac{125}{8} - 3\left(\frac{5}{2}\right)\left(\frac{7}{2}\right) = -\frac{85}{8}$ **(4)**

(c) find a quadratic equation that has roots

$$\frac{1}{\alpha^2 + \beta} \quad \text{and} \quad \frac{1}{\beta^2 + \alpha}$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined. **(4)**

(c) Sum: $\dfrac{\alpha+\beta + \alpha^2+\beta^2}{(\alpha^2+\beta)(\beta^2+\alpha)} = \dfrac{\frac{5}{2} - \frac{3}{4}}{\alpha^3 + \alpha^2\beta^2 + \beta^3 + \alpha\beta}$

$= \dfrac{7/4}{-\frac{85}{8} + \frac{7}{2} + \frac{49}{4}} =$

$$\frac{7\div4}{7\div2+49\div4-85\div8} = \frac{14}{41}$$

prod:

$$\frac{1}{7\div2+49\div4-85\div8} = \frac{8}{41}$$

$x^2 - \frac{14}{41}x + \frac{8}{41} = 0$

$41x^2 - 14x + 8 = 0$

Leave blank

4. $$f(z) = 2z^3 - z^2 + az + b$$

where a and b are integers.

The complex number $-1 - 3i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation. **(1)**

$z = -1 + 3i$

(b) Determine the value of a and the value of b. **(4)**

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram. **(2)**

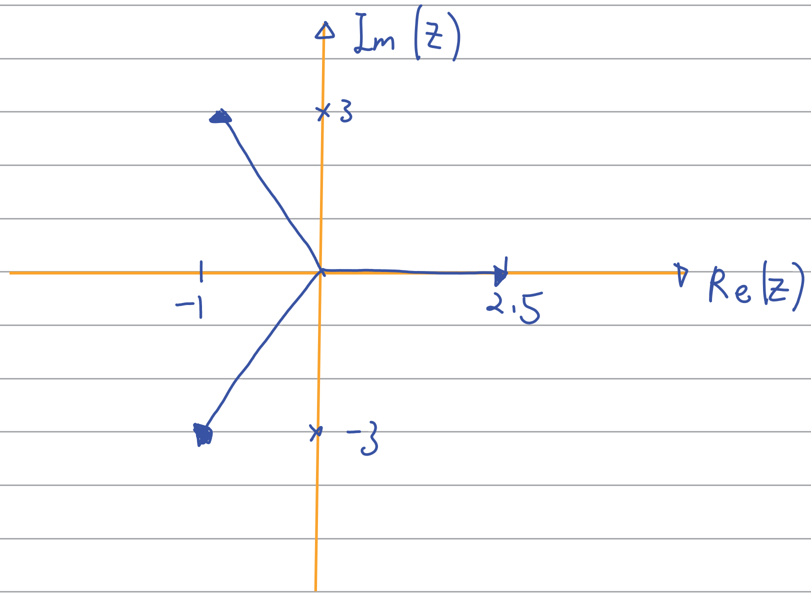
(b) $(-1+3i) + (-1-3i) = -2$ $\quad z^2 + 2z + 10$

$(-1+3i)(-1-3i) = 10$

$$\begin{array}{r|l} & 2z - 5 \\ z^2+2z+10 & 2z^3 - z^2 + az + b \\ & 2z^3 + 4z^2 + 20z \\ \hline & -5z^2 + (a-20)z + b \\ & -5z^2 - 10z - 50 \\ \hline & 0 \end{array}$$

$\begin{cases} a - 20 = -10 \\ b = -50 \end{cases}$ $\quad \begin{cases} a = 10 \\ b = -50 \end{cases}$

(c)



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5. (a) Use the standard results for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r-1)(r-3) = \frac{1}{12}n(n+1)(n-1)(3n-10)$$

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n+1} r(r-1)(r-3) = \frac{1}{12}n(n+1)(an^2+bn+c)$$

where a, b and c are integers to be determined.

(3)

(a) $\sum_{r=1}^{n} r(r^2-4r+3) = \sum_{r=1}^{n} r^3 - 4r^2 + 3r$

$= \frac{1}{4}n^2(n+1)^2 - 4\cdot\frac{1}{6}n(n+1)(2n+1) + 3\cdot\frac{n}{2}(1+n)$

$= \frac{1}{12}n(n+1)\left[3n(n+1) - 8(2n+1) + 18\right]$

$= \frac{1}{12}n(n+1)\left[3n^2+3n-16n+10\right]$

$= \frac{1}{12}n(n+1)(3n^2-13n+10)$

$= \frac{1}{12}n(n+1)(n-1)(3n-10)$

(b) $\sum_{r=n+1}^{2n+1} = S(2n+1) - S(n)$

$= \frac{1}{12}(2n+1)\overset{2(n+1)}{(2n+2)}\overset{\cdot 2n}{(2n)}(6n-7) - \frac{1}{12}n(n+1)(n-1)(3n-10)$

$= \frac{1}{12}n(n+1)\left[\underset{(48n^2-32n-28)}{(8n+4)(6n-7)} - \underset{(3n^2-13n+10)}{(n-1)(3n-10)}\right]$

$= \frac{1}{12}\cdot n(n+1)(45n^2-19n-38)$

Leave blank

6. The curve H has equation

$$xy = a^2 \qquad x > 0$$

where a is a positive constant.

The line with equation $y = kx$, where k is a positive constant, intersects H at the point P

(a) Use calculus to determine, in terms of a and k, an equation for the tangent to H at P **(4)**

The tangent to H at P meets the x-axis at the point A and meets the y-axis at the point B

(b) Determine the coordinates of A and the coordinates of B, giving your answers in terms of a and k **(2)**

(c) Hence show that the area of triangle AOB, where O is the origin, is independent of k **(2)**

(a) P: $x \cdot (kx) = a^2 \qquad x^2 = a^2/k \qquad x = a/\sqrt{k}$

$P\left(a/\sqrt{k},\ a\cdot\sqrt{k}\right)$

$y = a^2 x^{-1}$

$\frac{dy}{dx} = -\frac{a^2}{x^2} = -\frac{a^2}{a^2/k} = -k$

TANGENT: $y - a\sqrt{k} = -k\left(x - a/\sqrt{k}\right)$

OR $y = -kx + 2a\sqrt{k}$

(b) $y = 0$: $x = 2a/\sqrt{k} \qquad x = 0$: $y = 2a\sqrt{k}$

$A\left(2a/\sqrt{k}, 0\right) \qquad B\left(0, 2a\sqrt{k}\right)$

(c) AREA: $\frac{1}{2}\left(2a/\sqrt{k}\right)\left(2a\sqrt{k}\right)$

$= \frac{1}{2}(2a)(2a)$

$= 2a^2$

independent of k.

Leave blank

7. In part (i), the elements of each matrix should be expressed in exact numerical form.

(i) (a) Write down the 2×2 matrix that represents a rotation of 210° anticlockwise about the origin.

$$\frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix}$$

(1)

(b) Write down the 2×2 matrix that represents a stretch parallel to the y-axis with scale factor 5

$$\begin{array}{cc} x & y \end{array}$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$$

(1)

The transformation T is a rotation of 210° anticlockwise about the origin followed by a stretch parallel to the y-axis with scale factor 5

(c) Determine the 2×2 matrix that represents T

$$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -5 & -5\sqrt{3} \end{pmatrix}$$

(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} k & k+3 \\ -5 & 1-k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Find det $\mathbf{M}$, giving your answer in simplest form in terms of k.

(2)

A closed shape R is transformed to a closed shape R' by the transformation represented by the matrix $\mathbf{M}$.

Given that the area of R is 2 square units and that the area of R' is $16k$ square units,

(b) determine the possible values of k.

(3)

(ii) (a) $\det(M) = k(1-k) + 5(k+3) = -k^2 + 6k + 15$

(b) $-k^2 + 6k + 15 = 8k$

$k^2 + 2k - 15 = 0$

$(k+5)(k-3) = 0$

$k = 3, -5$

OR

$-k^2 + 6k + 15 = -8k$

$k^2 - 14k - 15 = 0$

$(k-15)(k+1) = 0$

$k = -1, 15$

$k = 3, -5, -1, 15$

Leave blank

8. The parabola C has equation $y^2 = 20x$

The point P on C has coordinates $(5p^2, 10p)$ where p is a non-zero constant.

(a) Use calculus to show that the tangent to C at P has equation

$$py - x = 5p^2$$

(3)

The tangent to C at P meets the y-axis at the point A.

(b) Write down the coordinates of A.

(1)

The point S is the focus of C.

(c) Write down the coordinates of S.

(1)

The straight line l_1 passes through A and S.

The straight line l_2 passes through O and P, where O is the origin.

Given that l_1 and l_2 intersect at the point B,

(d) show that the coordinates of B satisfy the equation

$$2x^2 + y^2 = 10x$$

(5)

(a) $\frac{dx}{dp} = 10p$ $\quad \frac{dy}{dp} = 10$ $\quad \frac{dy}{dx} = 1/p$

TANGENG: $\quad y - 10p = 1/p\,(x - 5p^2)$

$py - 10p^2 = x - 5p^2$

$py - x = 5p^2$

(b) $x=0$: $\quad y = 5p$ $\quad A\ (0, 5p)$

(c) $y^2 = 4 \cdot 5x^2$

$= 4 \cdot a x^2$ $\quad S\ (5, 0)$

(d): l_1: $\quad \frac{x}{5} + \frac{y}{5p} = 1$ $\quad px + y = 5p$

l_2: $\quad y = 2/p\ x$ $\quad \leftarrow P(5p^2, 10p)$

$p = \frac{y}{5-x} = \frac{2x}{y}$ $\quad \rightarrow y^2 = 10x - 2x^2$

$2x^2 + y^2 = 10x$

Leave blank

9. (i) A sequence of numbers is defined by

$$u_1 = 0 \qquad u_2 = -6$$

$$u_{n+2} = 5u_{n+1} - 6u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3 \times 2^n - 2 \times 3^n$$

(5)

(ii) Prove by induction that, for all positive integers n,

$$\mathrm{f}(n) = 3^{3n-2} + 2^{4n-1}$$

is divisible by 11

(5)

(i) $n=1 \qquad U_1 = 3\times 2 - 2\times 3 = 0$

$n=2 \qquad U_2 = 3\times 4 - 2\times 3 = 12-6=6$

ASSUME $P(n)$ TRUE: $U_n = 3\times 2^n - 2\times 3^n$

NOW $U_{n+1} = 3\times 2^{n+1} - 2\times 3^{n+1}$

$$U_{n+2} = 5[3\times 2^{n+1} - 2\times 3^{n+1}] - 6[3\times 2^n - 2\times 3^n]$$
$$= 15\cdot 2^{n+1} - 10\cdot 3^{n+1} - 18\cdot 2^n + 12\cdot 3^n$$
$$= 30\cdot 2^n - 30\cdot 3^n - 18\cdot 2^n + 12\cdot 3^n$$
$$= 12\cdot 2^n - 18\cdot 3^n$$
$$= 3\cdot 2^{n+2} - 2\cdot 3^{n+2} \rightarrow P(n+2)$$ by induction.

$P(n) \rightarrow P(n+2)$ TRUE FOR $P(n), P(n+1)$, THEN TRUE FOR $P(n+2)$

TRUE FOR $n=1,2$, SO TRUE FOR ALL n.

Leave blank

Question 9 continued

(ii) $P(1)$ $f(1) = 3^1 + 2^3 = 11$ TRUE

ASSUME $P(n)$ TRUE,

$$f(n) = 3^{3n-2} + 2^{4n-1} = 11k \quad k \in \mathbb{Z}.$$

CONSIDER $f(n+1) - f(n)$

$$= 3^{3n+1} + 2^{4n+3} - 3^{3n-2} - 2^{4n-1}$$
$$= 27 \cdot 3^{3n-2} + 16 \cdot 2^{4n-1} - 3^{3n-2} - 2^{4n-1}$$
$$= 26 \cdot 3^{3n-2} + 15 \cdot 2^{4n-1}$$

$$= 11 \cdot 3^{3n-2} + 15\left[3^{3n-2} + 2^{4n-1}\right]$$

$$= 11\left(3^{3n-2} + 15 \cdot 11k\right)$$ multiple of 11.

$f(n+1) - f(n)$ is a multiple of 11. and
$f(n)$ is a multiple of 11. so
$f(n+1)$ is a multiple of 11.

so $P(n) \longrightarrow P(n+1)$ by induction.

$P(1)$ is TRUE, $P(n)$ is true for $n \geq 1$