Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel**
**International**
**Advanced Level**

Centre Number | Candidate Number

**Time** 1 hour 30 minutes

**Paper reference** **WFM01/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F1

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Leave blank

**1.** (i) $$f(x) = x^3 + 4x - 6$$

(a) Show that the equation $f(x) = 0$ has a root $\alpha$ in the interval $[1, 1.5]$

**(2)**

(b) Taking 1.5 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of $\alpha$. Give your answer to 3 decimal places. Show your working clearly.

**(4)**

(ii) $$g(x) = 4x^2 + x - \tan x$$

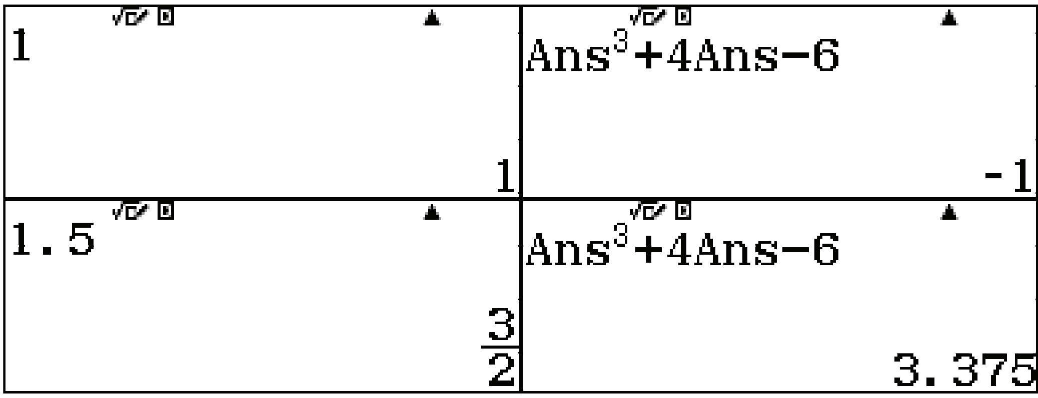
where $x$ is measured in radians.

The equation $g(x) = 0$ has a single root $\beta$ in the interval $[1.4, 1.5]$

Use linear interpolation on the values at the end points of this interval to obtain an approximation to $\beta$. Give your answer to 3 decimal places.

**(4)**

(i) (a)




(b) $f'(x) = 3x^2 + 4$

$x_2 = 1.5 - \frac{3.375}{10.75} = 1.186\ldots$

$x_3 = 1.186 - \frac{0.412}{8.220} \approx 1.1358$

$\alpha = 1.136$

Leave blank

**Question 1 continued**

(ii)

| | |
|---|---|
| 1.4 → $\frac{7}{5}$ | $4\text{Ans}^2+\text{Ans}-\tan(\text{Ans}$ → 3.442116285 |
| 1.5 → $\frac{3}{2}$ | $4\text{Ans}^2+\text{Ans}-\tan(\text{Ans}$ → -3.601419947 |

A

B


$$\frac{1.5-\beta}{B} = \frac{1.5-1.4}{B-A}$$

$\frac{1.5-1.4}{B-A}$

-0.01419741401

Ans×B

0.05113085003

1.5−Ans

1.44886915

$\beta = 1.449$

Leave blank

**2.** The complex numbers $z_1$, $z_2$ and $z_3$ are given by

$$z_1 = 2 - \mathrm{i} \qquad z_2 = p - \mathrm{i} \qquad z_3 = p + \mathrm{i}$$

where $p$ is a real number.

(a) Find $\dfrac{z_2 z_3}{z_1}$ in the form $a + b\mathrm{i}$ where $a$ and $b$ are real. Give your answer in its simplest form in terms of $p$.

**(3)**

Given that $\left|\dfrac{z_2 z_3}{z_1}\right| = 2\sqrt{5}$

(b) find the possible values of $p$.

**(4)**

(a) $\dfrac{(p-i)(p+i)}{2-i} = \dfrac{p^2+1}{2-i} \times \dfrac{2+i}{2+i}$

$= \dfrac{2p^2 + ip^2 + 2 + i}{4+1} = \boxed{\tfrac{1}{5}(2p^2+2) + \tfrac{i}{5}(p^2+1)}$

(b) $\left|\dfrac{z_2 \cdot z_3}{z_1}\right| = \sqrt{\dfrac{1}{25}\left[(2p^2+2)^2 + (p^2+1)^2\right]} = 2\sqrt{5}$

$20 = \frac{1}{25}\left[4(p^2+1)^2 + (p^2+1)^2\right]$

$20 = \frac{1}{5}(p^2+1)^2$

$100 = (p^2+1)^2$

$p^2 = 9$

$p = \pm 3$

Leave blank

**3.** The triangle $T$ has vertices $A(2,1)$, $B(2,3)$ and $C(0,1)$.

The triangle $T'$ is the image of $T$ under the transformation represented by the matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) Find the coordinates of the vertices of $T'$

**(2)**

(b) Describe fully the transformation represented by **P**

**(2)**

The $2 \times 2$ matrix **Q** represents a reflection in the $x$-axis and the $2 \times 2$ matrix **R** represents a rotation through 90° anticlockwise about the origin.

(c) Write down the matrix **Q** and the matrix **R**

**(2)**

(d) Find the matrix **RQ**

**(2)**

(e) Give a full geometrical description of the single transformation represented by the answer to part (d).

**(2)**

(a) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{pmatrix} 1 & 3 & 1 \\ -2 & -2 & 0 \end{pmatrix} \end{array}$

(b) ROTATION 270° ANTICLOCKWISE ABOUT THE ORIGIN.
90° CLOCKWISE

(c) $Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(d) $RQ = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(e) REFLECTION in $y = x$

Leave blank

**4.** A rectangular hyperbola $H$ has equation $xy = 25$

The point $P\left(5t, \dfrac{5}{t}\right)$, $t \neq 0$, is a general point on $H$.

(a) Show that the equation of the tangent to $H$ at $P$ is $t^2y + x = 10t$

**(4)**

The distinct points $Q$ and $R$ lie on $H$. The tangent to $H$ at the point $Q$ and the tangent to $H$ at the point $R$ meet at the point $(15, -5)$.

(b) Find the coordinates of the points $Q$ and $R$.

**(4)**

1a) $\dfrac{dx}{dt} = 5 \qquad \dfrac{dy}{dt} = -5t^{-2}$

$\dfrac{dy}{dx} = \dfrac{-5t^{-2}}{5} = -t^{-2}$

TANGENT: $y - 5/t = -t^{-2}(x - 5t)$

$t^2y - 5t = 5t - x$

$t^2y + x = 10t$

(b) R ON TANGENT: $t^2(-5) + 15 = 10t$

$t^2 + 2t - 3 = 0$

$(t+3)(t-1) = 0$

$t = 1, -3$

$t=1$ : $(5, 5)$

$t=-3$ : $(-15, -5/3)$

Leave blank

**5.** $$f(x) = (9x^2 + d)\left(x^2 - 8x + (10d + 1)\right)$$

where $d$ is a positive constant.

(a) Find the four roots of $f(x)$ giving your answers in terms of $d$.

**(3)**

Given $d = 4$

(b) Express these four roots in the form $a + ib$, where $a, b \in \mathbb{R}$.

**(2)**

(c) Show these four roots on a single Argand diagram.

**(2)**

(a) $9x^2 + d = 0$ $\quad x^2 = -d/9$ $\quad x = \pm \sqrt{d}/3\, i$

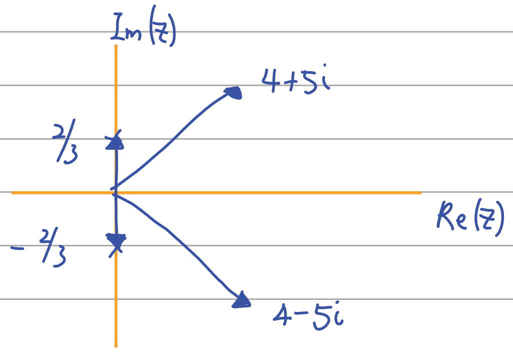
$$x = \frac{8 \pm \sqrt{64 - 4(10d+1)}}{2} = \frac{8 \pm \sqrt{60 - 40d}}{2}$$

$$= 4 \pm \sqrt{15 - 10d}$$

(b)

$x = \pm 2/3\, i$ $\qquad x = 4 \pm 5i$

(c)



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**6.** The parabola $C$ has Cartesian equation $y^2 = 8x$

The point $P(2p^2, 4p)$ and the point $Q(2q^2, 4q)$, where $p, q \neq 0$, $p \neq q$, are points on $C$.

(a) Show that an equation of the normal to $C$ at $P$ is

$$y + px = 2p^3 + 4p$$

**(5)**

(b) Write down an equation of the normal to $C$ at $Q$

**(1)**

The normal to $C$ at $P$ and the normal to $C$ at $Q$ meet at the point $N$

(c) Show that $N$ has coordinates

$$\left(2(p^2 + pq + q^2 + 2), -2pq(p + q)\right)$$

**(5)**

The line $ON$, where $O$ is the origin, is perpendicular to the line $PQ$

(d) Find the value of $(p + q)^2 - 3pq$

**(5)**

(a) At $P(2p^2, 4p)$ $\quad \frac{dy}{dx} = \frac{4}{4p} = 1/p$

NORMAL : $y - 4p = -p(x - 2p^2)$

$y + xp = 2p^3 + 4p$

(b) At $Q(2q^2, 4q)$ $\quad \frac{dy}{dx} = \frac{4}{4q} = 1/q$

NORMAL : $y - 4q = -q(x - 2q^2)$

$y + qx = 2q^3 + 4q$

(c) $2p^3 + 4p - xp = 2q^3 + 4q - qx$

$(p-q)x = 2(p^3 - q^3 + 2p - 2q)$

$= 2[(p-q)(p^2 + pq + q^2) + 2(p-q)]$

$x = 2(p^2 + pq + q^2 + 2)$

Leave blank

**Question 6 continued**

$$y = 2q^3 + 4q - q2(p^2 + pq + q^2 + 2)$$

$$= 2q^3 + 4q - 2qp^2 - 2pq^2 - 2q^3 - 4q$$

$$= -2pq(p+q)$$

(d)

$$m_{ON} = \frac{-2pq(p+q)}{2(p^2+pq+q^2+2)} = \boxed{-\frac{pq(p+q)}{p^2+pq+q^2+2}}$$

$$m_{PQ} = \frac{4(p-q)}{2(p^2-q^2)} = \frac{2(p-q)}{(p+q)(p-q)} = \frac{2}{p+q}$$

$$-\frac{pq(p+q)}{p^2+pq+q^2+2} \cdot \frac{2}{p+q} = -1$$

$$2pq = p^2 + pq + q^2 + 2$$

$$p^2 - pq + q^2 = -2$$

$$(p+q)^2 - 3pq = -2$$

Leave blank

**7.** (a) Prove by induction that for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

**(5)**

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + 2) = \frac{n}{6}(an^2 + bn + c)$$

where $a$, $b$ and $c$ are integers to be found.

**(4)**

(c) Using your answers to part (b), find the value of

$$\sum_{r=10}^{25} (r^2 + 2)$$

**(2)**

7(a) P(1) $n=1$ $\sum_{r=1}^{1} r^2 = 1^2 = 1 = \frac{1}{6}(2)(3)$

P(1) IS TRUE.

ASSUME P(n) TRUE : $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$

$$\sum_{r=1}^{n+1} r^2 = \frac{n}{6}(n+1)(2n+1) + (n+1)^2$$

$2 \quad +3$

$1 \quad +2$

$$= \frac{1}{6}(n+1)\left[2n^2 + n + 6n + 6\right]$$

$$= \frac{1}{6}(n+1)(2n^2+7n+6) = \frac{1}{6}(n+1)(2n+3)(n+2)$$

P(n) $\rightarrow$ P(n+1) by induction.

P(1) is true . so P(n) is true for all $n \in \mathbb{N}$.

Leave blank

**Question 7 continued**

(b) $\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} 2 = \frac{n}{6}(n+1)(2n+1) + 2n$

$= \frac{n}{6}\left[2n^2 + 3n + 1 + 12\right] = \frac{n}{6}\left[2n^2 + 3n + 13\right]$

(c)

$\frac{25}{6}\left[2(25)^2 + 3(25) + 13\right] - \frac{9}{6}\left[2(9)^2 + 3(9) + 13\right]$

| | |
|---|---|
| 25<br>25 | $\frac{\text{Ans}}{6}(2\text{Ans}^2+3\text{Ans}+1$<br>5575 |
| 9<br>9 | $\frac{\text{Ans}}{6}(2\text{Ans}^2+3\text{Ans}+1$<br>303 |

A

B

A−B

5272

Leave blank

**8.** Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers $n$.

**(6)**

$P(1)$ $\quad 4^3 + 5^3 = 64 + 125 = 189 = 27 \times 7$

$P(1)$ TRUE, ASSUME $P(n)$ TRUE

$$P(n) = 4^{n+2} + 5^{2n+1} = 7k \qquad k \in \mathbb{Z}.$$

NOW $\quad P(n+1) - P(n)$

$$= 4^{n+3} + 5^{2n+3} - 4^{n+2} - 5^{2n+1}$$

$$= 4 \cdot 4^{n+2} + 25 \cdot 5^{2n+1} - 4^{n+2} - 5^{2n+1}$$

$$= 3 \cdot 4^{n+2} + 24 \cdot 5^{2n+1}$$

$$= 3\left[4^{n+2} + 5^{2n+1}\right] + 21 \cdot 5^{2n+1}$$

$$= 3 \cdot 7k + 21 \cdot 5^{2n+1} = 7\left(3k + 3 \cdot 5^{2n+1}\right)$$

$P(n+1) - P(n)$ IS A MULTIPLE OF 7.
$P(n)$ IS A MULTIPLE OF 7, SO
$P(n+1)$ IS A MULTIPLE OF 7.

$P(n) \rightarrow P(n+1)$ by induction.

$P(n)$ is true.

$P(1)$ IS TRUE, SO $P(n)$ TRUE FOR ALL $n \in \mathbb{N}$.