Pearson Edexcel

A Level Mathematics 9MA0

Unit Test

8 Differentiation

Time allowed: 50 **minutes**

School:

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	6	
4	5	
5	5	
6	8	
7	6	
8	2	
9	4	
Total:	50	

1. (a) Given that $f(x) = \sin(x)$, show that [4]

$$f'(x) = \lim_{h\to 0}\left(\left(\frac{\cos(h)-1}{h}\right)\sin(x) + \frac{\sin(h)}{h}\cos(x)\right)$$

(b) Hence prove that $f'(x) = \cos(x)$. [2]

Total: 6

2. A toy soldier is connected to a parachute. The soldier is thrown into the air from ground level. The height, in metres, of the soldier above the ground can be modelled by the equation

$$H = \frac{4t^{\frac{2}{3}}}{t^2+1} \qquad 0 \leq t \leq 6s$$

where H is height of the soldier above the ground and t is the time since the soldier was thrown.

(a) Show that [4]

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8(1-2t^2)}{3\sqrt[3]{t}(t^2+1)^2}$$

(b) Using the differentiated function, explain whether the soldier was increasing or decreasing in height after 2 seconds. [2]

(c) Find the exact time when the soldier reaches a maximum height. [2]

Total: 8

3. A curve has the equation [6]

$$y = \ln(3x) - \mathrm{e}^{-2x}$$

Show that the equation of the tangent at the point with an x-coordinate of 1 is

$$y = \left(\frac{\mathrm{e}^2 + 2}{\mathrm{e}^2}\right) x - \left(\frac{\mathrm{e}^2 + 3}{\mathrm{e}^2}\right) + \ln(3)$$

4. Given that $x = \sec(4y)$, find

   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. [2]

   (b) Show that [3]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x\sqrt{x^2 - 1}}$$

   where k is a constant which should be found.

Total: 5

5. A curve C has equation $4^x = 2xy$ for $x > 0$. [5]

   Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point C with coordinates $(2, 4)$.

6. A curve has parametric equations

$$x = \cos(2t), \quad \text{and} \quad y = \sin(t), \qquad -\pi \leq t \leq \pi.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [3]

Leave your answer as a single trigonometric ratio.

(b) Find an equation of the normal to the curve at the point A where $t = -\frac{5\pi}{6}$. [5]

Total: 8

7. The curve C has equation $y = x^3 + 6x^2 - 12x + 6$.

   (a) Show that C is concave on the interval $[-5, -3]$. [3]

   (b) Find the coordinates of the point of inflection. [3]

Total: 6

8. In a rainforest, the area covered by trees, F, has been measured every year since 1990. [2]

   It was found that the rate of loss of trees is proportional to the remaining area covered by trees.

   Write down a differential equation relating F to t, where t is the numbers of years since 1990.

9. The volume of a sphere V cm^3 is related to its radius r cm by the formula $V = \frac{4}{3}\pi r^3$. [4]

   The surface area of the sphere is also related to the radius by the formula $S = 4\pi r^2$.

   Given that the rate of decrease in surface area, in cm^{2}s^{-1}, is $\frac{\mathrm{d}S}{\mathrm{d}t} = -12$, find the rate of decrease of volume $\frac{\mathrm{d}V}{\mathrm{d}t}$.